PRACTICAL LIMITS ON ACHIEVABLE PRECISION OF SOME NUCLEAR-PHYSICS PARAMETERS DETERMINATION

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Abstract

The status of experiments on determination of level density and partial widths of the nuclear reaction products emission in diapason of nucleon binding energy is presented. There are analyzed the sources and magnitude of probable systematical uncertainties of their determination. The maximally achievable precision of these parameters is estimated, as well.

There is considered ability of new method for determination of distribution parameters of neutron resonances reduced widths in order to distinguish their groups with the same structure of wave functions. It was obtained in both cases that the insufficient value of maximally achievable precision of the parameters of the experimental data analysis does not allow one to obtain reliable and detailed information on the studied nuclear properties – its entropy, strength functions of nuclear products emission and dominant level structure above $\approx 0.5B_n$.

1 Introduction

The problem of determination of level density ρ and partial widths Γ of gamma-quanta or nucleons interaction with excited nucleus is an object of numerous experiments. One can say the same and about determination of nuclear properties from parameters of neutron resonances of any stable or long-living target-nuclei.

In the first case, the special problem and importance is a necessity of their determination in the excitation energy region where resolution of existing spectrometers is insufficient for distinguishing of individual nuclear levels. Accordingly, only the spectra of reaction products S and their cross sections σ are available for experimental determination. Their observables are determined by product $\rho\Gamma$, entering as a parameter in corresponding functionals – the total gamma-spectra, averaged cross sections, spectra of two-step reactions. The ρ and Γ values can be extracted only from these data by solution of reverse task of mathematical analysis. If one does not consider the problem of precision of the model set functional relations $S = F(\rho, \Gamma)$ (or $\sigma = F(\rho, \Gamma)$), then unambiguous (asymptotically precise) determination of ρ and Γ is possible only under the condition that the matrix $L_m = J^t W J$ is not degenerated. Here is postulated that the Jacobi matrix J contains all the possible information on the case under consideration and the weight matrix W - diagonal.

If the number of real parameters in system of equations $S = F(\rho, \Gamma)$ (or $\sigma = F(\rho, \Gamma)$) is more than the number of experimental points, L_m is always degenerated. In practice, there were not found any systems of non-degenerated non-linear equations connecting ρ, Γ with the functions under study.

However, because of nonlinearity of function F, the region of the ρ and Γ values can be final even in this case. Minimization of uncertainty of the sought parameters in this case can be achieved only by choice of the most effective experiment. This conclusion follows from analysis of mathematical relations between the parameters and measured functions in one- and two-step experiments [1]. Estimation of the possible achievable precision in determination of both ρ and Γ can be obtained only at analysis of concrete possibilities of existing methods for determination of these parameters.

2 Partial widths and level density

1. The spectra of evaporation nucleons in unresolved energy region contain bigger number of unknown parameters than corresponding to them experimental points. By this, maximal width of the interval of possible values of the sought parameter ρ cannot be determined (or even limited) without involving of additional information. Practically in all performed within this method experiments, the not measured width Γ of nucleon emission at transition of a nucleus from level U_i to level $U_f > 0$ was changed by the calculated value of this parameter for $U_f = 0$.

Real precision of such subjective notions on the Γ values set in this way is unknown. Therefore, all accumulated from corresponding analysis information contains unknown uncertainty. Its estimation - 15% (published in [2]) can be related only to the widths observed in the spectra of evaporation nucleons as resolved peaks. This conclusion unambiguously follows from analysis of form of dependence of differential cross section of emission of evaporation nucleons with energy E_N by nucleus with excitation energy $U > E_N$.

In Hauser-Feshbach notion, the considered cross section is determined by sum over initial and final levels of products like $\Gamma_b(U, J, \pi, E, I, \pi)\rho_b(E, I, \pi)$ for final reaction product b [2, 3]:

$$\frac{d\sigma}{d\varepsilon_b}(\varepsilon_a, \varepsilon_b) = \sum_{J\pi} \sigma^{\text{CN}}(\varepsilon_a) \frac{\sum_{I\pi} \Gamma_b(U, J, \pi, E, I, \pi)\rho_b(E, I, \pi)}{\Gamma(U, J, \pi)} \tag{1}$$

where

$$\Gamma(U, J, \pi) = \sum_{b'} \Bigg(\sum_k \Gamma_{b'}(U, J, \pi, E_k, I_k, \pi_k) + \sum_{I'\pi'} \int_{E_c}^{U-B_{b'}} dE'\, \Gamma_{b'}(U, J, \pi, E', I', \pi')\, \rho_{b'}(E', I', \pi') \Bigg). \tag{2}$$

It follows from (1) and (2), that at presence of completely unknown systematical error δ of the calculated value of the width Γ_b, the experimental cross-section $\frac{d\sigma}{d\varepsilon}$ can be precisely reproduced only by use of the level density with adequate systematical error. If the calculated Γ_{cal} and unknown experimental Γ_{exp} widths are connected by the relation $\Gamma_{cal} = \Gamma_{exp}(1 + \delta)$, then the mistaken level density $\rho_{cal} = \rho_{exp}/(1 + \delta)$ is to be used in calculation for precise reproduction of cross section. Of course, the unknown relative error δ of the calculated width depends on excitation energy of final nucleus and can depend on spin of levels which are connected with each other by emission of reaction product. In this case, the error δ is the weight average. If one determines it by means of relation: $(1+\delta) = \rho_{es}/\rho_{2\gamma}$ which connects level density ρ_{es} from evaporation spectra [2, 3] with density $\rho_{2\gamma}$ determined from cascade intensity [4, 5], then it is possibly to estimate directly the error of calculated cross section $\frac{d\sigma}{d\varepsilon}$ at any excitation energy of final nucleus. The corresponding data were obtained for two nuclei and presented in [6, 7].

2. By now, the Norway collaboration has measured the total gamma spectra of depopulation of the levels in wide excitation energy interval. In correspondence with the method described in [8], the authors of the following experiments extract from them the spectra corresponding to the first gamma-quantum of cascade. By this, it is postulated that the strength function $f = \Gamma/(DE_\gamma^3)$ for arbitrary partial gamma-width is determined only by its multipolarity and does not depend on nuclear excitation energy. The authors suppose without a proof that the use of the known values of the total radiative width of neutron resonances, mean spacing between them D and density of low-lying levels permits one to get unbiased estimation of ρ and Γ. But, according to [9], the matrix L_m in this case is also degenerated. And the existing [5] experimental data on cascade population of large set of excited levels point to dependence of $k = f/A^{2/3}$ on structure of decaying and excited by gamma-transitions levels. Therefore, there is no possibility to obtain asymptotically zero uncertainty within the framework of the existing method [8]. Besides, the authors did not estimate neither the value of systematical error nor required precision of their experiment. It is done by us only in [10].

3. Any two-step reaction, for example, cannot give asymptotically zero uncertainty in determination of ρ and Γ. Nevertheless, even in presence the asymptotical uncertainty of the derived from it data provides obtaining of quite acceptable information on the gamma-decay parameters. In particular, the width of interval of the possible ρ and Γ values can be equal to some tens percents for the cascade of two gamma-quanta proceeding between neutron resonance and a group low-lying levels at zero total error of determination of the cascade intensity. Systematical error related with dependence of the radiative strength functions of the cascade dipole gamma-transitions on structure of initial and final levels, in two-step reactions can be taken into account, at least, partially [5].

4. Hence, it is necessary to develop and realize new independent methods of the ρ and Γ experimental determination. It is no sense to realize new experiment in the one-step variant. In practice, it is necessary to pass to registration of cascades of not only two successively emitted gamma-quanta but also of three and more [11]. The more general solution of this problem is to analyze the spectra of the two-step reactions with registration of nucleon products at the first step of nuclear reaction [6].

Serious problem in the experiments on determination of the most probable ρ and Γ values from two-step gamma-cascades is unknown influence of structure of the initial cascade level on the primary gamma-transition width – all corresponding data on ρ and Γ were obtained from analysis of intensity of cascades following thermal neutron capture (only 1 or 2 initial levels are excited with visible probability). As the most probable explanation, just the structure of wave function of neutron resonance is the cause of considerable variations of the strength function parameters in nuclei with different mass [12]. Dependence of strength functions on structure of intermediate cascade level in wide region of its energy unambiguously follows from the data [5]. It follows also a necessity to reveal a degree of dependence of strength functions and on structure of initial level. Id est, the results presented in [13, 14] require one to perform analysis of parameters of neutron resonances in order to discover the dependence of this type and its possible influence on dynamics of nuclear reaction. The total intensity of all the primary transitions, naturally, equals 100% per decay. Therefore, the expected effect can only change the form of energy dependence of cascade intensity in different resonances.

Level density in small, as compared with B_n excitation energy interval $\Delta U = U - B_n$, can be presented as decomposition in the row:

$$\rho(U) = \rho(B_n) + \frac{d\rho}{dU}\Delta U + ... \tag{3}$$

In the ideal case, the analysis of the reduced neutron widths must give the $\rho(B_n)$ and $\frac{d\rho}{dU}$ values with minimally possible error. The maximally precise $\rho(B_n)$ values are requred by normalization of its functional dependence for $U < B_n$. Reliably determined negative values of $\frac{d\rho}{dU}$ would testify to undoubted change in structure of neutron resonances and above B_n also.

3 Reduced neutron widths

The main grounds of this problem are the following:

(a) break, at least, of the first 3-4 Cooper pairs occurs in any nucleus discretely, with the interval by excitation energy being some less $2\Delta_0$ ($\Delta_0 = 12.8/\sqrt{A}$) [13, 14];

(b) moreover, there is observed excessive variation of parameters of the best approximation of radiative strength functions [12]. Besides, the forms of energy dependence of the two-step cascade intensities and total gamma-spectra following thermal neutron capture change, possibly, cyclically and, possibly, not accidentally.

1. Inevitable errors of experiment and random fluctuations of both the primary gamma-transition intensities and obtained ρ and Γ values do not allow one to connect break thresholds energy U_{th} of Cooper pairs and B_n . I. e., to fix the most probable structure of nucleus near B_n (number of broken pairs, difference of neutron binding energy and break threshold of the last pair and so on).

According to the results [14], correlation between level densities of vibration and quasi-particle types continuously changes at change of excitation energy. This conclusion follows from theoretical notions about form of energy dependence of density of levels of quasi-particle and vibration type and from the lowest χ^2 value at approximation of the Dubna data set on level density as compared with approximations [13, 15]. This circumstance must change strength function of the primary gamma-transitions following decay of the excited levels above B_n because of change in components of wave function which determine the value of its matrix element [16]. But the theory like quasi-particle-phonon model of nucleus cannot predict quantitatively details of this process now.

Therefore, one can accept determinate on the whole character of change of wave function of neutron resonances (and other high-lying levels) as a working hypothesis qualitatively explaining enumerated above aspects of nuclear investigation. This change can accordingly influence the distribution of the reduced neutron widths Γ_n^l for orbital momenta $l = 0, 1....$ Only experiment can determine whether this hypothesis is true or wrong.

2. According to conclusions [14], structure of resonances changes continuously (probably – not monotonously). That is why, potential dependence of Γ_n^l on the neutron (proton)

resonance energy, at worst for investigation cases is smooth and in some interval of their energy – weak.

In practice, it is impossible to obtain information on, for example, the value of the most important components of wave function of neutron resonance. Therefore, any data on appeared problem can be derived only from analysis of distribution parameters of reduced neutron widths measured in small energy interval with different type errors and distortions.

Id est, it is necessary to solve analogous to the previous case problem distorted by its nature, namely:

(a) to minimize a degree of model dependence at analysis of distributions Γ_n^l;

(b) to develop new method for analysis of available experimental information and to determine all region of its possible solutions;

(c) to develop new methods of investigations.

Random character of the Γ_n^l values observed in experiment is grounded experimentally and theoretically. The first part of the ground – the use of principles of mathematical statistics and its criterions, the second – development of model description of experimental data. In particular – theoretical investigation of fragmentation regularities of any nuclear state over higher-lying levels [17].

As a result, the Porter-Thomas hypothesis [18] transformed in immutable axiom that the random variation of reduced widths is described by χ^2- distribution with degree of freedom $f \approx 1$.

For absolute correctness of hypothesis [18], it is necessary that the neutron amplitude A ($A^2 = \Gamma_n^0$) would have normal distribution with zero average and dispersion $D(A) =< \Gamma_n^0 >$. Nobody tested these conditions and, therefore, it is necessary to begin analysis of the neutron width distributions just from their obligatory test.

Again, it is postulated, but not tested that a set of the experimental reduced neutron widths corresponds to the only one possible distribution and does not correspond to superposition of several functional dependences with unknown values $< A >$ and $D(A)$. Corresponding conclusion can be inexact within frameworks of the results obtained in [13, 14, 15].

Consequently, analysis of the distribution parameters of neutron widths aimed to derive from them information on the neutron resonance structure must test a possibility of presence of superposition of K distributions ($1 \leq K \leq 4$, for example). This condition automatically transfers the task of distribution analysis of neutron widths in category of search for badly stipulated or, most probably, degenerated solutions. Id est, serious analysis of Γ_n^l practically inevitably brings to multi-valued solution, and traditional analysis (for example, [19]) gives solution with unknown systematical error.

This conclusion was made from mathematical modeling of the problem under consideration. The expected $< A >$, $D(A)$ and K values for the accumulated by now information on parameters of neutron resonances are comparable with their random values obtained at approximation of relatively small sets of normally distributed amplitides with zero average and unit dispersion.

Nevertheless, the full-scale analysis of the Γ_n^l values showed that there are no grounds to accept distribution [18] in its classical form $< A >= 0$ and $D(A) = 1$ as the only true. This conclusion follows [20] from comparison of the most probable number of resonances in experimental width distribution for actinides at approximation of their distribution under condition that the neutron amplitude can have non-zero average and non-unit dispersion. The obtained in this way spacings between resonances noticeably differ from their estimations obtained from cumulative sums of resonances in function on neutron energy. The exit from this situation can be found only by means of obtaining of additional experimental information. Id est, by realization of the methodically independent experiment.

As a tested hypothesis, it is necessary to measure the total gamma-spectra in different resonances and/or their groups. The parameter for which is expected dependence on structure of wave function of resonance can be ratio of intensity of gamma-transitions to group of low-lying levels to mean intensity of primary gamma-transitions with energy $E_\gamma \sim 0.5B_n$. The experiment in spherical nuclei can be realized by use of scintillation detectors, in deformed – the use of Ge-detectors is more worth while.

4 Conclusion

Analysis of condition and possibilities of modern methods for determination of parameters of nuclear de-excitation process followed by emission of both nucleon products and gamma-quanta shows that the use of one-step reactions in region of high level density allows one to obtain the ρ and Γ values only with large systematical errors. Its maximal magnitude can be equal to 500 - 1000% [6, 7]. The two- and multi-step reactions permit one to decrease maximal systematical error to several tens percents. But, only under condition that the inevitable model notions about nucleus (on coexistence and interaction of fermi- and bose-excitations and their influence on the process under study, in the first turn) do not bring to noticeable and unknown errors in determination of ρ and Γ.

The volume and quality of the available data on the reduced neutron widths do not allow one to get unambiguous conclusions on the mean value, dispersion of neutron amplitudes and real number of groups of resonances with different structure. Besides, it is impossible to estimate degree of execution of conditions which are necessary for truth of [18].

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