PARAMETERS OF THE BEST APPROXIMATION FOR DISTRIBUTION OF THE REDUCED NEUTRON WIDTHS. THE MOST PROBABLE DENSITY OF NEUTRON RESONANCES IN ACTINIDES

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Abstract

In the frameworks of hypothesis of practical constancy of the neutron resonance number in small fixed intervals ΔE of neutron energy, their most probable value was determined for nucleus mass region $231 \leq A \leq 243$ from approximation of the reduced neutron widths by superposition of two or four independent distributions. This was done under assumption that a set of the measured neutron amplitudes can correspond to one or to superposition of some normal distributions with non-zero average and dispersion differing from $< \Gamma_n^0 >$.

The main result of the analysis: the mean D and S values can be determined only with unknown systematical uncertainty whose magnitude is determined by unknown precision of the Porter-Thomas hypothesis correspondence to concrete experimental sets of resonances and unknown experimental mean $< \Gamma_n^0 >$ widths.

1 Introduction

The density of neutron resonances $\rho_\lambda = D_\lambda^{-1}$ is one of the main results of analysis of the data of all experiments performed using the neutron time-of-flight method. It is the basis point for any experiments where nucleus level density is derived from the spectra of gamma-quanta or evaporation nucleons. High precision in determination of D_λ^{-1} is stipulated by excellent resolution of corresponding method, but it can be realized only by careful accounting or correction of all systematical errors of experiment.

The most serious and not removable from them is "omission" of resonances whose reduced neutron width Γ_n^0 (Γ_n^1...) is less than the sensitivity threshold of experiment. In principle, determination of the most probable value of D_λ^{-1} in this situation is possible only by means of the most precise approximation of distribution of Γ_n^0 in all region of their values and extrapolation of the obtained function into the region below threshold. Of course, precision of this procedure is determined by degree of correspondence of theoretical notions about distribution Γ_n^0 to experiment.

According to theoretical analysis [1], the value Γ_n^0 in nuclei of intermediate and large mass is determined by few-quasi-particle components of wave function whose square contribution in its normalization is estimated by value of about 10^{-6}-10^{-9}. Their small and chaotic values for different resonances are determined by strong fragmentation [2] of low-lying one- and two-quasi-particle states of a nucleus. There is the first necessary condition for description of fluctuations of Γ_n^0 by the Porter-Thomas distribution [3].

Another condition is that the mathematics expectation of mean value of amplitude $A = \sqrt{\Gamma_n^0}$ must be equal to zero, and its dispersion – to mean $< \Gamma_n^0 >$. Both conditions:

$$M(A) = 0,$$
$$D(A) = < \Gamma_n^0 > \quad (1)$$

are not tested in modern analysis of the experimental Γ_n^0 values [4]. I.e., applicability of the Porter-Thomas distribution is postulated but is not proved. The experimental width distribution is not tested also for possibility of existence of superposition of several distributions with different values $M(A)$ and $D(A)$. Approximation [5, 6] of level density derived from the two-step cascade intensities shows that the structure of any nucleus changes cyclically as increasing excitation energy. This fact is determined so far as at present there is the only methodically model-free method for determination of ρ – [7]. This occurs, at least, due to excitation of nucleus states with increasing number of quasi-particles and, probably, due to variation of number and type of phonons. Fragmentation of these complicating nucleus states inevitably changes coefficients of wave functions of neutron resonances (as it follows from basis notions of quasi-particle-phonon model of nucleus). As a result, it is possible violation of the Porter-Thomas distribution in existing today interpretation (1).

2 Data of analysis

The method for analysis of the data on Γ_n^0 accounting for these factors is described in [8], concrete results of the best fitting of the experimental data for actinides are given in [9]. Cumulative sum of Γ_n^0 in suggested there analysis is approximated by one or several distributions of the variables:

$$X = ((A - b)/\sigma)^2 \quad (2)$$

with the initial values of fitted parameters

$$b = M(A) \neq 0,$$
$$\sigma^2 = D(A) \neq < \Gamma_n^0 > . \quad (3)$$

Parameters of the best approximation of distribution of the experimental values of Γ_n^0 in actinides for variants of their one ($K = 1$) or, maximum, four ($K = 4$) distributions with different M and D are compared in [9] between themselves or with approximation of the distorted by given registration threshold pure model random values. This analysis brings to the conclusion that at present it is inadmissibly to exclude a possibility of existence of superposition of several differing by parameters b and σ width distributions in every nucleus. Although unambiguous conclusion about its presence cannot be made on the basis of the modern experimental data on the resonance neutron widths. Therefore, the mean spacing between resonances in actinides is determined below in different ($K = 2$ and $K = 4$) variants. The suggested in [9] possibility to estimate the most probable number of omitted resonances in any experiment calls no doubts if only the functional dependence of their part ΔS_{th} from the total number S was set on the grounds of some data for concrete intervals of resonance energies. Then the parameters of unknown distributions are determined by equation:

$$\chi^2 = (S - (\psi(A, b, \sigma) - \Delta\psi_{th}))^2 \quad (4)$$

Here $\psi(A, b, \sigma) = \int X * \Gamma(X)dX$ for gamma-function Γ with variable X. The value $\Delta\psi_{th}$ is determined only by difference $N_t - N_{exp}$ for the varied expected resonance N_t number in interval δE and the obtained experimentally N_{exp}. The number of these intervals practically was varied from 5 to 20 in dependence on quantity of experimental values of widths. Moreover, negative values $N_t - N_{exp}$ in all cases were changed by zero.

The calculated and experimental cumulative sums in this equation have differing values of variables: function S was obtained under assumption that the unknown mean value of neutron width corresponds to $\sum N_{exp}$, but the mean neutron width for the calculated value is determined by sum $\sum N_t$. Therefore, calculation of χ^2 is carried out after corresponding change in variable X for difference $\psi - \Delta\psi_{th}$.

The serious enough problem is setting of dispersion of cumulative sum for arbitrary value X. Methodically this problem has simple solution: there are generated large sets of cumulative sums of squares of normally distributed numbers with given b and σ values for each "partial" function number K and for them by means of usual relations of mathematical statistics is determined function $\sigma = f(X)$ for each magnitude of variable X. But, in practice, this procedure requires unacceptable expenditures of computer time. That is why, possible change of the χ^2 value for different densities of neutron resonances for realistic values of dispersions of cumulative sums was performed only for ^{232}Th, ^{233,235}U and ^{239}Pu (only in approach of validity of the Porter-Thomas distribution).

The difference of principle between the results of this approximation and the data given below was not revealed.

Function (4) has not real minimum and in this variant of analysis of distributions of reduced neutron widths. Comparison between the calculated and experimental cumulative sums shows that some small difference of χ^2 for tested N_t values is mainly caused by strong fluctuations of cumulative sums in region of the largest X values.

Naturally, function $\Delta\psi$ can take into account and other factors distorting experimental distribution of widths. This accounting can be performed in frameworks of both some model approaches and concrete experimental data. Of course, function $\Delta\psi$ cannot be set unambiguously for the majority of factors which distort the neutron widths distributions.

The desired $D = \sum \delta E / \sum N_t$ value corresponds to minimum of χ^2 for varied values D. Fluctuations of different strength in the found function $\chi^2 = f(D)$ are connected with ambiguity of the best fit in the region of the large X values or change in parameters for elements of the tested superposition at $K > 1$. In particular, at change of D in case $K = 2$, for example, the smaller values of χ^2 can be really realized not for two, but in fact – three distributions: sum of widths distributions for both spin values of resonances and additional distribution of widths corresponding to the largest values of Γ_n^0 and parameter $b >> 1$.

The example of concrete dependence of N_{exp} is shown in Fig. 1. The parameter of

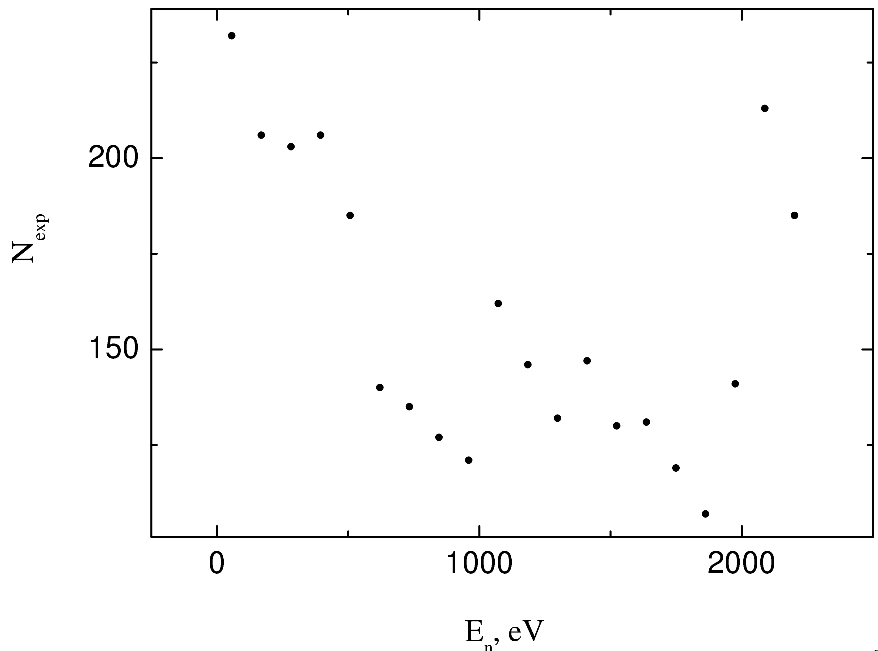

Fig. 1. Experimental number of resonances in interval $\delta E = 113$ eV for ^{235}U.

analysis (4) for this nucleus was tested for interval $110 \leq N_t \leq 5000$.

Comparison of experimental cumulative sum of widths in ^{235}U corresponding to different expected density of neutron resonances for $D = 0.1$ and $D = 0.7$ eV with the best approximation by expression (4) is presented in Fig. 2.

In this nucleus, as in all investigated here nuclei, is observed typical result: for $D \geq 0.1-0.2$ (odd) or $D \geq 1-2$ eV (even-even targets) is achieved the best and practically the same degree of correspondence of the experiment and model approximation. Any values of b and σ at noticeably smaller values D cannot give small χ^2 by use of superposition of both two and four different distributions. However, the values χ^2 for $K = 2$, respectively, increase with respect to $K = 4$. Sometimes – very essentially.

Besides, it should be taken into account that the practical search of parameters b and σ, which guarantees minimum of χ^2 in the used method of approximation cannot secure the best approximation of the experimental data in arbitrary variant of calculation. Only the repeated variation of initial values and ways of random processes can provide the sufficient for practical applications precision of determination of the lowest possible χ^2 value.

The obtained by us distributions $\chi^2 = f(D)$ for different variants of approximation of the experimental data for nuclei from the mass region $231 \leq A \leq 243$ are given in figures 3-4.

Estimated values of widths were taken from library ENDF/B-VII [10]. In order to compensate “omitted” resonances in ^{232}Th and ^{238}U, the authors of the neutron resonance evaluation included for these nuclei in the library data the resonances whose random widths are less than registration threshold. Naturally, presented here analysis of such

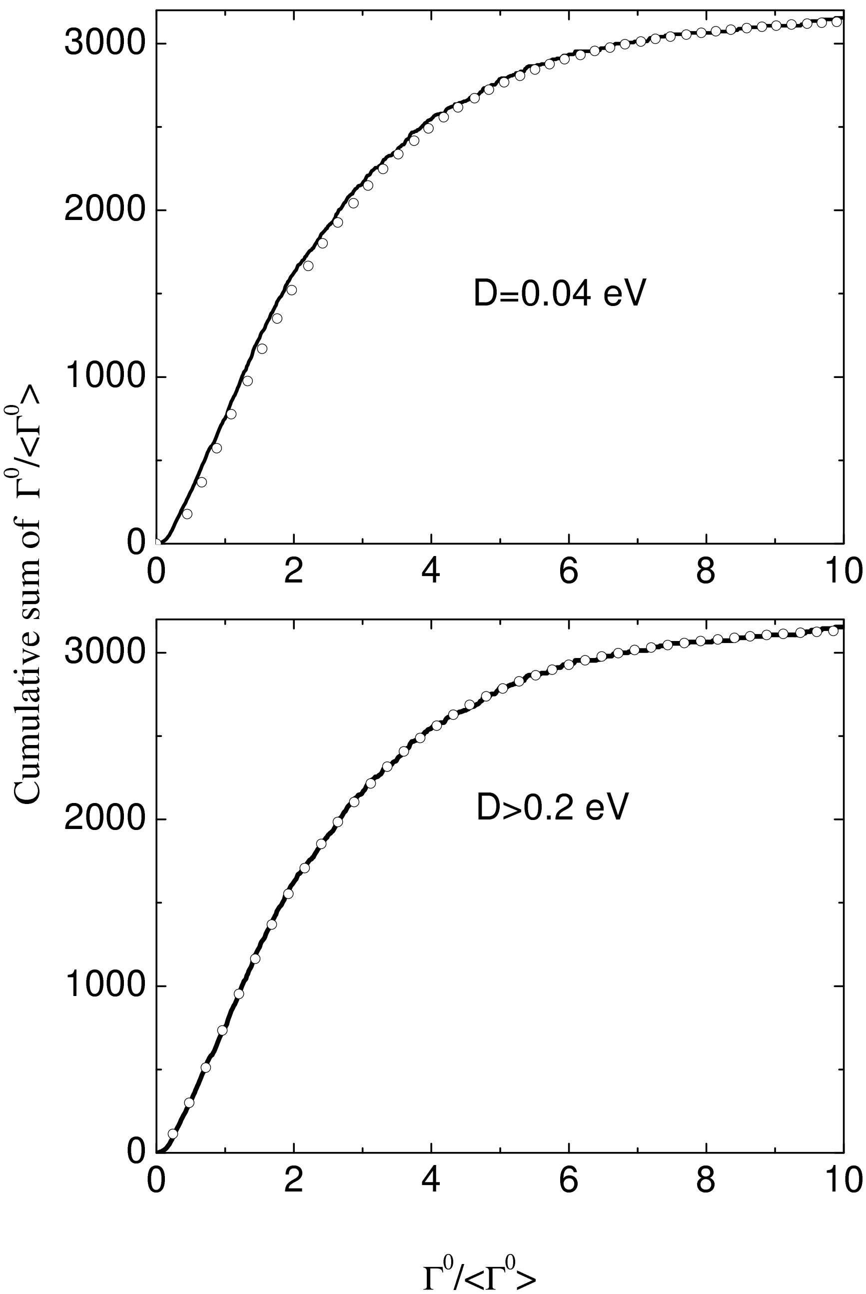

Fig. 2. Typical forms of the best approximations of cumulative sums for the experimental data on the reduced neutron widths. As an example, there are presented the data for $K = 4$ ^{235}U in region of strong increase of χ^2 and region of its practically constant value.

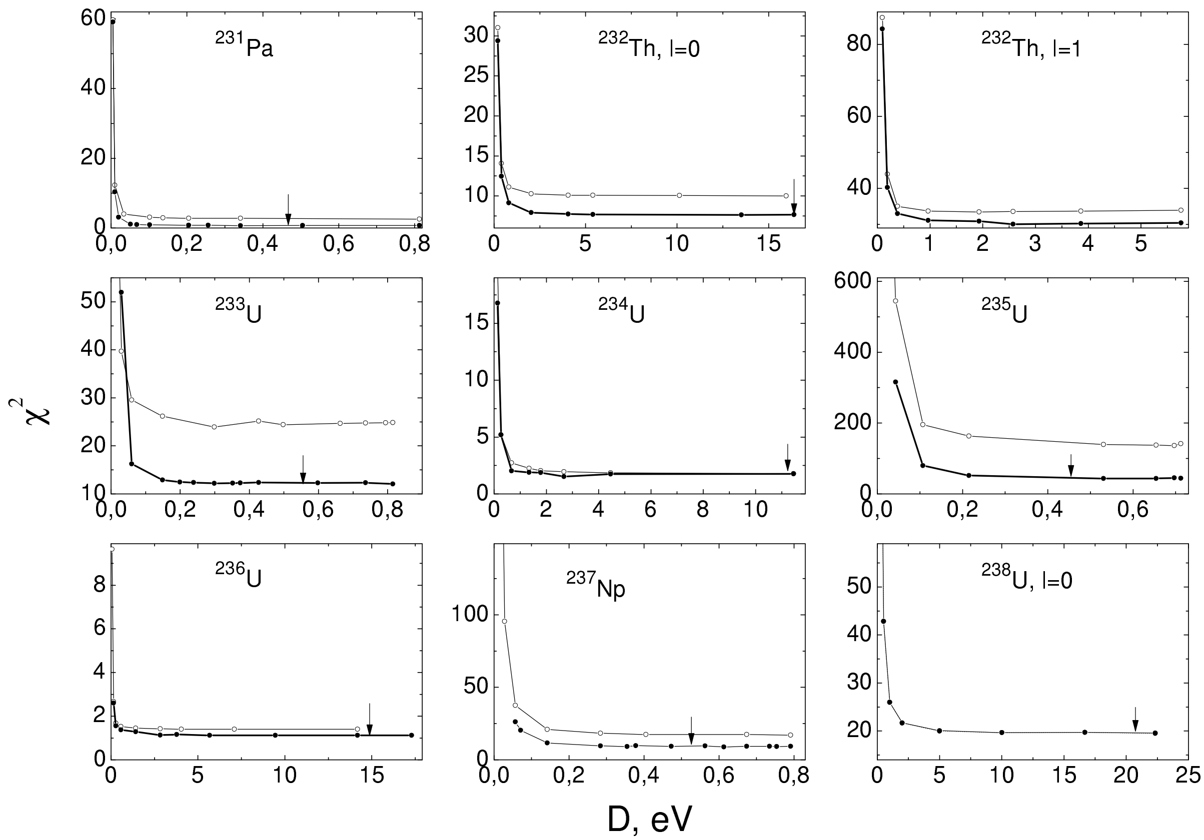

Fig. 3. The χ^2 value for the tested D parameter for the nuclei with mass $231 \leq A \leq 238$. The experimental sum of widths is approximated by two (open) or four (full circles) distributions with corresponding magnitudes of variable (2). The arrows correspond to D values from [11] or [10].

mixture may gives somewhat distorted information on density of neutron resonances and is added below, most probably, for demonstration of potential of the suggested method. Results of fitting of the D value, as it is seen from the data presented in figures 3-4 for each nucleus, depend on model notions. In practice, one can conclude that:

(a) the analysis gives wide spectrum of possible D values corresponding to either practically constant χ^2 value or – weakly fluctuating function of this parameter;

(b) weak local minima of χ^2 are caused by bad stipulation of approximation process for variant $K > 1$ distributions.

In both cases the number of fitted parameters is many times less than the number of analyzed resonances. Therefore, the data for four distributions can be adopted as the most probable ones.

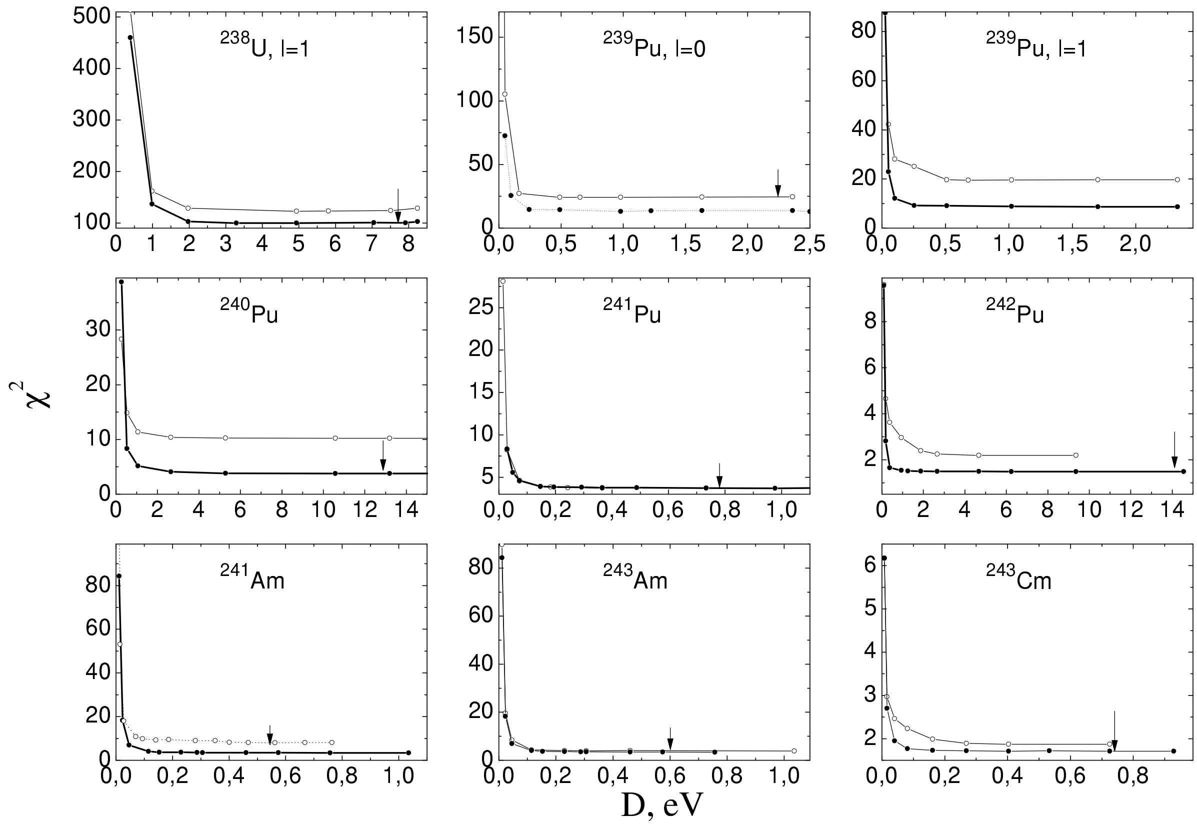

Fig. 4. The same, as in Fig. 3, for nuclei with mass $238 \leq A \leq 243$.

3 Some items of fundamental problem of determination neutron resonance widths distribution parameters

The most important result obtained in frameworks of described analysis of the experimental data on values of Γ_n^0 or Γ_n^1 – the mean Γ_n and ρ values are at present determined with on principle unknown systematical error. Really this result is expected: parameters of any process under study cannot be found even from mathematically strong extrapolation (or interpolation) of corresponding data (in given case – for the studied regions of nucleus excitation energy). The unexpected point was the found here possibility that the mean value of widths can be much less than registration threshold of experiment .

In original paper [3] is stated without any proof that: "As a consequence of experimental limitations, levels with small widths may escape detection, and also there may be only few of them...". Authors bring as an example for $X = 0.01$ the estimation of deviation in 9% between the average over measured widths and the expected one's average over the total distribution. These statements are quite true in case of small part of widths which are less than the threshold value. And they are absolutely mistaken – in case when the main part of neutron widths lie below registration threshold of experiment. Belonging of the tested set to one of these extreme (as and intermediate) cases is determined by value of $<\Gamma_n^0>$. In turn, it can be obtained only on the basis of necessary amount of additional experimental information.

Accordingly, all the published estimates of density of neutron resonances contain unknown systematical error. In the best case it is enough (for practical aims, for example) small; in the worse – changes the values of Γ_n and ρ by many times. The errors of parameters under consideration anticorrelate with each other. Accordingly, at calculation of, for example, averaged neutron-interaction cross-sections, their uncertainties can be negligibly small even for large $\delta\Gamma_n$ and $\delta\rho$. However, for understanding of occurring in nucleus processes of interaction and transition between Bose and Fermi systems and determining them properties of nuclear matter, the achieved precision for determination of level density can be insufficient.

Presentation of experimental data in form of cumulative sums of Γ_n^0 chosen for analysis has the lowest dependence on error of determination of $<\Gamma_n^0>$. Therefore, the result obtained here could not be determined earlier in the simplest analysis methods of distributions of $<\Gamma_n^0>$.

As a consequence, **any method for determination of mean parameters of neutron widths distributions can give only some their probabilistic values**.

4 Conclusion

The main result of the neutron widths distribution analysis: the D and S values can be determined only with unknown systematical uncertainty whose magnitude is determined now by unknown precision of the Porter-Thomas distribution correspondence to concrete experimental mean $<\Gamma_n^0>$ widths.

1. The suggested approximation of the total set of all the existing data on widths of neutron resonances does not allow one to find unambiguously determined absolute minimum of χ^2 for the unique value of D.

2. The use for this aim of superposition of several distributions with the different average and dispersion allows one to obtain the lowest value of χ^2, first of all, for the experimental data with number of widths exceeding ~ 100.

3. The analysis performed shows that the probability of correspondence of the distribution Γ_n^0 to the simple functional dependence in nuclei of different mass is less than that for the set of noticeably differing functions. Therefore, any quantitative determination of parameters of their distribution should be performed by comparison of two or more different model notions in maximum set of nuclei.

4. The obtaining of the more unambiguous conclusions with respect to the problem considered here requires very significant increase of sets of resonances with the experimentally determined values Γ_n^0 (Γ_n^1) at their correspondingly decreased distortions.

5. The increase of precision for determination of the mean parameters of the neutron

width distributions requires, most probably, considerable making more precise of model notions [3]. First of all of degree of influence of structure of the nuclear excited levels on level density and probability of emission of the nuclear reaction products in wide diapason of their energy. In particular – in region of neutron resonances.

6. Selection of neutron resonances by orbital momentum must be performed in common – by minimum sum value of χ^2 for obtained distributions with $l = 0$ and $l = 1$, for example.

References

[1] V.G. Soloviev, Sov. Phys. Part. Nuc. **3** (1972) 390.

[2] L.A. Malov, V.G. Solov'ev, Yad. Phys., **26(4)** (1977) 729.

[3] C.F. Porter, R.G. Thomas, Phys. Rev. **104** (1956) 483.

[4] H. Derrien, L.C. Leal, N.M. Larson, Nucl. Sci. Eng., 160, 149 (2008).

[5] A.M. Sukhovoj, V.A. Khitrov, Preprint No. E3-2005-196, JINR (Dubna, 2005).

[6] A. M. Sukhovoj, V. A. Khitrov, Physics of Paricl. and Nuclei, **37(6)** (2006) 899.

[7] A.M. Sukhovoj, V.A. Khitrov, Phys. Particl. and Nuclei, **36(4)** (2005) 359.

[8] A.M. Sukhovoj, V.A. Khitrov, XVIII International Seminar on Interaction of Neutrons with Nuclei, Dubna, May 2010, E3-2011-26, Dubna, 2011, p. 199.

[9] A.M. Sukhovoj, V.A. Khitrov, XVIII International Seminar on Interaction of Neutrons with Nuclei, Dubna, May 2010, E3-2011-26, Dubna, 2011, p. 208.

[10] http://www-nds.iaea.org.

[11] Reference Input Parameter Library RIPL-2. Handbook for calculations of nuclear reaction data. IAEA-TECDOC, 2002.

[12] S.F. Mughabghab, Neutron Cross Sections BNL-325. V. 1. Parts B, edited by Mughabhab S. F., Divideenam M., Holden N.E., N.Y. Academic Press, (1984)