# THE MEDIUM WITH POLARIZED NUCLEI AND EFFECTS OF LOW-ENERGY NEUTRON REFRACTION AND REFLECTION

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**Abstract**

The effects of refraction and reflection of primarily unpolarized low-energy neutrons in the medium with polarized nuclei have been considered. In such a medium, the neutron beam is characterized by two refraction indices, which correspond to the neutron spin projections onto the nucleus polarization vector, equaling 1/2 and –1/2 . It is shown that, taking into account this fact, at nonzero angle of incidence the beam of unpolarized neutrons is spatially separated into two beams of totally polarized neutrons. The angle between the beams increases when the angle of incidence $\theta_0$ approaches $\pi/2$ ( the angle of sliding $\pi/2 - \theta_0$ approaches zero) , as well as with the decrease of neutron energy. Besides, the polarization of reflected neutrons in the conditions of total internal reflection for one of the refraction indices is discussed .

Numerical estimates have been performed for the case of totally polarized liquid-hydrogen target. In particular, at the angle of incidence $\theta_0 = 85°$ and neutron energy $10^{-4}$ eV the angle between two beams of refracted polarized neutrons has the magnitude of the order of 1.5 ° .

## 1. Refraction index

The relation between the modulus of momentum $\hbar\chi$ of a particle with energy $E$, propagating through a macroscopic medium, and the modulus of its momentum in vacuum is as follows :

$$\chi = k\, n(k)\,, \tag{1}$$

where $n(k)$ is the refraction index and $k = \frac{1}{\hbar c}\sqrt{E^2 - (mc)^2}$ is the modulus of wave vector .

At $|n - 1| \ll 1$ we have a universal formula for the refraction index of a coherent wave in the medium :

$$n(k) = 1 + \frac{2\pi N\, a(k,0)}{k^2} \quad . \tag{2}$$

Here $N$ is the number of nuclei of the medium in the unit volume ; $a\,(k,\,0)$ is the “coherent” amplitude of scattering of the particle under consideration on the nuclei of the medium at zero angle in the laboratory frame – amplitude of “forward” scattering with quantum numbers remaining unchanged .

Under the condition $\dfrac{2\pi\, N\,|\,a(k{,}0)\,|}{k^2} << 1$ , the formula for the refraction index is valid at both low and high energies – since, irrespective of the wavelength $\lambda = {}^{2\pi}\!/_{k}$ , the “coherent” wave in the medium is formed due to the interference of the incident wave with secondary waves, scattered by all the nuclei of the medium in the “forward” direction without momentum transfer [1,2] .

## 2. Relations of neutron optics

In the case of low-energy neutrons, when in the c.m. frame of the neutron and nucleus the *s*-wave scattering takes place, the refraction index is determined by the expression :

$$n^2(k) = 1 + \frac{4\pi\, N\, a(k,0)}{k^2} \quad , \tag{3}$$

where $a\,(k,\,0)$ is the coherent amplitude of zero-angle scattering, specified in the rest frame of the nuclei of medium and connected with the angle-independent amplitude of elastic scattering in the c.m. frame of the neutron and nucleus by the following relation :

$$a(k,0) = \frac{m+M}{M}\,\tilde{a}(\tilde{k}) \quad . \tag{4}$$

Here $m$ and $M$ are masses of the neutron and nucleus, respectively,

$$\hbar\tilde{k} = \frac{M}{M+m}\;\hbar k \tag{5}$$

is the modulus of neutron momentum in the c.m. frame .

Formula (3) for slow neutrons is valid at $|\,n\,(k) - 1\,| \geq 1$ as well . In case if $|\,n\,(k) - 1\,| << 1$, Eq. (3) turns into the universal formula (2) .

Just as in the optics of electromagnetic waves, in the given case the following regularities are satisfied :

a) the angle of incidence $\theta_0$ is equal to the reflection angle $\theta'$ ;

b) at the incidence of the beam from vacuum, the angle of refraction $\theta$ is connected with the angle of incidence by the Descartes-Snell formula :

$$\sin\theta = \frac{\sin\theta_0}{n(k)} , \tag{6}$$

where $\theta_0$ , $\theta'$ , $\theta$ are the angles between the respective wave vectors and the normal to the plane of medium–vacuum separation ;

c) the components of wave vectors, aligned in the parallel and perpendicular directions to the plane of medium–vacuum separation, are as follows :

$$\chi_{||} = k'_{||} = k_{||} = k\sin\theta_0 ;$$

$$\chi_{\perp} = k\,n(k)\cos\theta = k\sqrt{n^2(k) - \sin^2\theta_0} ; \quad k' = -k_{\perp} = -k\cos\theta_0 . \tag{7}$$

The amplitudes of the refracted and reflected waves are determined by the formulas :

$$D = \frac{2\cos\theta_0}{\sqrt{n^2(k) - \sin^2\theta_0} + \cos\theta_0} , \tag{8}$$

$$R = D - 1 = -\frac{\sqrt{n^2(k) - \sin^2\theta_0} - \cos\theta_0}{\sqrt{n^2(k) - \sin^2\theta_0} + \cos\theta_0} ; \tag{9}$$

the reflection coefficient is equal to $|R|^2$ .

The coherent scattering amplitude has a positive imaginary part ; this leads to the attenuation of the refracted wave, connected with the fact that neutrons leave the beam on account of absorption and elastic scattering, as well as to some uncertainty of the refraction angle at the fixed angle of incidence, which will further be neglected .

On account of a small magnitude of the imaginary part of low-energy neutron scattering amplitude, in most cases the amplitude $a\,(k, 0)$ , the wave vectors and the refraction index may be considered as real quantities .

At small angles of sliding $\varphi_0 = \frac{\pi}{2} - \theta_0 << 1$ we obtain :

$$R = \frac{\varphi_0 - \sqrt{\varphi_0^{\,2} + \frac{4\pi\, N\, a(k,0)}{k^2}}}{\varphi_0 + \sqrt{\varphi_0^{\,2} + \frac{4\pi\, N\, a(k,0)}{k^2}}} \quad . \tag{10}$$

If the amplitude $a\,(k, 0) < 0$, then at the sliding angles $\varphi_0 < \sqrt{\frac{4\pi\, N\, |a(k,0)|}{k^2}}$ the radicand becomes negative . Then $|\,R\,|^2 = 1$, i.e. the total internal reflection of neutrons takes place .

**3. The medium with polarized nuclei**

In the medium with polarized nuclei, the propagation of neutrons is characterized by two refraction indices, corresponding to the orientations of neutron spin along and against the direction of the vector of polarization of nuclei .

In case of the nonzero spin of a nucleus, the spin structure of the coherent *s*-wave neutron scattering amplitude in the nucleus rest frame has the following form :

$$\hat{a}(k,0) = b(k)\,(\,\hat{I}^{(1)} \otimes \hat{I}^{(2)}) + \frac{d(k)}{j}\,(\,\hat{\boldsymbol{\sigma}} \otimes \hat{\mathbf{j}}\,) \quad , \tag{11}$$

where $\hat{I}^{(1)}$ and $\hat{I}^{(2)}$ are the two-row and $(\,2\,j + 1\,)$ - row unit matrices , $j$ is the nucleus spin, $\hat{\boldsymbol{\sigma}}$ is the doubled operator of neutron spin ( vector Pauli operator ) , $\hat{\mathbf{j}}$ is the operator of nucleus spin .

In accordance with (4), the coefficients $b\,(k)$ and $d\,(k)$ in Eq. (11) are connected with the respective coefficients $\tilde{b}(\tilde{k})$ and $\tilde{d}(\tilde{k})$ in the analogous expression for the scattering amplitude in the c.m. frame of the neutron and nucleus by the relations :

$$b\,(k) = K\,\tilde{b}(\tilde{k}) \;\; , \;\; d\,(k) = K\,\tilde{d}(\tilde{k}) \quad , \tag{12}$$

where the quantity

$$K = \frac{m + M}{M} \qquad (13)$$

has the meaning of coherence factor .

The coherent amplitudes are eigenstates of the two-row matrix :

$$\hat{a}(k) = b(k)\,\hat{I}^{(1)} + d(k)\,(\hat{\boldsymbol{\sigma}}\,\mathbf{P}) \quad , \qquad (14)$$

where $\mathbf{P} = \frac{\langle \hat{\mathbf{j}} \rangle}{j}$ is the nucleus polarization vector ( the averaging is performed over the spin state of nuclei in the medium ) .

For unpolarized nuclei ( $\mathbf{P} = 0$ ) we have : $a\,(k) = b\,(k)$ .

If the projection of neutron spin onto the polarization vector **P** equals (+ 1/2 ), then the coherent amplitude is

$$a_{+}(k) = b(k) + d(k)\,|\,\mathbf{P}\,| \quad , \qquad (15)$$

and the refraction index amounts to :

$$n_{+}(k) = 1 + \frac{2\pi\, N\,(b(k) + d(k)\,|\,\mathbf{P}\,|)}{k^2} \quad . \qquad (16)$$

Meantime, if the projection of neutron spin onto the polarization vector **P** equals ( – 1/2 ), then the coherent amplitude is

$$a_{-}(k) = b(k) - d(k)\,|\,\mathbf{P}\,| \quad , \qquad (17)$$

and the respective refraction index amounts to :

$$n_{-}(k) = 1 + \frac{2\pi\, N\,(b(k) - d(k)\,|\,\mathbf{P}\,|)}{k^2} \quad . \qquad (18)$$

Taking into account that the coherent amplitudes for low-energy neutrons incorporate small additional imaginary terms , the refraction indices $n_{+}$ and $n_{-}$ are, strictly speaking, complex – and, in accordance with this, the neutron beams with spin projections $+1/2$ and $-1/2$ onto the polarization vector of nuclei $\mathbf{P}$ are characterized by different coefficients of attenuation .

## 4. Baryshevsky – Podgoretsky effect

Now let us consider the Baryshevsky – Podgoretsky effect [3], i.e. the precession of neutron spin in the medium with polarized nuclei .

Let the spin of a neutron entering the medium with polarized nuclei be aligned along the axis $x$ , being perpendicular to the nucleus polarization vector $\mathbf{P}$ directed along the axis $z$ . Then the spin state of the neutron upon entering the medium is :

$$\left|+\frac{1}{2}\right\rangle_x = \frac{1}{\sqrt{2}}\left(\left|+\frac{1}{2}\right\rangle_z + \left|-\frac{1}{2}\right\rangle_z\right) \quad , \tag{19}$$

and the beams corresponding to the states $\left|+\frac{1}{2}\right\rangle_z$ and $\left|-\frac{1}{2}\right\rangle_z$ – if they go together, being not separated spatially – propagate with different refraction indices and acquire different phases. As a result, at a finite distance $l$ along the beams, the state with spin aligned against the axis $x$ emerges :

$$\left|-\frac{1}{2}\right\rangle_x = \frac{1}{\sqrt{2}}\left(\left|+\frac{1}{2}\right\rangle_z - \left|-\frac{1}{2}\right\rangle_z\right) \quad . \tag{20}$$

In doing so, at the distance $l$ we have :

$$\left|+\frac{1}{2}\right\rangle_x \to \frac{1}{\sqrt{2}}\left(\left|+\frac{1}{2}\right\rangle_z \exp(ik\, n_+(k)\, l) + \left|-\frac{1}{2}\right\rangle_z \exp(ik\, n_-(k)\, l)\right) =$$

$$= \left[\cos\left(k\,\frac{n_+(k)-n_-(k)}{2}\,l\right)\left|+\frac{1}{2}\right\rangle_x + i\sin\left(k\,\frac{n_+(k)-n_-(k)}{2}\,l\right)\left|-\frac{1}{2}\right\rangle_x\right] \times \tag{21}$$

$$\times\ \exp\left(i\,k\,\frac{n_+(k)+n_-(k)}{2}\,l\right)$$

The angle of rotation of the plane of neutron polarization at the length $l$ amounts to

$$\Omega = k\,[\,n_+(k) - n_-(k)]\,l = 2\pi\,N\,\frac{a_+(k) - a_-(k)}{k}\,l = 4\pi\,N\,\frac{d(k)\,|\,\mathbf{P}\,|}{k}\,l \quad . \qquad (22)$$

Let us introduce the time of neutron passage over the distance $l$ : $l = \frac{\hbar\,k}{m_n}\,t$ ,
where $m_n$ is the neutron mass . Then $\Omega = \omega\,t$ , where

$$\omega = \frac{4\pi\,N\,\hbar\,d(k)\,|\,\mathbf{P}\,|}{m_n} \qquad (23)$$

is the frequency of neutron spin precession .

One may introduce an effective pseudomagnetic field, in which the neutron spin – due to the interaction of this field with the neutron magnetic momentum – would undergo the precession with the same frequency as in the medium with polarized nuclei . This pseudomagnetic field is determined from the condition :

$$-\frac{2\,\mu_n\,\mathbf{H}_{eff}}{\hbar} = \frac{4\pi\,N\,d(k)\,\mathbf{P}}{m_n} \quad , \qquad (24)$$

where $\mu_n = \frac{e\hbar g}{2m_n c}$ is the neutron magnetic momentum ( $\frac{e\hbar}{2m_n c}$ is the nuclear magneton , $g = -1.91$ is the hyromagnetic multiplier ) .

So, we find :

$$\mathbf{H}_{eff} = \frac{4\pi\,\hbar c}{e\,|\,g\,|}\,N\,d(k)\,\mathbf{P} \quad . \qquad (25)$$

Taking into account the numerical values
( $e = 4.8\cdot 10^{-10}$ CGSE units , $\hbar = 1.05\cdot 10^{-27}$ erg $\cdot$ sec, $c = 3\cdot 10^{10}$ cm / sec ) ,
we obtain:

$$\mathbf{H}_{eff} = 4.31\cdot 10^{-7}\,N\,d(k)\,\mathbf{P} \quad .$$

The Baryshevsky–Podgoretsky effect arises under the condition that the neutron beams with different refraction indices are not separated spatially . This situation holds, strictly speaking, only in case of normal incidence of neutrons to the plane of medium–vacuum separation ; but at nonzero angles of incidence such a joint passage does not take place, in general .

## 5. Separation of a beam of unpolarized neutrons into totally polarized beams upon refraction .

Let a beam of unpolarized neutrons fall onto the plane vacuum–medium boundary at the angle $\theta_0 = \pi / 2 - \varphi_0$ . As it is known, the beam of unpolarized neutrons represents an incoherent mixture of two neutron beams with spin projections ( + 1 /2 ) and ( – 1 /2 ) onto an arbitrary direction, which have equal intensities. In the medium, neutrons with the spin projection ( + 1/2 ) onto the direction of nuclear polarization vector $\mathbf{l} = \mathbf{P} / |\mathbf{P}|$ deviate from the normal by the angle $\theta_+$, which is determined from the Descartes–Snell law :

$$\sin\theta_+ = \frac{\sin\theta_0}{n_+(k)} \quad .$$

Taking into account that $n_+(k) = 1 + \frac{2\pi N\, a_+(k)}{k^2}$ and $\frac{2\pi N\, |a_+(k)|}{k^2} << 1$ ,

we find :

$$\sin\theta_+ - \sin\theta_0 = -\frac{2\pi N\, a_+(k)}{k^2} \sin\theta_0 \ . \tag{26}$$

Meantime, neutrons with the spin projection ( –1/2 ) onto the direction $\mathbf{l} = \mathbf{P} / |\mathbf{P}|$ deviate by the angle $\theta_-$, for which :

$$\sin\theta_- = \frac{\sin\theta_0}{n_-(k)} \ , \quad \sin\theta_- - \sin\theta_0 = -\frac{2\pi N\, a_-(k)}{k^2} \sin\theta_0 \quad . \tag{27}$$

Thus, $\sin\theta_+ - \sin\theta_- = \sin\theta_0 \frac{2\pi N}{k^2} (a_-(k) - a_+(k))$ , or

$$\sin\theta_+ - \sin\theta_- = -\sin\theta_0 \frac{4\pi N\, d(k)\, |\mathbf{P}|}{k^2} = -\sin\theta_0 \frac{2\pi \hbar^2 N\, d(k)\, |\mathbf{P}|}{m_n\, E_n} \ , \tag{28}$$

where $m_n$ is the neutron mass and $E_n = \frac{\hbar^2 k^2}{2\, m_n}$ is the laboratory neutron energy .

Under the condition $\mathrm{tg}\,\theta_0\, |\theta_+ - \theta_-| << 1$ , we obtain :

$$\theta_- - \theta_+ = \mathrm{tg}\,\theta_0 \frac{4\pi\, N\, d(k)\,|\mathbf{P}|}{k^2} \quad . \tag{29}$$

For the sliding angles $\varphi_+ = \pi/2 - \theta_+$ and $\varphi_- = \pi/2 - \theta_-$ , this formula gives :

$$\varphi_+ - \varphi_- = \mathrm{ctg}\,\varphi_0 \frac{4\pi\, N\, d(k)\,|\mathbf{P}|}{k^2} \quad . \tag{30}$$

We see that the angle between two totally polarized beams increases with decreasing energy and with increasing the angle of incidence .

At the angles of incidence being close to $\pi/2$ , i.e. at small angles of sliding $\varphi_0 = \pi/2 - \theta_0 \ll 1$, $\varphi_+ = \pi/2 - \theta_+ \ll 1$, $\varphi_- = \pi/2 - \theta_- \ll 1$ , we have :

$$\sin\theta_0 = \cos\varphi_0 \approx 1 - \frac{\varphi_0{}^2}{2},$$

$$\sin\theta_+ = \cos\varphi_+ \approx 1 - \frac{\varphi_+{}^2}{2}\ ,\ \sin\theta_- = \cos\varphi_- \approx 1 - \frac{\varphi_-{}^2}{2} \quad .$$

Then

$$\varphi_+ = \sqrt{\varphi_0{}^2 + \frac{4\pi\, N\, a_+(k)}{k^2}} \quad , \quad \varphi_- = \sqrt{\varphi_0{}^2 + \frac{4\pi\, N\, a_-(k)}{k^2}} \quad . \tag{31}$$

If $\varphi_0{}^2 \gg \frac{4\pi\, N\,|a_+(k)|}{k^2}$ , $\varphi_0{}^2 \gg \frac{4\pi\, N\,|a_-(k)|}{k^2}$ , then we obtain:

$$\varphi_+ = \varphi_0 + \frac{2\pi\, N\, a_+(k)}{\varphi_0\, k^2} \quad , \quad \varphi_- = \varphi_0 + \frac{2\pi\, N\, a_-(k)}{\varphi_0\, k^2} \ ,$$

$$\varphi_+ - \varphi_- = \frac{2\pi\, N}{\varphi_0\, k^2}\,(a_+(k) - a_-(k)\,) = \frac{4\pi\, N\, d(k)\,|\mathbf{P}|}{\varphi_0\, k^2} \quad . \tag{32}$$

This approximation corresponds to the condition : $|\varphi_+ - \varphi_-| \ll \varphi_0$ , which follows from the former condition $\mathrm{tg}\,\theta_0\,|\theta_+ - \theta_-| \ll 1$ at $\theta_0 \approx \pi/2$, $\theta_+ \approx \pi/2$, $\theta_- \approx \pi/2$ .

Thus, we have two beams of totally polarized neutrons with the intensities $|D_+|^2 k n_+$ and $|D_-|^2 k n_-$, where $D_+$ and $D_-$ are the amplitudes of refracted waves for the refraction indices $n_+$ and $n_-$, respectively, which are determined according to Eq. (8). The angle between these beams is equal to

$$|\Delta\theta| = |\Delta\varphi| = |\varphi_+ - \varphi_-| \quad .$$

Since the refraction indices are very close to unity, the component of wave vector difference $\Delta\mathbf{k}$, being perpendicular to the wave vector of the incident beam $\mathbf{k}$, amounts to $|\Delta\mathbf{k}| = k|\varphi_+ - \varphi_-|$. The beams can be separated, if the uncertainty of wave vectors, connected with the spatial width $d$ of the beam, is much smaller than the wave vector difference $|\Delta\mathbf{k}|$. This leads to the condition :

$$k|\varphi_+ - \varphi_-| >> \frac{1}{d} \quad . \tag{33}$$

If this condition is satisfied, the beams are spatially separated on the base, where $l|\varphi_+ - \varphi_-| > d$ .

## 6. Reflection of neutrons from the boundary between the medium with polarized nuclei and vacuum .

The reflected neutrons with the refraction indices $n_+$ and $n_-$ move in the same direction, but they are characterized by different probabilities of reflection . The vector of polarization of the reflected beam is as follows :

$$\zeta = \frac{|R_+|^2 - |R_-|^2}{|R_+|^2 + |R_-|^2}\,\mathbf{l} \quad , \tag{34}$$

where $\mathbf{l} = \frac{\mathbf{P}}{|\mathbf{P}|}$ . If $\cos^2\theta_0 >> \frac{4\pi N|a_+(k)|}{k^2}$ , $\cos^2\theta_0 >> \frac{4\pi N|a_-(k)|}{k^2}$ ,

we obtain :

$$|R_+|^2 = \frac{\pi^2 N^2 |a_+(k)|^2}{k^4\cos^4\theta_0} \quad , \qquad |R_-|^2 = \frac{\pi^2 N^2 |a_-(k)|^2}{k^4\cos^4\theta_0} \quad , \tag{35}$$

$$\zeta = \frac{|a_+(k)|^2 - |a_-(k)|^2}{|a_+(k)|^2 + |a_-(k)|^2}\,\mathbf{l} = \frac{2\,b(k)\,d(k)}{|b(k)|^2 + |d(k)|^2|\mathbf{P}|^2}\,\mathbf{P} \quad . \tag{36}$$

Let us emphasize that, in the conditions under consideration, at small sliding angles $\cos^4\theta_0 \approx \varphi_0{}^4$ , and

$$|R_+|^2 = \frac{\pi^2 N^2 |a_+(k)|^2}{k^4 \varphi_0{}^4} \quad , \quad |R_-|^2 = \frac{\pi^2 N^2 |a_-(k)|^2}{k^4 \varphi_0{}^4} \quad .$$

Let us suppose now that the sliding angle is very small and the coherent amplitude $a_+(k) = b(k) + d(k)|\mathbf{P}|$ is negative . If, in doing so,

$$\varphi_0 < \sqrt{\frac{4\pi N |a_+(k)|}{k^2}} \quad ,$$

then for neutrons with the spin projection onto the vector **P** , equaling (+ 1/2 ) , the total reflection takes place and, in accordance with this, $|R_+|^2 = 1$ . Meantime, let the coherent amplitude $a_-(k)$ for neutrons with the spin projection (– 1/2 ) onto the vector **P** be positive. Then, assuming that $\varphi_0 \ll 1$, we may write for the reflection probability :

$$|R_-|^2 = \left( \frac{\varphi_0 - \sqrt{\varphi_0{}^2 + \frac{4\pi N a_-(k)}{k^2}}}{\varphi_0 + \sqrt{\varphi_0{}^2 + \frac{4\pi N a_-(k)}{k^2}}} \right)^2 \quad . \qquad (37)$$

In doing so, the polarization vector of reflected neutrons is as follows :

$$\zeta = \frac{1 - |R_-|^2}{1 + |R_-|^2}\, \mathbf{l} \quad . \qquad (38)$$

**7. The liquid-hydrogen medium with polarized protons .**

In order to perform numerical estimates, let us consider the incidence of unpolarized neutrons from vacuum onto the totally polarized liquid-hydrogen medium,. In the case under consideration, $N = 7 \cdot 10^{22}$ cm$^{-3}$ . The parameters of low-energy *np*- scattering in the c.m. frame take the following values :

$$\tilde{b} = 1.89 \text{ fm}, \quad \tilde{d} = -7.27 \text{ fm} \quad .$$

In doing so, the coherence factor (13) is $K \approx 2$ , and in accordance with this

$$a_+ = 2(\tilde{b} + \tilde{d}) = -10.76 \text{ fm}, \quad a_- = 2(\tilde{b} - \tilde{d}) = 18.32 \text{ fm}.$$

At the neutron energy $E_n = 10^{-4}$ eV = $1.6 \cdot 10^{-16}$ erg and the angle of incidence $\theta_0 = 85\,°$, calculations give the following value for the angle of divergence of the totally polarized neutron beams :

$$|\Delta\theta| = 2.846 \cdot 10^{-2} \text{ rad} = 1.63\,° .$$

In doing so, the component of the wave vector difference for the refracted beams , being perpendicular to the primary wave vector, is equal to $|\Delta k_\perp| = k\,|\Delta\theta| = 6.43 \cdot 10^5 \text{ cm}^{-1}$ , which is surely much larger than the uncertainty connected with the spatial width of the beams . Thus, the polarized beams of refracted neutrons can be separated .

In these conditions the reflection coefficients take the values :

$$|R_+|^2 \approx 3.84 \cdot 10^{-3} , \quad |R_-|^2 \approx 1.01 \cdot 10^{-2} ,$$

and the polarization vector for reflected neutrons is as follows :

$$\zeta = \frac{|R_+|^2 - |R_-|^2}{|R_+|^2 + |R_-|^2}\mathbf{P} = -0.485\,\mathbf{P} \quad , \quad |\mathbf{P}| = 1 \quad .$$

In the same totally polarized liquid-hydrogen medium, the critical sliding angle for neutrons with the spin projection onto the medium polarization vector, equaling ( + 1/2 ), amounts to ( $a_+ = -10.76$ fm $< 0$ ) :

$$\varphi_{cr}^{+} = 4.2 \cdot 10^{-2} \text{ rad} = 2.4\,°$$

( the critical angle of incidence is $\theta_0^{(cr)} = 87.6\,°$ ) .

At $\varphi < \varphi_{cr}^{+}$ ( for example, at $\varphi = 1.5\,°$ ) , the total reflection of neutrons with the spin projection ( + 1/2 ) onto **P** takes place . Meantime, at $\varphi = 1.5\,°$ the reflection coefficient for neutrons with the spin projection ( – 1/2 ) onto **P** equals ( $a_- = 18.32$ fm $> 0$ ) :

$$|R_-|^2 = 0.166 .$$

Thus, the resulting polarization of reflected neutrons is :

$$\zeta = \frac{1 - 0.166}{1 + 0.166}\,\mathbf{P} = 0.715\,\mathbf{P} \quad .$$

## 8. Summary

1. The study of effects of low-energy neutron refraction and reflection in a medium with polarized nuclei has been performed . In such a medium, the neutron beam is characterized by two refraction indices, corresponding to the neutron spin projections (+ 1/2) and (–1/2) onto the nucleus polarization vector .

2. The existence of two refraction indices leads to the precession of the neutron spin in the medium with polarized nuclei ( under the condition that neutrons with different spin projections are not separated spatially ) .

3. It is found that, at nonzero angles of incidence onto the boundary between the polarized medium and vacuum, the beam of unpolarized neutrons is spatially separated, in general, into two refracted beams of totally polarized neutrons with opposite polarizations. The angle between the beams is proportional to the spin-dependent part of the amplitude of neutron elastic scattering on the nucleus, and it increases when the angle of incidence approaches $\pi/2$ , as well as with the decrease of neutron energy .

4. The polarization of reflected neutrons is discussed .

5. Numerical estimates have been made for the case of totally polarized liquid-hydrogen target .

## References

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