

# TESTING SIMPLE MODELS OF E1 RADIATIVE STRENGTH FUNCTIONS

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## Astract

Different Lorentzian-type models of E1 radiative strength functions (RSF) were tested. New ready-to-use table of giant dipole resonance (GDR) parameters and their uncertainties were obtained from fitting the theoretical calculations of photoabsorption cross sections to the experimental data. Renew systematics for GDR parameters are given. It is demonstrated that closed-form approaches with asymmetric shape of the RSF provide the most reliable simple method for estimation of dipole RSF of  $\gamma$ -decay.

## 1 Introduction

Gamma-emission is one of the most universal channels of the nuclear de-excitation which accompany any nuclear reaction. The photoabsorption and  $\gamma$ -decay processes can be described by means of radiative strength functions (RSF) ([1]-[3]). They are involved in calculations of the observed characteristics of most nuclear reactions. Thus, RSF are of considerable importance for investigation of nuclear structure (nuclear deformations, energies and widths of the giant dipole resonances, contribution of velocity-dependent force, shape-transitions, etc) and mechanisms of decay processes. However, microscopic calculations of the RSF, as a rule, are time-consuming. Therefore, simple closed-form expressions are often preferable for their evaluation.

In this contribution we present overview and testing of the simple practical methods for the calculation of E1 radiative strength function for photoabsorption and  $\gamma$ -decay. Dipole electric gamma-transitions are dominant when they occur simultaneously with transitions of other multipolarities. Therefore, we focus here on the dipole RSF. Theoretical calculations of photoabsorption cross sections and gamma-decay strength functions performed within the Lorentzian-type RSF models are compared with experimental data to test proposed closed-form expressions. However, for simplified RSF calculations it is important to have reliable set of giant dipole resonance (GDR) parameters. The new ready-to-use table of GDR parameters and their errors were obtained from fitting the theoretical calculations for photoabsorption cross sections  $\sigma(\gamma, abs)$  to the experimental data. Strength  $\gamma$ -decay functions for the middle-weight and heavy atomic nuclei obtained by the use of new parameters were compared with corresponding experimental data. Complex analysis of obtained results allowed to make conclusions about reliability of proposed methods for RSF calculations and renewed values of GDR parameters.

## 2 Main features of the tested RSF models

Phenomenological models assume the dipole RSF to have a Lorentzian-like shape with different expressions for the “width”  $\Gamma_\gamma(E_\gamma)$  of the curve [2, 3]. The Standard Lorentzian model (SLO [4, 5]) is based on the Brink hypothesis. The dipole RSF in the SLO model is Lorentzian with an energy-independent width  $\Gamma_\gamma(E_\gamma)$  equal to the GDR width  $\Gamma_r$ :

$$\overleftarrow{f}(E_\gamma) = \overrightarrow{f}(E_\gamma) \equiv f_{SLO}(E_\gamma) = 8.674 \cdot 10^{-8} \sigma_r \Gamma_r \frac{E_\gamma \Gamma_r}{(E_\gamma^2 - E_r^2)^2 + [\Gamma_r \cdot E_\gamma]^2}, \quad MeV^{-3}, \quad (1)$$

where  $\sigma_r$  and  $E_r$  are the peak cross section (in  $mb$ ) and the GDR energy (in  $MeV$ ), respectively.

The SLO approach is probably the most appropriate method for describing photo-absorption data for medium-weight and heavy nuclei [6, 8]. However, the SLO model for  $\gamma$ - emission significantly underestimates the  $\gamma$ -decay spectra at low energies [9].

The Enhanced Generalized Lorentzian (EGLO) [10, 11] and Generalized Fermi-Liquid (GFL) models [12] are based in low energy range on Kadenskij-Markushev-Furman [13] and give more correct description of the  $E1$  strengths at energies  $E_\gamma$  close to zero. For spherical nuclei the EGLO RSF is given by the following expression:

$$\overleftarrow{f}(E_\gamma) \equiv \overleftarrow{f}_{EGLO}(E_\gamma) = 8.674 \cdot 10^{-8} \cdot \sigma_r \Gamma_r \left[ \frac{E_\gamma \Gamma_K(E_\gamma)}{(E_\gamma^2 - E_r^2)^2 + E_\gamma^2 \Gamma_K^2(E_\gamma)} + \frac{0.7 \Gamma_K(E_\gamma = 0)}{E_r^3} \right] \quad (2)$$

with width equal to

$$\Gamma_k(E_\gamma) = \Gamma_r \frac{E_\gamma^2 + 4\pi T_f^2}{E_r^2} K(E_\gamma),$$

where empirical factor  $K(E_\gamma)$  is obtained from fitting to experimental data.

The dipole  $\gamma$ -decay RSF within the GFL model has the following form [12, 14]:

$$\overleftarrow{f}(E_\gamma) \equiv \overleftarrow{f}_{GFL}(E_\gamma) = 8.674 \cdot 10^{-8} \cdot \sigma_r \Gamma_r \frac{K \cdot E_r \cdot \Gamma_m(E_\gamma)}{(E_\gamma^2 - E_r^2)^2 + K [\Gamma_m(E_\gamma) E_\gamma]^2}, \quad (3)$$

The quantity  $K$  is determined by the Landau parameters of the quasi-particle interaction in the isovector channel of the Fermi system. The energy-dependent width  $\Gamma_m(E_\gamma)$  is taken to be a sum of a collisional damping width  $\Gamma_{coll}$  and the additional term  $\Gamma_{dq}$ :

$$\Gamma_m(E_\gamma) = \Gamma_{coll}(E_\gamma) + \Gamma_{dq}(E_\gamma) \quad . \quad (4)$$

The collisional component is taken as:

$$\Gamma_{coll}(E_\gamma) \equiv C_{coll} (E_\gamma^2 + 4\pi^2 T_f^2) \quad , \quad (5)$$

with  $C_{coll}$  determined by normalizing the total width (4) at  $E_\gamma = E_r$  and  $T_f = 0$  to the GDR width of a cold nucleus, i.e.  $\Gamma_m(E_\gamma = E_r) = \Gamma_r$ . The component  $\Gamma_{dq}$  is taken in the following form [15]:

$$\Gamma_{dq}(E_\gamma) = C_{dq} \sqrt{E_\gamma^2 \bar{\beta}_2^2 + E_\gamma s_2}, \quad (6)$$

where  $C_{dq} = \sqrt{5 \ln 2 / \pi} = 1.05$ ;  $s_2 = E_{2+} \bar{\beta}_2^2 \approx 217.16/A^2$  with  $E_{2+}$  being the energy of the first vibrational quadrupole state, and  $\bar{\beta}_2$  is the effective deformation parameter characterizing the nuclear stiffness with respect to surface vibrations.

It can be noted that the SLO, EGLO and GFL expressions for the  $\gamma$ -decay strength function of heated nuclei are not consistent with the general relations between a RSF and the imaginary part of the response function [16]. To avoid this shortcoming, at least approximately, the Modified Lorentzian approach (MLO) was proposed [17, 18]. Shape of MLO RSF results from a semi-classical approach based on the Landau-Vlasov equation with a non-Markovian collision term [19].

The  $\gamma$ -decay RSF within the MLO model has the following form:

$$\overleftarrow{f}(E_\gamma) = \overleftarrow{f}_{MLO}(E_\gamma) = 8.674 \cdot 10^{-8} \sigma_r \Gamma_r \frac{E_\gamma}{1 - \exp(E_\gamma/T_f)} \frac{\Gamma_\gamma(E_\gamma)}{(E_\gamma^2 - E_r^2)^2 + [\Gamma_\gamma(E_\gamma) \cdot E_\gamma]^2}, \quad (7)$$

where  $\Gamma_r = \Gamma_\gamma(E_\gamma = E_r)$  at zero excitation energy; the width  $\Gamma_\gamma(E_\gamma)$  depends on the assumptions on the damping mechanism for the collective states.

Different semi-empirical expressions for the width were used in the MLO approach (MLO1, MLO2, MLO3) [3], but, as a rule, corresponding RSF are in rather close agreement. We also test the RSF description for modified Lorentzian model given by Eq.(7) with the simplified expression for  $\Gamma_\gamma(E_\gamma)$  ([20]-[22]). This model is denoted as the Simplified Modified Lorentzian (SMLO) model and it closely agrees with MLO1-model.

### 3 Calculations and discussions

For the calculation of the radiative strength functions within simplified models it is necessary to determine GDR parameters. We obtained new ready-to-use table of GDR parameters and their uncertainties for the SLO, MLO1(SMLO) models by fitting the theoretical calculations for photoabsorption cross sections  $\sigma(\gamma, abs)$  to the experimental data from the EXFOR library (<http://www-nds.iaea.org/exfor/>). If experimental or evaluated data for some nuclei were absent in database, total photoabsorption cross section was approximated by sum of partial cross-sections of total photoneutron emission  $\sigma(\gamma, sn)$ , photoproton emission  $\sigma(\gamma, p)$  and photofission  $\sigma(\gamma, F)$  ([20]-[22]). New values of GDR parameters and their errors were used to obtain renewed systematics in forms (8) and (9), which can be used for more reliable description of gamma-decay and average GDR properties. For the SLO model the following systematics were obtained:

$$\bar{E}_r = 27.469/A^{1/3} + 22.063/A^{1/6} (MeV), \Gamma_r = 0.027 E_r^{1.91} (MeV); \quad (8)$$

$$\begin{aligned} \bar{E}_r &= 4.755(1 + 108I^2)/A^{1/3} + 32.788(1 - 7.5899I^2)/A^{1/6} (MeV), I = (N - Z)/A, \\ \Gamma_r &= 0.37E_r - 0.14E_r\beta_2 - 0.6E_{2_1^+}, \end{aligned} \quad (9)$$

where  $E_r$  and  $\Gamma_r$  are GDR energy and width respectively, average GDR energy  $\bar{E}_r$  is equal to  $E_r$  for spherical nuclei and  $\bar{E}_r = (E_1\sigma_1 + E_2\sigma_2)/(\sigma_1 + \sigma_2)$  for axially deformed nuclei [6],

$\sigma_{1,2}$  - cross section values in the first and second peak respectively,  $E_{2_1^+}$  - energy of the first collective  $2^+$  state,  $\beta_2$  - parameter of quadrupole deformation,  $N$  and  $Z$  - numbers of neutrons and protons in nuclei with mass number  $A$ . Similar systematics were obtained for the SMLO model [21, 22].

The mean GDR energies and widths as well as different systematics are presented in Fig.1.

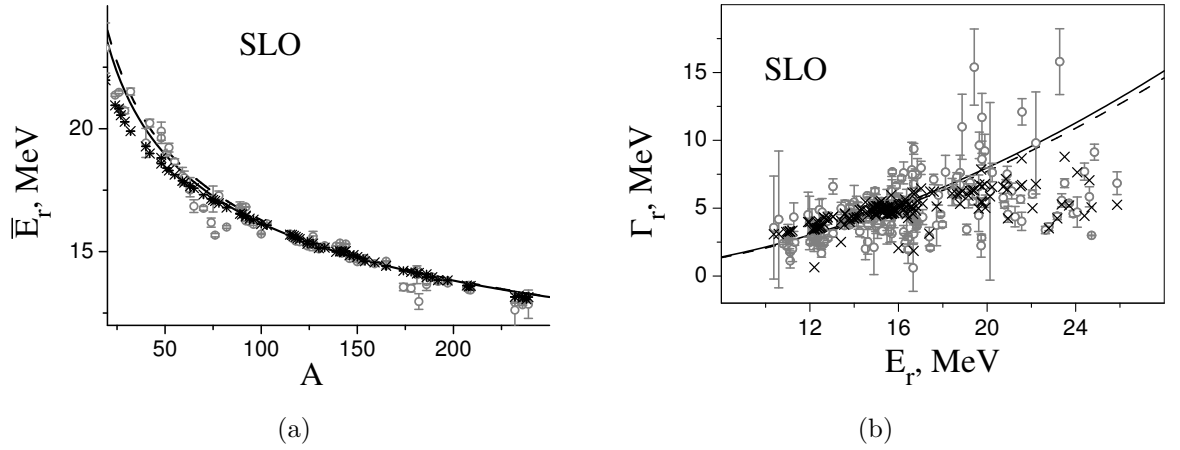


Fig.1. Mean GDR energies (a) and widths (b) calculated by the use of SLO model: open circles - renewed data fitting; solid line - parameters obtained within systematics (8), crosses - parameters from systematics (9), dashed curve - parameters from systematics [6](a) and [7](b).

As one can see from Fig.1, GDR energies and widths within renewed systematics in general agree with results of previous systematics [6, 7] for the middle-weight and heavy atomic nuclei. Differences in range with  $A < 50$  can be explained by ignorance of cross sections of the photo-charged-particle reactions which give important contribution to  $\sigma(\gamma, abs)$ . Fig.2 demonstrates ratio of renewed GDR energies (a) and widths (b) within SLO and SMLO models. It can be seen that the SLO(MLO1) and SMLO parameters are in rather close agreement.

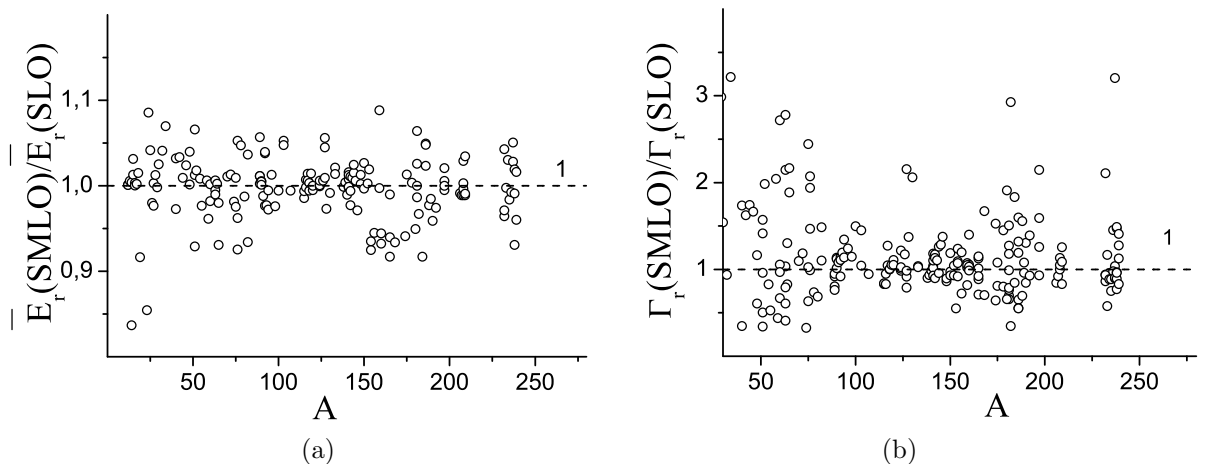


Fig.2. Ratio of renewed GDR energies (a) and widths (b) within SMLO(MLO1) and SLO models.

In order to test simplified RSF models described in previous section, the  $\gamma$ -decay radiative strength functions calculated by the use of renewed GDR parameters were compared with experimental data. SMLO parameters were used for RSF calculations within MLO models, and for other models (EGLO, GFL and SLO) GDR parameters of SLO model were applied. Different variants of MLO model give similar trend for photoabsorption cross sections. Therefore, only the MLO1 calculations are shown in the figures.

Fig.3 shows dipole  $\gamma$ -decay strength functions for  $^{150}\text{Sm}$  and  $^{148}\text{Sm}$  within different RSF models in comparison with experimental data from Refs.[23](a) and [24](b).

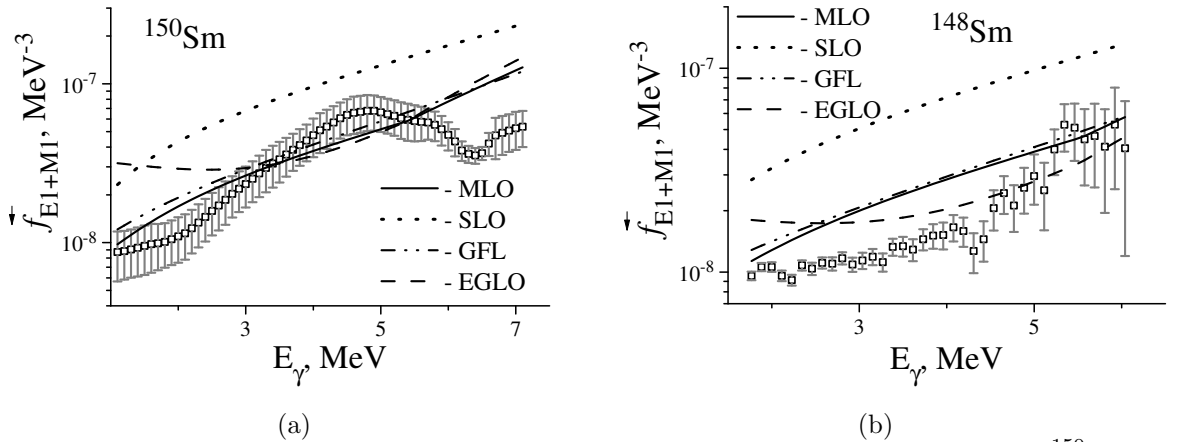


Fig.3. The  $\gamma$ -decay strength functions within different RSF models for  $^{150}\text{Sm}$  (left panel) and for  $^{148}\text{Sm}$  (right panel). The experimental data are taken from [23](a) and [24](b).

As one can see from Fig.3, in the low-energy region considered models with asymmetric shape of the RSF (EGLO, GFL, MLO1, SMLO) describe the experimental data much better than the SLO model, which predicts a vanishing strength function at zero  $\gamma$ -ray energy. The EGLO, GFL, MLO and SMLO results of the calculations for  $\gamma$  – decay are all characterized by a non-zero limit and a temperature dependence at low  $\gamma$ -ray energies. It can be also noted that different variants of the MLO (SMLO) approach are based on general relations between the RSF and the nuclear response function [16]. Therefore, they can potentially lead to more reliable predictions among simple models.

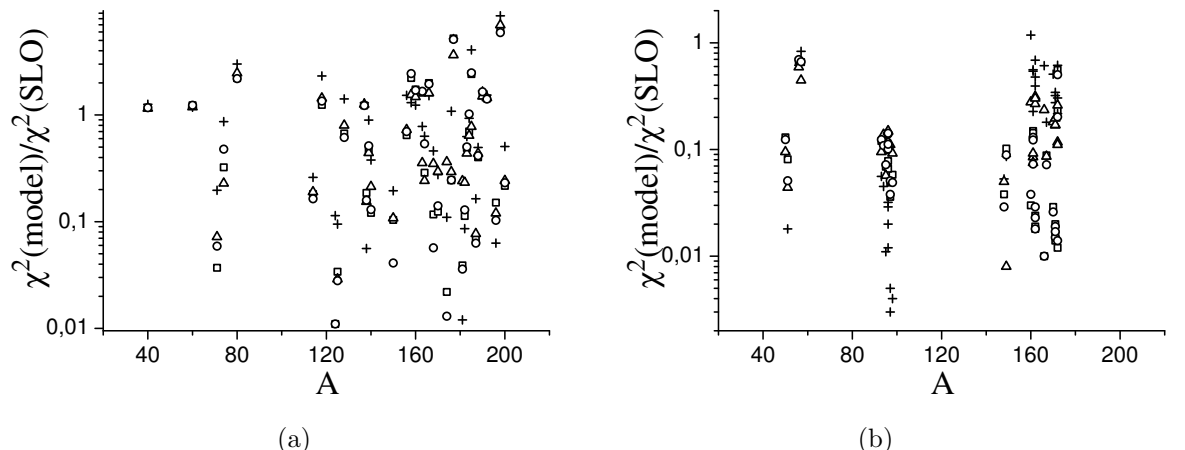


Fig.4. Relative deviations  $\chi^2(\text{model})/\chi^2(\text{SLO})$  of the  $\gamma$ -decay strength functions within different models from experimental data presented in [23](a) and [24](b) for different nuclei: + - EGLO;  $\Delta$  - GFL;  $\square$  - MLO;  $\circ$  - SMLO.

Fig.4 shows the ratio  $\chi^2(model)/\chi^2(SLO)$  of  $\chi^2$  deviations of the  $\gamma$ -decay strength functions within different models from experimental data of [23](a) and [24](b). The average values of the ratio for approximately 40 nuclei with  $25 < A < 200$  were obtained and results are presented in Table 1.

Table 1. The average  $\chi^2(model)/\chi^2(SLO)$  ratio of  $\chi^2$  deviations of the calculated  $\gamma$ -decay strength functions from experimental data.

Exp. data, Ref.	Model			
	EGLO	GFL	MLO	SMLO
[23]	1.219	0.888	0.982	0.998
[24]	0.292	0.165	0.123	0.122
[3]	0.12	0.115	0.194	0.189

As one can see from Fig.4 and Table 1, strength functions within EGLO, GFL, MLO models give better results than the SLO one. It enables us to conclude that asymmetric shape of the RSF gives better agreement with the experimental data. The MLO and GFL models give the best agreement with experimental data for heavy nuclei.

Fig.5(a) shows dipole  $\gamma$ -decay strength functions calculated by the use of different RSF models for nuclei with  $25 < A < 250$ . The RSF were calculated for  $\gamma$ -ray energies that approximately correspond to the mean energy of E1 transitions in the file “gamma-strength-exp.dat” from the RIPL2 library [3] provided by J. Kopecky. GDR parameters were obtained from fitting the experimental photoabsorption data, as described above.

In agreement with the previous investigations [2, 3, 20, 21, 22], Fig.5(a) demonstrates that all considered models with asymmetric shape of the RSF (EGLO, GFL, MLOS, SMLO) describe the experimental  $\gamma$ -decay data with  $E_\gamma \sim S_n$  better than the standard SLO model.

Fig.5(b) shows excitation function of the  $^{183}W(n, \gamma)^{184}W$  reaction calculated by the use of different RSF models in  $\gamma$  channel.

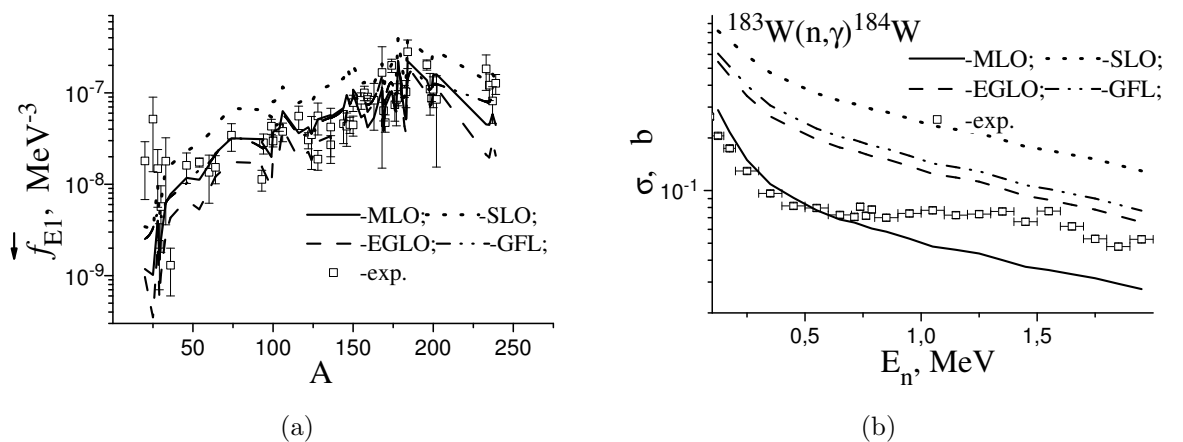


Fig.5. The E1  $\gamma$ -decay strength functions calculated by the use of different RSF models as function of mass number(a) and excitation function of  $^{183}W(n, \gamma)^{184}W$  reaction using different RSF models(b). The experimental data are taken from “gamma-strength-exp.dat” file of the RIPL2[3](a) and from [25](b).

The cross section calculations were performed by the use of EMPIRE code [26]. It should be mentioned that calculated results were not normalized on the experimental  $\Gamma_\gamma$  values at the neutron binding energy. The calculations within the MLO model in general gives better agreement with the experimental data for middle-weighted and heavy nuclei.

The overall comparison of the calculations within different simple models and experimental data shows that the EGLO and MLO (SMLO) approaches with asymmetric shape of the RSF provide a unified and rather reliable simple method to estimate the dipole RSF both for  $\gamma$ -decay and for photoabsorption over a relatively wide energy interval ranging from zero to slightly above the GDR peak.

Reliable experimental information is needed for more precise determination of the temperature and energy dependence of the RSF, so that the contributions of the different mechanisms responsible for the damping of the collective states can be further investigated. This should help us to discriminate between the various closed-form models describing the dipole RSF.

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