INTERFERENTIAL MINIMA IN RESONANCE CROSS SECTIONS

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Abstract

The peculiarities of total neutron cross section energy dependence near to the interference minima with accounting for the Doppler effect are considered. We present the results for one-level and simplified many-level cases. Some of those are in analytical form. The practical possibilities for applications of our results in the analysis of resonance averaged transmissions <exp(-nσ)> and neutron free path ~<1/σ> are discussed. The importance of resonance neutron transmission measurements with thick samples is pointed out.

In the investigations of total neutron cross sections resonance structure usually the most attention is paid to determination of the resonance parameters and in less degree to the analysis of minima between levels that is connected mainly to the interference between the resonance and potential scattering.

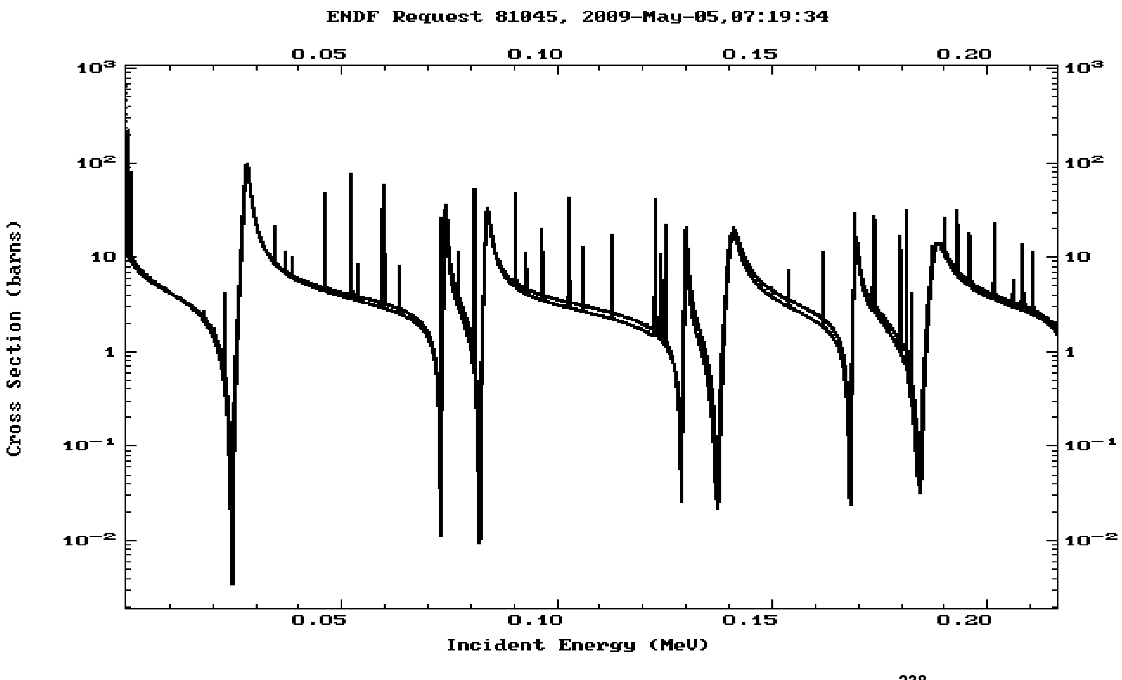

Figure 1: Resonances and interferential minima in the cross section of ^{238}U [1].

In the same time the minimal cross sections are important from fundamental point of view – containing unique information about the phases of p-wave potential scattering. These are much more important for practical problems e.g. for the estimation of free path length in the medium proportional to 1/σ that is maximal in the minima, neutron transmission exp(-nσ) at big thickness and related basic characteristics of neutron transport in the media with resonance cross sections (the Green function of the transport equation is proportional to exp(-nσ)). All this shows the importance of methodical background of the resonance analysis stressing on the possibly correct account for the interference.

Some variants are discussed in this work as an illustration of the character of the problem.

ONE LEVEL PRESENTATION

Let us consider the simplest variant for the shape of resonance cross section one level formula of Breit-Wigner

$$\sigma(x) = \sigma_p + \sigma_0 \frac{\cos 2\varphi + x \sin 2\varphi}{1+x^2} = \sigma_{\min} + \left(\sigma_{\max} - \sigma_{\min}\right) \frac{\left(1 + x tg\varphi\right)^2}{1+x^2} \qquad (1)$$

where σ_p is potential scattering,

$$\sigma_0 = 4\pi k^{-2} g(J) \Gamma_n / \Gamma, \qquad x = \left(E - E_\lambda\right) 2/\Gamma,$$

φ is the phase of potential scattering (s-wave).

The cross sections

$$\sigma_{\min} = \sigma_p - \sigma_0 \sin^2 \varphi$$

$$\sigma_{\max} = \sigma_p + \sigma_0 \cos^2 \varphi \qquad (2)$$

correspond to the minimum of (1) at x=-ctg φ and to maximum at x=tg φ.

Let us note here

$$\sigma_{\max} / \sigma_p = \alpha, \qquad \sigma_{\min} / \sigma_p = \beta,$$

so that

$$tg^2\varphi = \left(1-\beta\right) / \left(\alpha - 1\right).$$

Then the expression for $\sigma(x)/\sigma_p$ can be presented via our parameters as follows

$$\frac{\sigma}{\sigma_p} = \frac{\left(x+\delta\right)^2 + \alpha\beta}{1+x^2}, \qquad \delta^2 = \left(1-\beta\right)\left(\alpha-1\right) \qquad (3),$$

and correspondingly for the inverse function ~ 1/σ we have a similar form

$$\frac{\sigma_p}{\sigma} = \frac{1+x^2}{\left(x+\delta\right)^2 + \alpha\beta} \qquad (4)$$

On the Figures 2 and 3 are presented these functions for the resonances at 28 keV in ^{56}Fe and 192 eV in ^{238}U. For ^{56}Fe resonance the variants for monoisotope and natural iron (92% of ^{56}Fe) are given. It is seen that the resonance shape is changing sharply in dependence of the value of σ_{min} (Figure 2). This effect is well known in applications and leads to considerable results in the evaluation of macroconstans. So, the ratio of effective resonance integral in medium to that of separate nucleus J_R is

$$J_{ef}^{R} / J^{R} = 1/\sqrt{\alpha\beta} \qquad (5)$$

and neglecting the interference this ratio becomes equal to 0.23. With accounting for the interference this value for natural iron rises up to 0.85 and in monoisotope ^{56}Fe reaches 5.2 [2].

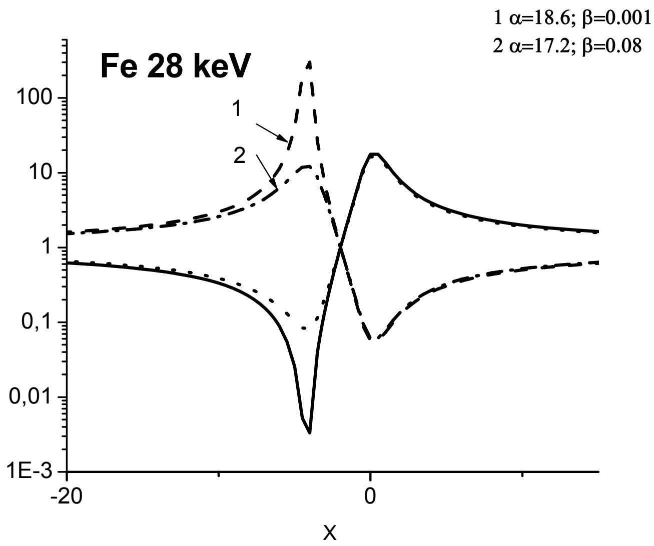

Figure 2: ^{56}Fe resonance at 28 keV – monoisotope α=18.6; β=0.001 and natural iron α=17.1; β=0.08

Next example – the level with E_λ=192 eV in ^{238}U illustrates the interference effect itself as well as the dependence on temperature. Formally the temperature dependence is accounted by averaging of (3) over Maxwell distribution:

$$\left(\frac{\sigma}{\sigma_p}\right)_T = \frac{\xi}{\pi}\int_{-\infty}^{\infty} \frac{(y+\sigma)^2+\alpha\beta}{y^2+1} e^{-\xi^2(x-y)^2} dy =$$

$$= \psi(x)\left[\left(\delta+\chi(x)/\psi(x)\right)^2+\alpha\beta\right]+q(x) \qquad (6)$$

with $q(x) = 1-\left[\psi^2(x)+\chi^2(x)\right]/\psi(x)$, where $\psi(x), \chi(x)$ are usual Doppler functions.

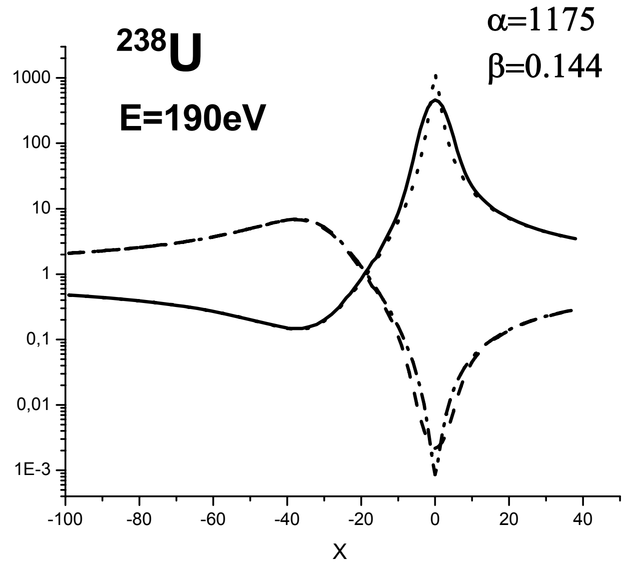

Figure 3: [238]U resonance at 190 eV α=1170; β=0.145; — T=0°K; --- T=300°K (ξ=0.3)

On Figure 3 is presented the shape of this cross section with and without account for the Doppler effect ($\xi = \Gamma/2\Delta = 0.3$). It is seen that the temperature does not affect the cross section value in the interference minimum. The ratio J_{ef}/J_R =0.160 for ξ=0.3, σ_p=10 b, φ=0.023, but neglecting interference J_{ef}/J_R =0.063 [2].

MULTILEVEL CASE

A. Resolved resonances (RRR)

Detailed energy structure of total cross section in wide intervals, where are observed hundreds commonly influencing each other resonances is described rather convincingly by the R-matrix formalism in Reich-Moore approach [1]. So, in the case of one-channel scattering (s-resonances e.g.) the cross section is presented as

$$\sigma(E) = 2\pi k^{-2} g\left[1 - \mathrm{Re}\,U(E)\right] \qquad (7)$$

with $U(E) = e^{-2i\varphi}\dfrac{1+iK(E)}{1-iK(E)}$

and $K(E) = \sum\limits_{\lambda} \dfrac{\Gamma_{\lambda n}/2}{E_\lambda - E - i\Gamma_{\lambda\gamma}/2} = K_1 + iK_2$.

This relation for σ can be transformed to the form similar to (1), although there x should be substituted by

$$\tilde{x}(E) = -K_1(E) / \left[K_2(E) + |K(E)|^2 \right] \qquad (8)$$

and σ_o by

$$\tilde{\sigma}_0 = 4\pi k^{-2} g(J) \left\langle 1 - K_2(E) \Big/ \left[K_2(E) + |K|^2 \right] \right\rangle \cdot$$

Moreover, here the experimental points on energy are the roots of the equations:

$$\tilde{x}(E_{\max}) = tg\varphi; \qquad \tilde{x}(E_{\min}) = -ctg\varphi \cdot$$

This approach has been used before in the applications, where is needed a multilevel presentation of resonance cross sections for low absorption nuclei like Fe, Ni, Cr,…[4]. Here has been assumed K=K_1 and $\tilde{\sigma}_o = \sigma_o$.

From our transformations can be seen that there are not principal complications in the case of one channel scattering with competing multichannel radiation capture. The effect of parameterization scheme (7) in RRR is illustrated by the code SAMMY, that includes in the resonance analysis hundreds level simultaneously (800 s-resonances for ^{238}U). However, here also the new transmission measurements could be rather useful for précising and systematization of cross section data in resonance minima.

B. Unresolved resonances (URR)

The region of unresolved resonances (URR) is dispersed over quite spacious energy interval above observable levels and up to clearly flat energy dependence described by the optical model with complex potential. There is no more possibility for identification of the parameters E_λ and $\Gamma_{\lambda n}$ in K(E) (7), however from the analysis of experimental data are determined only averaged over many resonances parameters $\bar{\Gamma}_n$ and $\bar{D}$ (strength functions). Usually here are observed intermediate energy structures that are not identified physically in the frame of optical model. The Hauser-Feshbach scheme is used here for data analysis and the strength functions are determined not by optical model, but only by the results of parameterization.

Other and probably more important, by our opinion, problem in unresolved region is the disagreement of the results of transmission measurements with the evaluation using the cross section data

$$\left\langle \exp(-\mathrm{n}\sigma) \right\rangle \neq \exp\left[-\mathrm{n} \left\langle \sigma \right\rangle \right],$$

that is evidently a revelation of interference minima. For physical analysis of this difference we are using a variety of models [3,5,6]. Now for this purpose we use the code HARFOR, where the main idea is related with the formation of periodically repeated ladder of N-resonances, that models the energy dependence of function R(E)=K(E)/√E in the scheme (7).

$$R \approx \tilde{R}(\varepsilon) = S_n^0 \frac{1}{N} \sum_{\lambda=1}^{N-1} \xi_\lambda ctg\left[\left(\varepsilon_\lambda - \varepsilon - iS_\gamma \right) \Big/ N \right], \qquad (9)$$

,

where $S_n^0 = \pi \cdot \tilde{\Gamma}_n^0 / 2D$, $\varepsilon_\lambda - \varepsilon = \pi(E_\lambda - E)/D$, $S_\gamma = \pi\,\Gamma_\gamma / 2D$.

The problem of choice of the parameters ξ_λ and ε_λ and the number of levels N is solved here by the criterion of best agreement of the model with the statistical characteristic function

$$X(t,t') = \left\langle \exp\left(iRt - iR^* t'\right)\right\rangle \tag{10}$$

calculated for big sets of N by using the statistics of Porter-Thomas for ξ_λ^2 and that of Wigner for $|\varepsilon_\lambda - \varepsilon_{\lambda-1}|$ [2].

In such model of R-function the cross sections and different functionals of those – transmission, 1/σ, σ_γ/σ are presented as periodical functions of energy and their averages are average over the period 0<ε<N. The results estimated with the code HARFOR have been presented earlier at ISINN [3].

The equal resonances variant (N=1 in (9)) can be considered as a simplest model of resonance structure R(E) in the unresolved region, where

$$\tilde{R}(E) = S_n^0 ctg(\varepsilon - iy) \tag{11}$$

Here for the model resonance cross sections we have simple, by form, expressions as periodical functions of ε, similar to (1)

$$\sigma(E) = \sigma_p + \sigma_0 \frac{\cos 2\varphi + S^{-1} tg\varepsilon \sin 2\varphi}{1 + S^{-2} tg^2 \varepsilon} \tag{12}$$

with

$$S = \left(S_n + S_\gamma\right) \Big/ \left(1 + S_n S_\gamma\right), \quad S_n = S_n^0 \sqrt{E},$$

and

$$\sigma_\gamma(\varepsilon) = \sigma_{\gamma 0} \frac{1 + tg^2\varepsilon}{1 + S^{-2} tg^2 \varepsilon}$$

For these models some energy averaged cross section functionals can be calculated as corresponding averaged over the period (0<ε<1) values. So, <σ> and <σ$_\gamma$> here are the same as in Hauser Feshbach approach (without account for the fluctuations of parameters).

For the self indication cross section we have:

$$\left\langle \sigma_\gamma e^{-n\sigma} \right\rangle = \left\langle \sigma_\gamma \right\rangle e^{-n\sigma_{\min}} e^{-n\sigma_0/2} I_0\left(\frac{n\sigma_0}{2}\right) \tag{13}$$

the transmission for $n\sigma_o$>>1 is looking in the following way:

$$\left\langle e^{-n\sigma}\right\rangle \to \frac{S\sigma_0}{\sigma_{min}+S^2\sigma_{max}}\frac{1}{\sqrt{\pi n\sigma_0}}e^{-n\sigma_{min}}, \qquad (14)$$

where σ_{max} and σ_{min} are maximal and minimal values of $\sigma(\varepsilon)$ (12) ($\sigma_o=\sigma_{max}-\sigma_{min}$). For other funcionals used in practice are obtained also not complicate by form expressions. These analytical results are used in applications with redetermination of the parameters σ_{max} and σ_{min} on the results of more precise evaluations[4]. Though, the obligatory add-up of interferential minima and parameter σ_{min} for different intervals of URR is an essential moment of our models.

CONCLUSION

The data about resonance minima have important scientific and practical outline. In the region of resolved resonances the identification of level positions and cross sections in the minima permits to improve the accuracy of resonance parameters and gives useful information about the phases of s-wave potential scattering at corresponding energies and contribution of p-wave in the cross section as well. The R-matrix formalism in the Rich-Moore approach that is used here corresponds to the problems of resonance analysis near the minima.

In the unresolved resonances region, the choice of unite methodology of cross sections parameterization here meets some difficulties and most important here is the lack of experimental data for the structure characteristics of average cross sections and the search of interferential minima from measurements with relatively thick targets ($n\sigma_{max}$>>1). The cross section values in the minima and their energies are of interest indubitably, that means the intensity and spectra of filtered neutrons [7] should be measured with better accuracy.

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