Interaction of waves with ordinary and birefringent media moving with acceleration

A.I.Frank[1,] , P.Geltenbort[2], M.Jentschel[2], G.V.Kulin[1], D.V.Kustov[1,3] and A.N.Strepetov[4]

[1]*Frank Laboratory of Neutron Physics, Joint Institute for Nuclear Research, Dubna, Russia*
[2]*Institut LaueLangevin, Grenoble, France*
[3]*Institute for Nuclear Research, Kiev, Ukraine*
[4]*National Research Centre "Kurchatov Institute", Moscow, Russia*

Abstract

Already at the end of the last century theory predicted that the wave number and frequency of any wave will change when passing through an accelerating refractive medium. The effect was calculated both for electromagnetic and neutron waves. As a refractive index may be introduced for waves of any nature one can speak about a very general Accelerating Medium Effect. As far as we know this effect has not yet been observed for light. Here we report on a neutron-optics experiments performed with ultra-cold neutrons where this effect has been demonstrated for the first time ever. The maximum energy transform in the experiment was ± (2÷6) $\times 10^{-10}$ eV, which agrees with theory within less than 10%. Possibilities for future investigations of the Accelerating Medium effect will be discussed.

1. Introduction

The investigations of light propagation through a moving matter are continuing, starting from the historical paper of Fizeau [1] and up to the nowadays. In the 1970's the similar line of investigation has appeared also in the neutron optics [2-9]. By analogy with the light optics, the neutron experiments with moving matter are called Fizeau neutron experiments. Some results of the light and neutron optical investigations were reviewed and compared in [10]. In most of the mentioned papers, authors were mainly interested in the phase shift of a wave transmitted through a moving sample. Less attention was paid to the question of wave frequency in matter.

However, if a layer of some matter and its boundaries moving as a single whole, the wave frequency in matter measured in the laboratory frame will be different from that in vacuum. This is the case both in light [11] and neutron optics [2,12], where the frequency shift is caused by the Doppler effect and is determined by the following relation $\Delta\omega = (n-1)k_0 v$, where n is the index of refraction in the moving coordinate frame with matter at rest, k_0 is the wave number of the incoming wave, and v is the speed of the moving matter.

In the case of uniform motion of a material layer the Doppler shift resulting from passage of the wave through two boundaries are equal in magnitude but opposite in sign. The total effect is zero and the frequency of the wave transmitted through the moving matter does not change. For a long time the role of uniform velocity has been underestimated, although it is a decisive one. In the case of arbitrary motion frequency shifts on the sample boundaries do not cancel each other and the frequency of the transmitted wave is not equal to the incoming one. This was first shown by Tanaka [13] who solved the problem of electromagnetic wave transmission through a linearly accelerated dielectric matter on the basis of a covariant generalization of the Maxwell equations. As far as we know, the corresponding light optics experiment has not been carried out till now, although the possibility of doing that has been discussed [14].

The problem of neutron transmission through a layer of matter moving with linear acceleration was considered by Kowalski [15] in the context of a new type of experiment to verify the equivalence principle. The author comes to the conclusion that the energy of neutrons transmitted through such a layer must change. Later, the same result was obtained by Nosov and Frank [16] who calculated the velocity of the neutron transmitted through the accelerated boundaries of the sample.

The first brief note of experimental observation of neutron energy change on transmission through accelerated matter can be found in [17]. A more detailed investigation of the accelerated medium effect (AME) is given in [18]. Recently, acceleration and slowing down of neutrons on transmission through an oscillating sample was observed by a peculiar time-of-flight method [19]. The detected energy change was in a good agreement with the theory.

In the next sections we shall present rather briefly the main results of the experiments [18,19]. Then we discuss some new aspects of the neutron AME for the birefringent media.

2. Observation of the accelerating matter effect in experiments with ultracold neutrons.

2.1 Spectroscopy with UCN

The equation determining energy change due to AME was obtained by the different methods in [15,16] in the form

$$\Delta E \cong mwd\left(\frac{1}{n}-1\right) \tag{1}$$

Where m is the neutron mass, d – thickness of the sample, n – refraction index and w – sample acceleration. As shown in [18] eq. (1) can be obtained with good approximation from the equivalence principle without any detailed calculation. It was tested with reasonable accuracy in spectroscopic experiment with ultracold neutrons (UCN) [18].

The change of energy by transmission through an accelerating sample was firstly measured several years ago in experiments with ultracold neutrons. Neutrons passed through a silicon sample, which oscillates. Accordingly the energy change of the quasi-monochromatic neutrons is given as

$$\Delta E \cong -mA\Omega^2 d\frac{1-n}{n}\sin\Omega t, \qquad \Omega << \frac{v_i}{d}, \tag{2}$$

where A and Ω – denote the oscillation amplitude and frequency of the sample and v_i – their velocity inside the sample material. The layout of the experimental setup, a modified spectrometer of [15,16], is shown in Figure 1.

Ultra-cold neutrons enter the top part of the spectrometer horizontally before falling through an annular corridor. At the lower end of the corridor, a monochromator (1) is placed. It is a neutron interference filter and acts as a kind of Fabry-Perrot interferometer (FPI). It transmits only UCN with a narrow spectrum of vertical velocities. The sample (2), silicon plates of 0.6 and 1.85 mm thickness respectively, are located just below the monochromator. It can be harmonically moved up and down by means of a special driver (3). The sample was oscillating with 40 or 60 Hz. The maximal tunable acceleration of the sample was 90m/s^2.

Passing the monochromator and sample, neutrons arrive at a vertical mirror neutron guide, where the second FPI, analyzer *4*, is located and whose position can be varied in height. A scintillation detector for UCNs *5* is placed below the analyzer. The transmission line maxima of the monochromator and analyzer correspond to 107 and 127neV, respectively. The dependence of the count rate on the distance between the filters is qualitatively presented in Figure 2. The sign-alternating change in the neutron energy caused by passage through a sample moving with variable acceleration leads to the periodic variation in the count rate, as shown by the dashed straight lines in Figure 2.

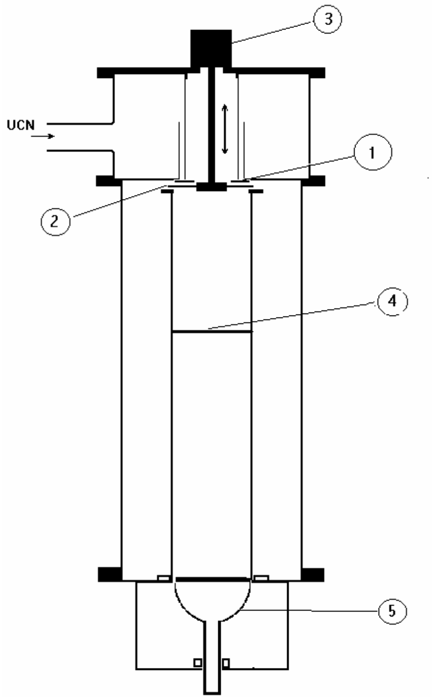

Figure 1. Layout of the experimental setup

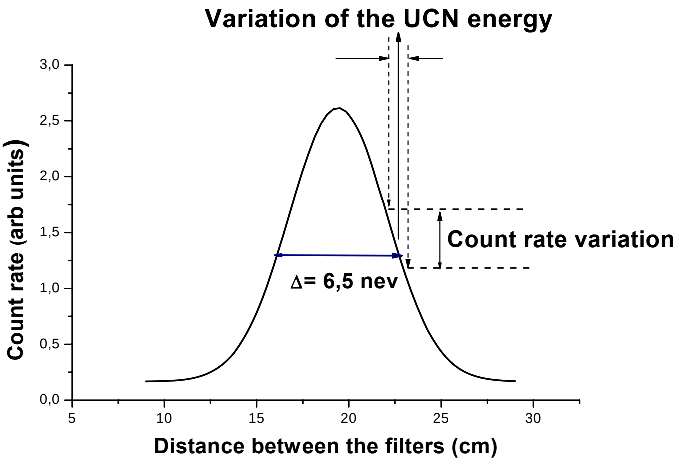

Figure 2. Count rate vs. the distance between the filters (scanning curve) and the detection principle of periodical variation of energy

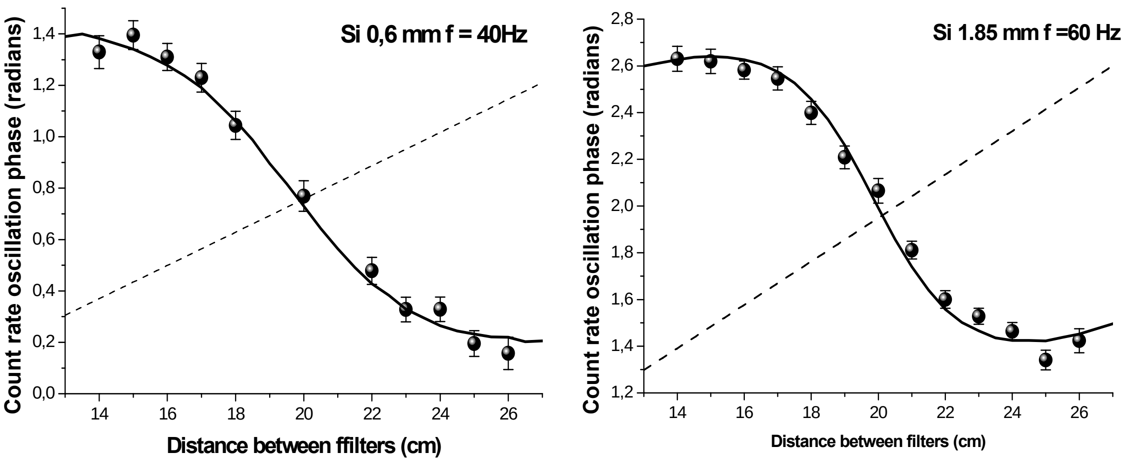

Figure 3. Measured phase of the counting-rate oscillation versus the analyzer position. Solid (red) curve – Monte-Carlo simulation. Dashed (blue) line – theoretical prediction for the phase in the absence of the Accelerating medium effect

The maximum change in the energy of UCNs, which is determined by Eq. (6), reaches approximately 0.6neV.

The phase and amplitude of the count rate modulation were measured in the experiment for various positions of the analyzer filter. The time dependence of the count rate was determined in a time interval equal to the oscillation period. The origin of the scale was specified by a generator controlling the sample motion. The data were normalized to the averaged count rate and fitted to the function $f(t)=1+B\sin(\Omega t-\varphi)$. The amplitude B and phase φ of the count rate oscillation were determined by such a way in each value of the position of the analyzer. Some results from the phase measurements are shown in Figure 3.

Monte Carlo simulation of the oscillation phases were repeated using the relation $\Delta E \cong -KmA\Omega^2 L\left[(1-n)/n\right]\sin\Omega t$ which differs from equation (2) by the correction factor K. For the latter one a value of $K=0.94\pm0.06$ was found. Thus, the results of the measurements of the counting-rate oscillation phase are in reasonable agreement with the existing theory of the accelerating medium effect.

2.2. The Accelerating Medium Effect and time focusing of neutrons.

2.2.1. Experimental Principle

Recently a new type of experiment for the observation of the AME was carried out [19]. Here a plate, vibrating in space, was used as periodic modulator to change the velocity of ultra-cold neutrons. The main idea of this experiment is rather similar to experiments on time focusing [20] and its main principle is illustrated in Figure 4. Monochromatic neutrons are transmitted through a modulator – acting as a time lens. The neutron velocity was changed periodically such that in the ideal case all neutrons arrive simultaneously in the detector L. The lens is working in a cyclic regime and time focuses those neutrons passing through it in one cycle. In reference [20] an aperiodic diffraction grating moving across the neutron beam was used as a lens. In the current experiment we used a plate of silicon oscillating along the propagation direction of the ultra-cold neutron beam. Moving with periodic acceleration it was periodically accelerating or slowing down the neutrons due to the AME. Here the momentum transfer to the neutrons was insufficient for an efficient focusing and the focal plane was significantly away (to the right side) from the plane L. However, in the detection plane there occurs a substantial concentration in time of flight of the incoming neutrons. This leads to a time modulation of the count rate at the detector. It can be shown that in this case the time dependence of the detector count rate is determined by the derivative of the modulation function Δv(t). The change of velocity by passing the oscillating plate is according to equation (2) given by $\Delta \mathrm{v}_n = -A\Omega^2 \frac{d}{\mathrm{v}_0}\left(\frac{1-n}{n}\right)\sin\Omega t$ and therefore the weak focusing leads to the following time dependence of the count rate

$$N_a(t) = N_0 + C_a A\Omega^3 \left(\frac{1-n}{n}\right)\cos\Omega(t+\tau) \qquad (3)$$

where N_0 – is the mean count rate, τ - is the average time of flight and C_a – is a constant coefficient. Obviously the harmonic movement of the sample leads to a periodic variation of the neutron velocity with respect to the sample. As the dependence of all absorbing processes in the case of ultra-cold neutrons is proportional to 1/v the transmission of the sample is also changing accordingly. Therefore, additionally to the AME given by equation (8) there is another systematic effect described by the following equation $N_v(t) = N_0 + C_v A\Omega\cos(t+\tau)$.

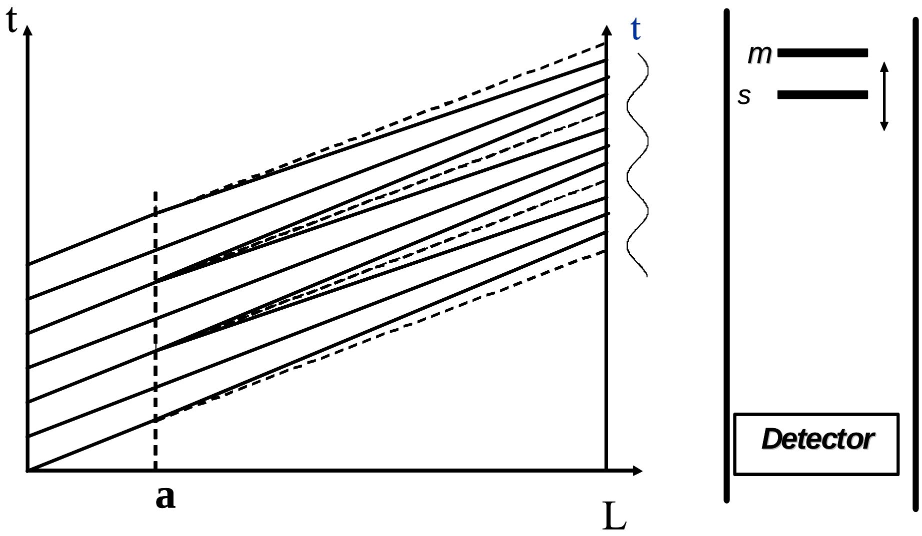

Figure 4.

A coordinate versus time diagram illustrating the idea of time focusing.

As both effects are synchronous, the harmonic modulation of the count rate is obtained by summing the amplitudes of both

$$\Delta N\left(\Omega\right) = A\Omega^2\left(C_a\Omega + \frac{C_v}{\Omega}\right) \tag{4}$$

Expression (4) is written such, that it underlines the main principle of the given experiment: the measurement is carried out such that the quantity $A\Omega^2$ stays constant and the AME is growing proportionally while the systematic velocity effect if inversely proportional to the frequency of modulation.

The systematic velocity effect was encountered in former experiments [17,18]. In these measurements it was already found that if the monochromatisation happens after transmission through the sample the AME is excluded. All experimentally observed modulations are due to the systematic velocity effect. This finding has been explored for the current measurement.

The experimental strategy consisted in the measurement of the modulation amplitude of the ultra-cold neutron count rate transmitted through an oscillating sample for a large set of frequencies Ω and fixed values of $A\Omega^2$. The measurement was carried out in two geometries. In the first case the monochromatic neutrons were transmitted through the sample and the amplitude modulation was given by equation (4). In the second case the neutrons were transmitted through the oscillating sample and only after through the mononchromator. In the second case the modulation amplitude is defined by the systematic velocity effect only. The frequency dependence of the difference of both effects is defined exclusively by the AME and should depend linearly on the frequency of modulation.

2.2.2. Experimental realisation and results

For the experiment the same spectrometer as in reference [18] was used. A schematic illustration is given in Figure 1. The only difference consists in the absence of the analyzer 4. As in earlier works we used a Fabri-Perot interferometer as monochromator. It transmits a single wave length at about *107 neV* with a relative width of $\Delta E/E \approx 0.04$. As a sample we used a wafer with *1.85 mm* thickness, which was put into oscillation by the same driving stage as in [18]. The time depending acceleration was permanently measured by a piezoelectric sensor, the sinusoidal signal of which was also used for the stabilization of the amplitude of the driving stage. This allowed to stabilize the quantity $w_{max} = A\Omega^2$ on the level of *2%*. The measurement was carried out for two values w_{max} = 57 m/s^2 and w_{max} = 72 m/s^2 and for a frequency range of f = 20 ÷100Hz. The obtained results are shown in Figure 5.

The results show without any doubt the presence of weak time focusing, i.e. the acceleration/slowing down of neutrons when passing the oscillating sample. The modulation amplitude in the geometry excluding the AME effect (lower blue points) is substantially lower than the amplitude of modulation sensitive to both effects (upper red points). Further, it can be seen that the difference between both effects is growing with an increase of the modulation frequency.

Unfortunately it is difficult to directly compare these data with calculations. The reason is that the modulation amplitude is also depending on the dispersion of the time of flight, i.e. on the width of the spectrum transmitted by the monochromator. Additionally it turned out that the background of the spectrometer is slightly changing, when the monochromator is changed from one position into another. Therefore a number of calibration measurements were added, in which the modulation of the ultra-cold neutron flux was realized via a mechanical chopper. In these measurements the non moving silicon sample was also present and the monochromator was changed between different positions. Such calibrations allowed to normalize the data of Figure 6. The difference of the two curves should be a straight line, corresponding to the first term of equation (4). These results are shown in Figure 6 and it is possible to compare them directly to theoretical calculations. The main parameter here is the inclination angle of the straight line. The calculations were done by assuming several origins of the background. The dispersion of the calculated background values was included into a systematic error.

The error of the linear fit of the data in Figure 6 was considered to be the statistical error. From the obtained results it was possible to estimate the agreement of the calculated velocity change when neutrons are transmitted through an oscillating refractive sample.

For the factor K = $\Delta v_{exp}/\Delta v_{exp}$ and values of $w_{max} = A\Omega^2$ as mentioned above the agreement of experiment and calculation was: K_1= 0.95 ± 0.10_{stat} ± 0.05_{syst} and K_2 = 0.95 ± 0.05_{stat} ± 0.05_{sys}

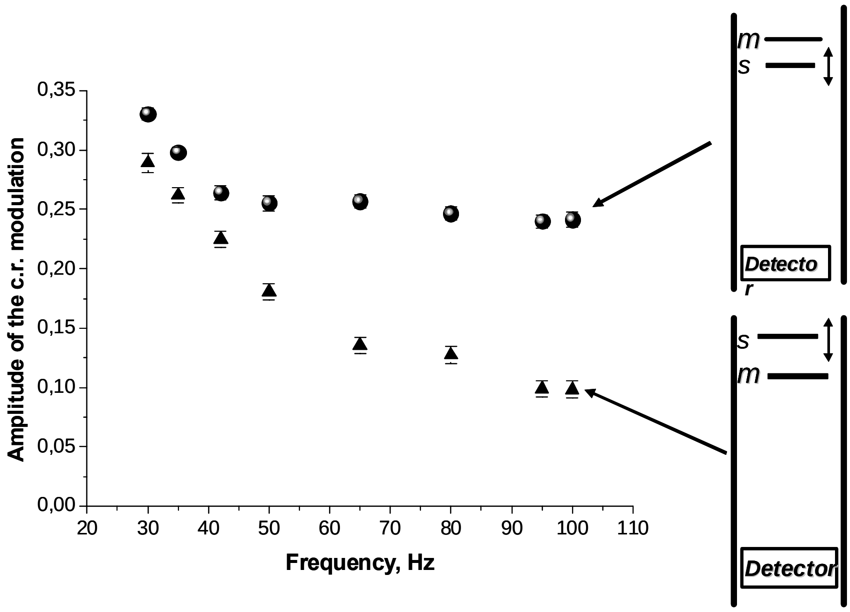

Figure 5. The count rate oscillation amplitude as a function of frequency for a fixed value of $w_{max} = A\Omega^2$ =72m/s^2. The measurements were done in two geometries, which symbolically are indicated on the right. Here M – is the monochromator and S – is the sample.

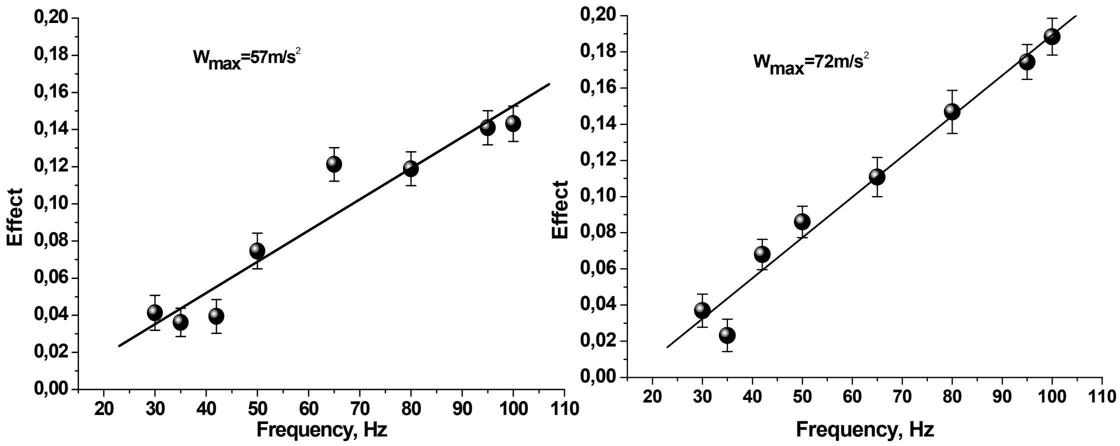

Figure 6. The difference between normalized amplitudes of the count rate oscillation versus frequency. The value $w_{max} = A\Omega^2$ was fixed. On the left $w_{max} = 57m/s^2$, on the right $w_{max} = 72m/s^2$.

3. The Accelerating Matter Effect in the case of double refraction

In the present works concerning the AME the polarization of waves was not considered so far. In order to partially fill this gap we will consider briefly the case of matter with double refraction, which is characterized by two indices of refraction $n_{\pm}$, according to two polarizations of the incoming wave.

In neutron optics the quantities $n_{\pm}$ correspond to two different projection of the neutron spin on a physical axis. Accordingly we will rewrite equation (1) as

$$\Delta E_{\pm} \cong mwd\left(1-n_{\pm}\right)/n_{\pm}\,, \quad \Delta\omega_{\pm} = \Delta E_{\pm}/\hbar\,.$$

After transmission through an accelerated birefringent sample the two spin components of the neutron wave function differ by a frequency and form a non-stationary superposition. In the case of an arbitrary polarization of the original wave function $\Psi_0(x,t)$ the final state will have the form

$$\Psi(x,t) = A_{+}\exp\left[-i\left(\Delta k_{+}x + \Delta\omega_{+}t + \chi_{+}\right)\right]\left|\uparrow\right\rangle + A_{-}\exp\left[-i\left(\Delta k_{-}x + \Delta\omega_{-}t + \chi_{-}\right)\right]\left|\downarrow\right\rangle \qquad (5)$$

where $\Delta k_{\pm} = \left(m/\hbar^2\right)\left(\Delta E_{\pm}/k_0\right)$ and t is the time counting from the moment when the wave escapes the matter. The constant phase angles, $\chi_{\pm}$, that are irrelevant to what follows, determine spin directions on transmission through a moving sample [21]. The wave function in equation (5) describes the state with spin precession. The precession angle is obtained from the difference between the phase angles of two spin components:

$$\varphi(x,t) = \left(\Delta k_{+} - \Delta k_{-}\right)x - \left(\Delta\omega_{+} - \Delta\omega_{-}\right)t + \chi_{+} - \chi_{+}$$

Assuming here the simplifying condition $\Delta E_{\pm}/E \ll 1$ it is possible to write that $\left(\Delta k_{+} - \Delta k_{-}\right)x = \left(\Delta\omega_{+} - \Delta\omega_{-}\right)x/v$, from which follows that in the reference system moving with the velocity of the neutron the direction of the spin vector is unchanged. However, in a fixed point of observation, *x*=*L*, the spin direction changes periodically in time with the frequency $\Omega = \Delta\omega_{+} - \Delta\omega_{-}$. This periodic change of the polarization direction can be measured. The beat frequency Ω and respective energy transfers $\hbar\Omega$ can be quite small.

In neutron optics there can be several physical reasons for double refraction. First of all one should mention the rather simple case of interaction of the magnetic momentum of the neutron μ with a magnetic field B. Obviously any space in which the magnetic field has a nonzero value is acting with two refraction indexes $n_{\pm} = \left(1 \mp \mu B/E\right)^{1/2}$. If this space moves together with its field boundaries it will generate according to what was mentioned above a non-stationary state (10). Sample material, put into a magnetic field can also act as double refracting material for neutron waves [22,23]. This will be caused by different neutron wave numbers due to the presence of the magnetic field and the dispersion of the material itself. An accelerated motion of the sample in a constant magnetic field will also lead to states with different frequencies for the different spin components of the wave function. Double refraction might also occur in the absence of a magnetic field. First of all one might focus here on nuclear pseudomagnetism, which takes place when a neutron wave is propagating in matter with polarized nuclei. [24,25]. Due to the spin dependence of the nuclear interaction the coherent scattering length $b_{\pm}$ is different for the two values of the total spin. As a result, the medium has two refractive indices.

Finally, double refraction might be caused by parity violation in neutron-nucleon interaction. The forward scatter length and consequently the refractive index will be depending on the neutron spin orientation [26,27]

4. Conclusion

It was shown, that the Accelerating Medium Effect is closely related to the equivalence principle (EP). Consequently, the equations which describe the frequency change after their passage through accelerated refractive samples, may be derived not only from first principles but from the EP too. This can be interpreted as additional evidence for the general nature of this effect which exists for waves and particles of different nature. Two experiments detecting the AME in neutron optics were described. The measured energy variations were equal or less than 5×10^{-10} eV while the velocities changed by about 1 cm/s. The measurements agree with theoretical predictions better than 10%. New possibilities for the detection and for applications of the AME may be possible by the use of birefringent material.

This work is partly supported by the Russian Foundation for Basic research (project 11-02-00271).

References

1. Fizeau H. L., Acad. Sci. Ser. B, 33 (1851) 349; Ann. Chim. Phys., 57 (1859) 385.
2. Horn M. A. and Zeilinger A., Neutron Interferometry, edited by U. Bonse and H. Rauch, Vol. 9 (Clarendon, Oxford) 1979, p. 350.
3. Klein A. G. et al., Phys. Rev. Lett., 46 (1981) 1551.
4. Horne M. A., Zeilinger A., Klein A. G. and Opat G. I., Phys. Rev. A, 28 (1983) 1.
5. Sears V. F., Phys. Rev. A, 32 (1985) 2524.
6. Arif M. et al., Phys. Rev. A, 31 (1985) 1203.
7. Bonse U. and Rump A., Phys. Rev. Lett., 56 (1986) 2411.
8. Arif M. et al., Phys. Rev. A, 39 (1989) 931.
9. Bonse U. and Rump A., Phys. Rev. A, 37 (1989) 1059.
10. Neutz R. J., Stedman G. E. and Bilger H. R., J. Opt. Soc. Am., 13 (1996) 2065.
11. Yeh J., J. Appl. Phys., 36 (1965) 3513.

12. Peres A., Am. J. Phys., 51 (1983) 947.
13. Tanaka K., Phys. Rev. A, 25 (1982) 385.
14. Neutze R. and Stedman G. E., Phys. Rev. A, 58 (1998) 82.
15. Kowalski F. V., Phys. Lett. A, 182 (1993) 335.
16. Nosov V. G. and Frank A. I., Yad. Fiz., 61 (1998) 686 [Phys. Atom. Nucl., 61 (1998) 613].
17. Frank A. I. et al., Pisma Zh. Eksp. Teor. Fiz., 84 (2006) 131 [JETP Lett., 84 (2006) 363]
18. Frank A. I. et al., Yad. Fiz., 71 (2008) 1686 [Phys. Atom. Nucl., 71 (2008) 1656].
19. Frank A. I. et al., Pisma Zh. Eksp. Teor. Fiz., 93 (2011) 403 [JETP Lett., 93 (2011) 361].
20. Frank A I, Geltenbort P, Kulin G V and Strepetov A N *JETP Letters* 78 **(**2003) 188
21. Littrell K. C., Werner S. A. and Allman B. E., J.Phys. Soc. Jap. Suppl., 65A (1996) 98.
22. Baryshevskii V G, Cherepitsa S V and Frank A I 1991 *Phys.Lett*. A **153** 299
23. Frank A I, Bondarenko I V, Kozlov A V, Høghøj P and Ehlers G 2001 *Physica B* **297** 307
24. Baryshevkii V G, Podgoretsky M I 1964 *JETP* **47** 1050
25. Abragam A, Bacchella G L, Glätti H, Meriel P, Pinot M, and Piesvaux J 1973 *Phys.Rev.Lett* **31** 776
26. Forte M, Heckel B R, Ramsey N F, Green K, Greene G L, Byrne J and Pendlebury J M 1980 *Phys.Rev.Lett*. **45** 2088
27. Sushkov O P and Flambaum V V 1982 Sov. Phys. Usp. **25** 1