PARAMETERS OF THE BEST APPROXIMATION FOR DISTRIBUTION OF THE REDUCED NEUTRON WIDTHS . ACTINIDES

A.M. Sukhovoj, V.A. Khitrov

Joint Institute for Nuclear Research, Dubna, Russia

Abstract

The data of ENDF/B-VII library on Γ_n^0 (Γ_n^1) for nuclei ^{231}Pa, ^{232}Th, ^{233,234,235,236,238}U, ^{237}Np, ^{239,240,241,242}Pu, ^{241,243}Am and ^{243}Cm (including p-resonances of ^{232}Th, ^{238}U, ^{239}Pu) in form of cumulative sums in function on $\Gamma_n^0/<\Gamma_n^0>$ were approximated by variable number K of partial items ($1 \leq K \leq 4$). Parameters of approximation – mean value of neutron amplitude, its dispersion and portion of contribution of part of widths of distribution number K in their total sum. The problems of their determination from distributions of different number of squares of normally distributed random values with variable threshold of loss of some part of the lowest Γ_n^0 values were studied.

It was obtained for some part of neutron resonances that their mean amplitudes can considerably differ from zero value, and dispersions – from $<\Gamma_n^0>$. And it is worth while to perform any quantitative analysis of distributions Γ_n^0 by means of comparison of different model notions with obligatory estimation of random dispersion of the desired parameters.

1 Introduction

The experimental data on reduced neutron widths Γ_n^0 (Γ_n^1) of s- or p-resonances potentially contain diversiform information. In particular, of interest are the data on their real density ρ and on few-quasi-particle components of wave function of high-lying level [1]. Restoring of this information from the data of any experiment on the neutron time of flight method cannot be made without the use of hypothesis on form of the frequency distribution of neutron widths. Porter and Thomas first assumed [2], on the grounds of large fluctuations of Γ_n^0, that it corresponds to χ^2-distribution with one degree of freedom. In the other words – to distribution of squares of normally distributed random values ξ with mathematics expectation $M(\xi) = 0$ and dispersion $D(\xi) = 1$. These conditions are realized to higher or lower extent in case when wave function of neutron resonance contains many small items of different sign which determine the value Γ_n^0. Within frameworks of notions of quasi-particle-phonon nuclear model, this means that fragmentation of any nuclear states like m quasi-particles $\otimes$ n phonons over nuclear levels in region of neutron resonances (for truth of [2]) must be very strong.

Theoretical analysis of fragmentation process of nuclear states of different type over the levels with different excitation energy [3] showed that this condition can be not realized in case of large enough values m and n. This conclusion follows and from results of approximation below B_n [4, 5] of methodically more precise experimental data for ρ [6, 7]

on density ρ_n of n-quasi-particle levels by Strutinsky model [8]. It can be performed at wide enough variation of assumptions on shape of correlation function of nucleon pair in heated nucleus and on coefficients of its vibration enhancement. Nevertheless, results of approximation already gave some notion on structure of high-lying levels of any nuclei.

The results [4, 5] unambiguously show that at the simplest hypotheses of correlation functions δ_n of nucleon Cooper pairs in heated nucleus, the number n increases by 2 quasi-particles with excitation energy interval being some less than $\Delta E_{\rm ex} \approx 2\delta_0$. Id est, structure of wave function of highly-excited levels (and, it is not excluded, of neutron resonances) can cyclically change. These data are enough for qualitative explanation of change in form of radiative strength functions as increases mass of nucleus A [9]. Id est, there are theoretical and experimental grounds for detailed and methodically independent analysis of the data on Γ_n^0. The primary goal of this analysis – discovery of possible deviations of neutron width distribution from the Porter-Thomas distribution and estimation of reliability extent for their observation in experiment.

Reanalysis of the data on gamma-transition intensities from reaction $(\overline{n},\gamma)$ [10, 11] revealed strong influence of nuclear structure on distribution of parameters of radiative widths of primary gamma-transitions from neutron resonances. This is an additional argument for independent full-scale analysis of the data on neutron widths with less quantity of the used model ideas.

2 Modern status of the problem

Direct determination of structure of arbitrary nuclear levels above some MeV usually is inaccessible for all the known experiments. Therefore, any information on this account can be derived only from indirect data (as it was shown in [1]). First of all – from analysis of the results of model approximation of experimental distributions of Γ_n^0 ($2g\Gamma_n^0$ – for resonances with different spins). Numerous problems of analysis of experimental data of this type are described, for example, in [12]. Analysis of modern data on parameters of neutron resonances of ^{238}U in neutron energy region up to 20 keV is presented, for example, in [13], ^{232}Th - [14].

The basis for all the performed earlier analyses is an assumption that the hypothesis [2] describes tested set of the data on Γ_n^0 with a precision exceeding accuracy of the experiment. By this it was assumed that in analysis is in some form realized correct accounting (or exclusion) of experimental distortions of the data under consideration (omission of weak levels, unresolved multiplets, admixture of resonances with other orbital momentum l and so on). Or observed discrepancies of experiment with distribution [2] are completely explicable by enumerated factors. In practice, it is tested up to now only the hypothesis of deviation of experimental distribution of Γ_n^0 from the expected theoretical one owing only to deviation of parameter ν of theoretical distribution from unit.

Whereas, the form of distribution of Γ_n^0 strongly depends on degree of execution of condition of equality to zero of mean amplitude A_n ($\Gamma_n^0 = A_n^2$) of the tested set of resonances. It is absolutely impossible also to exclude a possibility that the experimental data are superposition from K distributions even for their set with precisely determined spins and orbital momenta of resonance neutron. Qualitatively, the possibility $M(A_n) \neq 0$ directly follows from [3], $K > 1$ – from [4, 5].

3 Necessity in full-scale analysis of experimental data

The fitted function in full-scale analysis is the sum of K distributions $P(X)$ of squares of normally distributed random values with independent variables X_k each. The desired parameters in compared variants are the most probable value b_k of amplitude $A = \sqrt{\Gamma_n^0 / < \Gamma_n^0 >}$, dispersion σ_k and total contribution C_k of function number k for the variable

$$X_k = ((A_k - b_k)^2)/\sigma_k^2 \tag{1}$$

in the total experimental sum of widths. Statistically significant result $b_k \neq 0$ allows one to state that the neutron resonance is not completely chaotic system, $\sigma_k < 1$ means, in particular, that interaction of neutron with non-excited nucleus is the more or less determinate process.

The proof of the notion that the most precise approximation of dispersion of widths by several distributions ($K > 1$) is not caused by random fluctuations of X, can give new information on nuclear structure in region B_n. First of all – information on possible existence of neutron resonances with different structure of their wave functions and on regions of Γ_n^0 values, where radiative strength function (the total gamma-spectrum) have essentially different form (see, for example [9]).

Cyclic change in structure of neutron resonances at different neutron energies (directly following from successive break up of Cooper nucleon pairs [4, 5]) can stipulate non-monotonous character of change in density of nuclear excited levels and above neutron binding energy. This means additional systematical error of the data on ρ in the most important for this nuclear-physics parameter point.

As it was obtained by modeling [16], the values of b_k, σ_k and C_k with small statistical error for accumulated by now sets of neutron widths cannot be get not only for $K > 1$, but also for $K = 1$. Most probably, this circumstance has principle character and appears itself, first of all, by extraction of level density and emission probability of the nuclear reaction product from the spectra (cross sections) of nuclear reactions.

4 Non-removable uncertainty of experimental data analysis of some nuclear-physics experiments

By analysis of experimental data in low energy nuclear physics (at least in some its sections) is really used the postulate on principle possibility of unambiguous determination of desired nuclear parameters. For example, level density in fixed interval ΔE for given nuclear excitation energy and emission probability of some nuclear reaction product at their de-excitation. Or excitation – at decay of higher-lying levels. However, the experience of determination of ρ and radiative strength functions k from intensities of two-step gamma-cascades [6, 7] together with analysis of possibilities of existing methods of analogous experiments [15] shows that their unambiguous determination is impossible. It is true, at least, for the present and for region of high level density. Practically, it follows from this circumstance that the ρ and k values can be determined only with inevitable systematical error or there can be found only final interval of values of these parameters which contains desired parameter. And its asymptotical width is not equal to zero.

The task under consideration obviously belongs to the same class. Id est:

(a) the parameter ν of χ^2-distribution can be unambiguously determined (with precision up to experimental uncertainty and statistical fluctuations), but there cannot be tested all the necessary conditions of applicability of hypothesis [2], or

(b) there can be found only asymptotically non-zero interval of values of parameters of expression (1). Below is realized only the second possibility. The more detailed description the analysis method of and results of its test are presented in [16].

5 Results of analysis

Comparison of experimental cumulative sums with approximating curves for 15 sets of s- and three sets of p-resonances in variants:

the distributions $K = 1$ and superposition of four possible distributions ($K = 4$) is presented in figures 1 and 2.

The ratios of χ^2 in function of nuclear mass for two variants of approximation are shown in Fig. 3. (Approximation in all the cases was performed, as a minimum, over X=1000 values.

It follows from these figures that approximation of the experimental data by superposition of four distributions improved precision for the greater part of experimental data. And maximally – for s-resonances of A-odd nuclei and all sets of available p-resonances.

The simplest possible explanation is obvious: the $2g\Gamma_n^0$ values for different spins of resonances differ by parameters of distributions of their neutron amplitudes. It is enough for their appearance in the obtained experimental data as a superposition of two distributions with different σ and/or b values. But, the tendency of change of ratio of approximation parameter χ^2 for different nuclei allows and possibility of change in shape of width distribution when nucleus mass changes.

Large dispersion of random values X brings to large fluctuations of cumulative sums of both experimental data and model distributions [16]. And, respectively, to essential variations of the best values of parameters (1). That is why, the conclusions about possible deviations of b and σ parameters from expected values 0 and 1, respectively, can have, as it was mentioned above, only probabilistic character. In Fig. 4 are compared frequency distributions of these parameters for modeling sets with N=150, 500 and 2000 random

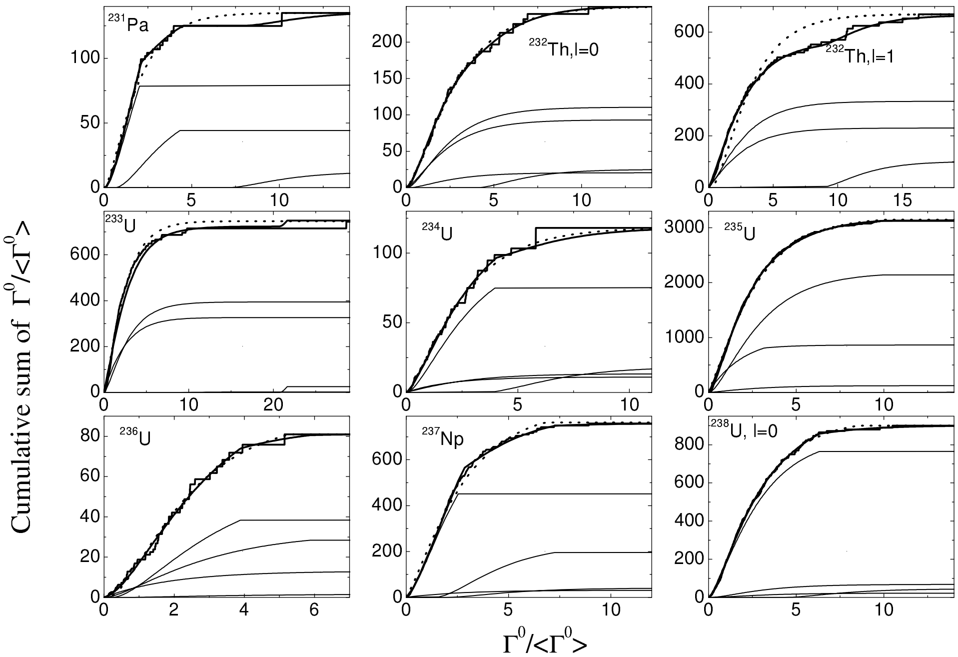

Fig. 1. Histogram - cumulative sum of $\Gamma_n^0/<\Gamma_n^0>$ for their values, less than given magnitude. Thick solid curve – the best approximation by four distributions, dotted curve – by one distribution for nuclei with mass $231 \leq A \leq 238$. Thin curves – the most probable values of approximating functions for $1 < K \leq 4$.

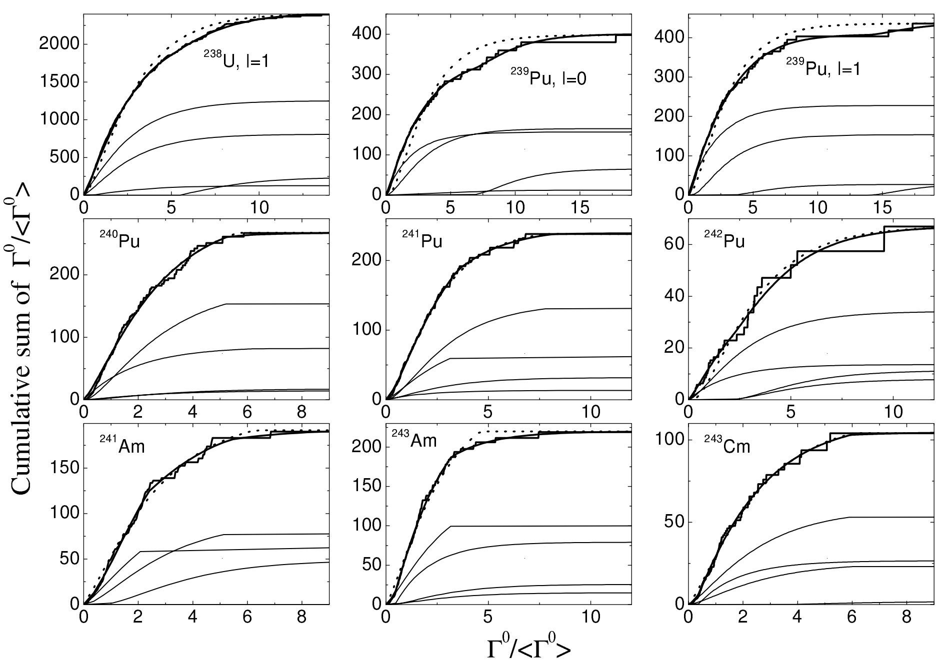

Fig. 2. The same, as in Fig. 1, for $238 \leq A \leq 243$.

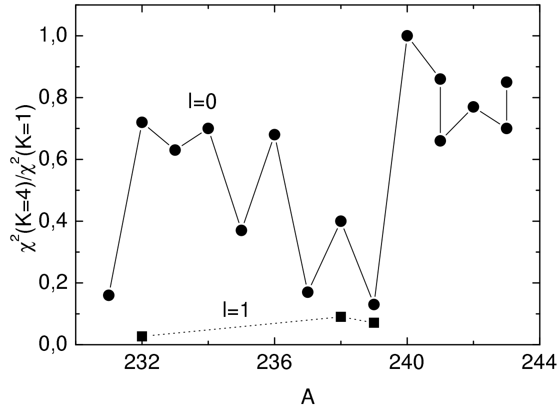

Fig. 3. The ratio of the lowest χ^2 values for the sets from K approximating distributions for actinides under consideration.

X values. Modeling was performed for the variant of absence of omission of small X values and for omission corresponding to exclusion of $L = 30\%$ of their lowest magnitudes (linearly changing with number of random value).

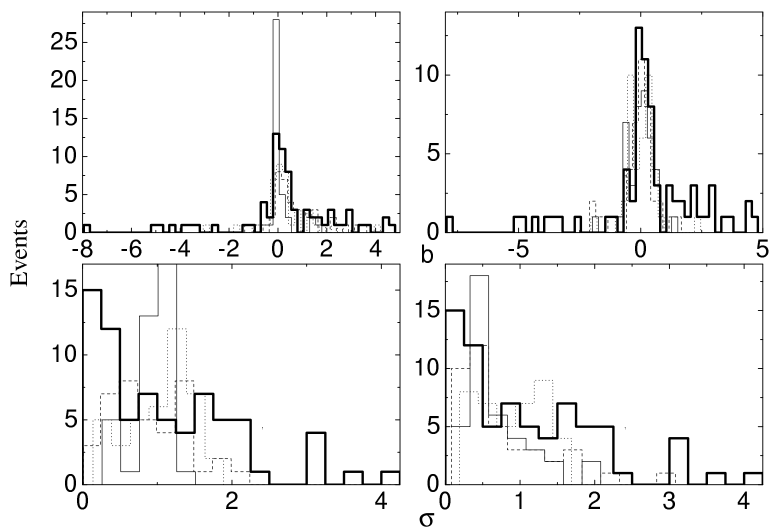

Fig. 4. The comparison of the frequency distributions of appearance of given values of b (upper) and σ (lower) row accordingly for $K = 1$. Left column – in modeling are included all the possible random values; right column – there are excluded $L = 30\%$ of the lowest random values in each tested set. Thick solid curve – experimental data set, thin solid, dashed and dotted curves – the data for N=2000, 500 and 150 random values in modeled sets.

The widths of corresponding distributions decrease when N increases. They are minimal for $L = 0$ and in all practical cases – less than the maximal width of b_k and σ_k experimental frequency distribution. Figures 1 and 2 permit one to conclude that deviation of experimental width distribution from the Porter-Thomas distribution appears itself mainly at $X = (\Gamma_n^0 / < \Gamma_n^0 >) > 2 - 5$. Discrepancy between experimental data and hypothesis [2] at less values of X can be related, first of all, to omission of weak resonances

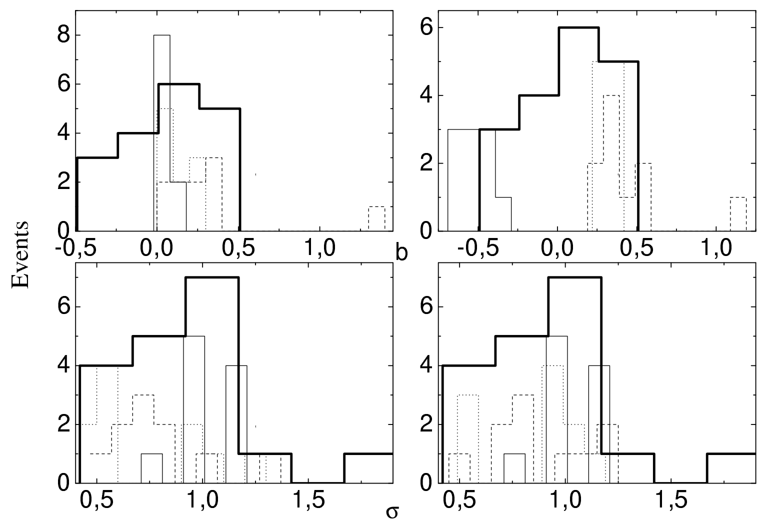

Fig. 5. The same, as in Fig. 4, for the case $K = 4$.

or to other systematical errors of experiment. But, it is not excluded and possibility of real deviations of parameters b and σ from the values corresponding to hypothesis [2].

Although deficiency of experimental Γ_n values for analyzed here actinides did not allow one to get unambiguous conclusions on real parameters of expression (1), the information from the data of estimation of mean spacing between their resonances is much more unexpected [17].

6 Conclusion

1. The analysis performed shows that the probability of correspondence of distribution $2g\Gamma_n^0$ to the unique functional dependence ($K = 1$) in nuclei of different mass is less than to the set of visibly different functions ($K = 4$). Therefore, any quantitative test of hypothesis [2] should be performed by comparison of two or more different model notions in maximal set of nuclei.

2. The results of performed analysis, probably, do not contradict to hypothesis [2] on equality of mean value of amplitude to zero for the main part of the determined Γ_n^0 values. More precise statement on this account can be made only after significant decrease of experimental threshold of resonance registration.

3. The more unambiguous conclusions on equality of dispersion of experimental distribution of Γ^0 to the fitted value cannot be made on basis of present analysis.

4. Parameters of approximation of the experimental data permit a presence of, at least, superposition of two sets of resonances for $K > 1$ with different structure of wave functions.

5. Possible deviation of parameter ν of the Porter-Thomas distribution from $\nu = 1$ for

the majority of experimental sets of neutron widths can be interpreted only after model estimation of its random fluctuations.

6. Unambiguous conclusions on problems considered here require very significant increase of sets of resonances with experimentally determined Γ_n^0 (Γ_n^1) values at their accordingly decreased distortions.

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