MODIFIED MODEL OF NEUTRON RESONANCES WIDTH DISTRIBUTION. ESTIMATION OF THE QUALITY OF RESULTS AND INTERPRETATION OF AVIALABLE PHYSICS INFORMATION

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Abstract

In the work was performed analysis of parameters of approximation of 157 sets of resonances in the mass region of nuclei $35 \leq A \leq 249$. It was shown that the experimental values of widths can correspond with high probability to superposition of several independent neutron widths with their non-zero mean values and non-unit dispersion.

1. Introduction

Extraction of any nuclear-physics information is impossible, at least, at present without the use of model ideas. These ideas are formed on the grounds of the limited by volume and distorted by errors of experiment information. Moreover, usually there is realized the analysis algorithm which provides objective unambiguity of the obtained conclusions on determined parameters of a nucleus. As a consequence, determined with unknown error of a model.

One can bring as an obvious example the problem of determination of penetrability T for product emission of the used reaction and level density ρ of complicated nuclei in excitation region where the mean spacing between them $D=\rho^{-1}$ is less than resolution of spectrometer *FWHM*. The known methods for determination of this value [1] and [2] from one-step reactions (registration of reaction products without regime of coincidences) the use, for example, the not tested model notion on independence of the $T=f(E_{\rm prod})$ values on excitation energy of final nucleus. Evident mistakenness of this hypothesis was revealed [3,4] by analysis of the experimental data of the two-step reaction *(n,2γ)*. This fact could be obtained more earlier. For this the authors of corresponding experiments had to perform analysis of obtained data in comparison of its results by means of different nuclear models. Here – of two long ago known, but principally different models of level density [5,6].

There is rather considerable and the other aspect of this problem: connection between the measured spectra (cross-sections) and determined parameters of reaction under study is nonlinear, and systems of corresponding equations are badly stipulated or degenerated. I. e., the desired data, at least in some cases, cannot be unambiguous even in principle. However, the interval of their possible values, corresponding to maximum of the likelihood function, can be final [3,4].

2. The problems of the use of new method for analysis of the neutron width distributions

This situation is observed and by analysis of the data on the reduced neutron widths of resolved resonances in frameworks of the modified model of their distribution (several independent neutron amplitudes with different mean values and dispersions can correspond to the existing set of the experimental Γ_n^0 values). There were used in the corresponding method [7] the model ideas which are equivalent to the widest set of the fitted functions (the Porter-Thomas distribution [8] –their only particular case) and selection of the most suitable one is performed by the criterion χ^2. In this case, to the same minimum of χ^2 in variant of the best approximation usually correspond the functions with different values of the parameters. But only their superposition in any set from 157, presented in [7] results of analysis of the concrete Γ_n^0 values, was the only which reproduced cumulative sums of the experimental widths with a precision which is higher than that provided by model [8].

Just so was obtained the main conclusion of analysis [7] – neutron resonances of any nuclei with $A \geq 35$ for $0 \leq l \leq 3$, as a minimum, can have different structure of their wave functions. And number of probable groups of like by structure neutron resonances can achieve the value k=4. Unfortunately, at present this conclusion is not, as a minimum, absolute. As a consequence, it is necessary, first of all, to take into account and analyze a possibility that the interpretation of the experimental width distribution as a superposition of several identical functions with different values of parameters of their neutron amplitudes can be the result of random (first of all – nuclear) fluctuations of widths of resonances.

Although the number of the experimental values of neutron widths strongly exceeds the number of parameters of approximating curve (4 –for variant k=1, 13 – for k=4) for the main mass of nuclei by ≈80 and more units, the number of degrees of freedom decreases when X increases. This circumstance excludes any possibility of simple application of the criterions of mathematical statistics for estimation of probability of random appearance of small values of $R=\chi^2(k=4)/\chi^2(k=1)$. Just small volume of the data on the Γ_n^0 values does not allow one to make absolute conclusion on this account only from analysis of ratios χ^2 for competing notions.

Therefore, besides the analysis of the experimental data it is worth while in addition completely to model the algorithm of the method [7] for random values which correspond by their quantity to typical experimental sets. One variant must by parameters of distribution correspond to model [8] , the other - to typical values [7].

3. Analysis of results of approximation of experimental data in modified model

The statement on reliability of the main results of approximation [7] requires, first of all, to proof their independence on mass of nuclei and number of resonances in the approximated sets. This can be done only by means of analysis of ratios $R = \chi^2(k=4)/\chi^2(k=1)$ as unitary set of random values. The data in Fig. 1 do not reveal evident dependence of the R values for nuclei with different parity of nucleons and orbital momentums of neutrons. Its maximal scattering for the lowest numbers of resonances N_r in any of 157 data sets has, most probably, clearly statistical nature. The mean value of R for s–resonances varies from 0.34(16) to 0.42(22) for nuclei with different parity of nucleons , and from 0.30(16) to 0.21(15) for their p- and d-resonances. (Discrepancy of this scale for different l is, most probably, caused by imperfection of the methods for determination of the momentum values of concrete resonances.)

In Fig. 2 are given the ratios R of parameters of approximation, averaged over several intervals of nucleus mass A and numbers of resonances N_r. Evident absence of dependence on both parameters and their considerably less than unity experimental and model values for big enough N_r quite correspond to preliminary conclusion [7] on existence in arbitrary nucleus of superposition of neutron resonances with different structure.

Relatively small random fluctuations of form of the width distribution at truth of the Porter-Thomas distribution [8] can be expected for the sets of resonances with their number N_r≈400-500 and more. That is why, the approximate conservation of form of the analyzed cumulative sum of widths in different energy intervals of neutron resonances of large enough width would be an additional argument in favor of hypothesis on superposition of neutron resonances of different structure in their experimentally obtained set.

Really such analysis (although with insufficient data set) can be performed only for *s*- and *p*-resonances of ^{235}U and ^{238}U respectively. Although in compilation [9] and library ENDF/B-VII [10] are given the data on of spins of resonances, but at absence of the quantitative data on reliability of their determination it is preferably to use in the testing analysis the values $g\,\Gamma_n$. Both sets contain resonances with two possible spins. Therefore, the possibility of difference between the mean values of $g\,\Gamma_n$ can bring to superposition of two distributions with the expected and practically constant relation of their contributions in the total function at any neutron energies E_n, but, in principle, with different parameters of their neutron amplitudes. (The evaluated data of ^{238}U for *p*-resonances contain "fictitious resonances" of small width, introduced by authors [11] for reproduction of the capture cross sections of neurons. But they, probably, increase the ratio *R*.)

The energy interval of the studied resonances E_n for the nuclei under consideration equals 2.26 and 20.0 keV, respectively. Cumulative sums of $g\,\Gamma_n$ were obtained in two variants in the intervals of energy ΔE_n=0.45 and 4.0 keV for the data presented in Fig. 3 and ΔN_r=450 and 400 (Fig. 4) for ^{235}U and ^{238}U respectively. Approximation of these cumulative sums was performed completely by analogy with [7]. Id est., by singular distribution *(k=1)* with varied magnitudes of the mean value of neutron amplitude *b* and its dispersion σ. The variant with superposition of four such distributions was used for comparison of the obtained results. The obtained ratios $\chi^2(k=4)/\ \chi^2(k=1)$ for each interval are shown in Fig. 5, and approximated parameters *b* and σ – in Fig. 6.

As it is seen from Fig. 4, cumulative sums for ^{235}U change from interval to interval more strongly than for ^{238}U. In correspondence with the experimental data [7] and theoretical analysis [12], one can expect, from the one hand, noticeable change of structure of resonances in ^{235}U just inside of the accessible to the experiment by the time-of-flight method region of neutron energies. On the other hand, one cannot exclude and possibility of resulting influence of omission of *s*-resonances and increase of portion of the mistakenly identified *p*-resonances at increase of E_n.

The comparison of the values of ratio $\chi^2(k=4)/\ \chi^2(k=1)$ for different intervals of neutron energies with the values from [7] allows one to conclude that, with the high probability, the set of the experimental widths corresponds to superposition of several distributions, but it is not the result of random grouping of the widths at some their values. Also, the *b* and σ parameters undoubtedly change at change of E_n (as a mass of a nucleus), as it follows from theoretical analysis [12] by V.G. Soloviev and L.A. Malov of main principles of fragmentation of the complicated nuclear states. Making more precise reliability this conclusion or its refutation requires the data on some thousands of resonances for many nuclei with different parity of nucleons and from different diapasons of their masses.

In Fig. 7 are compared the best parameters *b* and σ of distribution of neutron amplitudes of all 157 nuclei in the variant of approximation *k*=1 with analogous values of the partial distribution, which gives the greatest contribution in approximation of the experimental cumulative sum. Noticeably lesser scattering of the latter is indirect confirmation of conclusion [7] on presence in any nucleus of levels with different structure and above the neutron binding energy.

As it can be seen from comparison of Fig. 7 and the data of Table 1, considerable fluctuations of parameters *b* and σ point to presence in the tested sets of the reduced neutron widths of noticeable systematical errors, as a minimum. And, as a maximum – on presence of evident deviations of these parameters from assumptions [8]. Therefore, the available data do not allow one to make the final choice between the variants $k=1$ and $k\geq 2$.

The results of approximation [3] of the experimental level density below the neutron binding energy, determined from the spectra of intensity of cascade gamma-transitions show that one can expect noticeable change in structure of neutron resonances at change of nucleus excitation energy by $\Delta E_{ex} \cong 1\text{-}2$ MeV. So, one can expect small enough variations of magnitudes of the parameters b and σ in the interval of neutron energy of about some tens keV or somewhat less. And the best parameters of width distribution in neighboring intervals of neutron energy differ only in limits of errors of approximation. As a result, there is the main basis of quantitative modeling of the approximation process. This was done for two possible variants of the initial model set. It is assumed that such set corresponds [8] to:
(a) the unique (k=1) partial distribution, or to
(b) superposition of several (k=4) partial distributions of neutron amplitudes.

Table 1. The averages of the best values of approximation parameters of variant (a).

parameter	B	b_1	b_2	b_3	b_4
model	0	-	-	-	-
N_r =150	-0.14(43)	-0.05(19)	1.60(46)	1.78(28)	1.58(52)
N_r =800	-0.02(11)	0.08(8)	2.07(42)	1.62(55)	1.88(46)
N_r =3000	0.03(6)	0.015(86)	0.73(55)	2.06(77)	1.49(60)
parameter	Σ	σ_1	σ_2	σ_3	σ_4
model	1	-	-	-	-
N_r =150	0.90(14)	0.19(4)	0.07(8)	0.05(4)	0.01(3)
N_r =800	0.88(19)	0.66(24)	0.58(34)	0.82(41)	0.66(38)
N_r =3000	0.90(29)	0.96(11)	0.42(23)	0.64(22)	0.66(36)
parameter	C	c_1	c_2	c_3	c_4
model	1	-	-	-	-
N_r =150	1	0.52(9)	0.26(8)	0.15(4)	0.07(2)
N_r =800	1	0.90(5)	0.07(4)	0.025(19)	0.004(3)
N_r =3000	1	0.79(14)	0.17(14)	0.03(3)	0.027(37)

Table 2. The averages of the best values of approximation parameters of variant (b).

parameter	B	b_1	b_2	b_3	b_4
model	-	0	1	1.4	1.7
N_r =100	-0.4(4)	-0.2(2)	1.5(3)	1.7(2)	2.1(5)
N_r =500	-0.2(1)	-0.2(2)	1.4(2)	1.4(2)	1.8(2)
N_r =3000	-0.3(1)	-0.2(1)	1.3(1)	1.5(2)	1.6(2)
parameter	Σ	σ_1	σ_2	σ_3	σ_4
model	-	0.4	0.3	0.2	0.1
N_r =100	0.45(13)	0.50(1)	0.28(16)	0.23(8)	0.04(4)
N_r =500	0.31(8)	0.50(1)	0.21(5)	0.24(10)	0.11(5)
N_r =3000	0.3(1)	0.50(1)	0.31(4)	0.21(3)	0.09(2)
parameter	C	c_1	c_2	c_3	c_4
model	-	0.45	0.25	0.2	0.1
N_r =100	1	0.54(8)	0.10(11)	0.17(11)	0.18(8)
N_r =500	1	0.49(10)	0.20(12)	0.08(7)	0.22(12)
N_r =3000	1	0.55(7)	0.10(9)	0.06(7)	0.29(4)

Accordingly, the quite random approximation parameters in the first variant will be C, b, σ with indexes 1-4, in the second – B and Σ. The analysis was performed for three sets of the random X values which cover the interval of number N_r of the included in analysis resonances.

It should be noted that the mean value $<g \Gamma_n^0>$ in method [7] will be always shifted at presence of superposition of several ($2 \le k \le 4$) types of the resonance wave functions even at absence of any experimental errors. Therefore, all the parameters of the best approximation will be shifted relatively to the initial values b and σ. The same concerns and parameter Σ of the variant k=1: its significant deviations from unit are always presented in the used normalization to the experimental mean value $< \Gamma_n^0>$.

The random differences between the approximation of the modeled distribution and their initial values are small enough as at determination of B and Σ (Table 1), as for b_1 - b_4 and $\sigma_1 - \sigma_4$ (Table 2) even for the set from N_r=100 resonances (random values X).

The dependence of B and Σ on random set number ("mass of nucleus") for variants k=1 and k=4 is shown in Fig. 8. It is possible to estimate from this modeling the number of resonances N_r, which is necessary in order to obtain reliable conclusions on real parameters of neutron amplitudes by quantity of about 3000-4000 and more.

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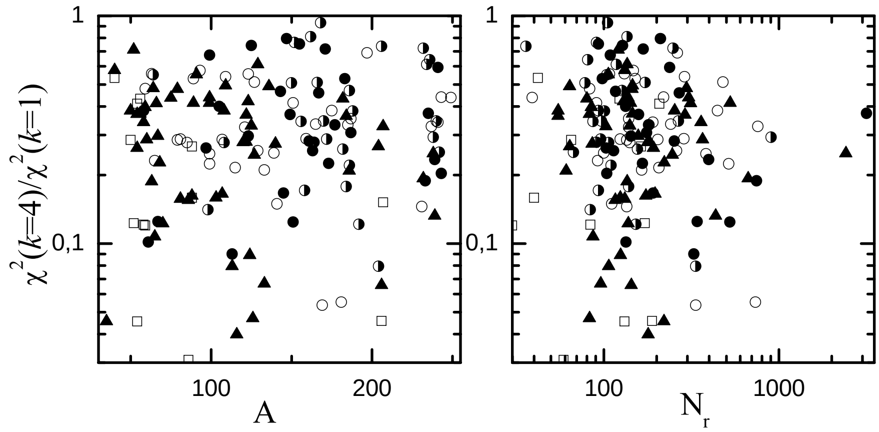

Fig. 1 . The ratio of criterions χ^2 for two variants of analysis in function on nuclear mass *A* or number of resonances N_r. The circles: closed – even-odd, open – odd-even, semi-open – even-even target nuclei. Triangles – *p*-, squares – *d*-resonances of any nuclei.

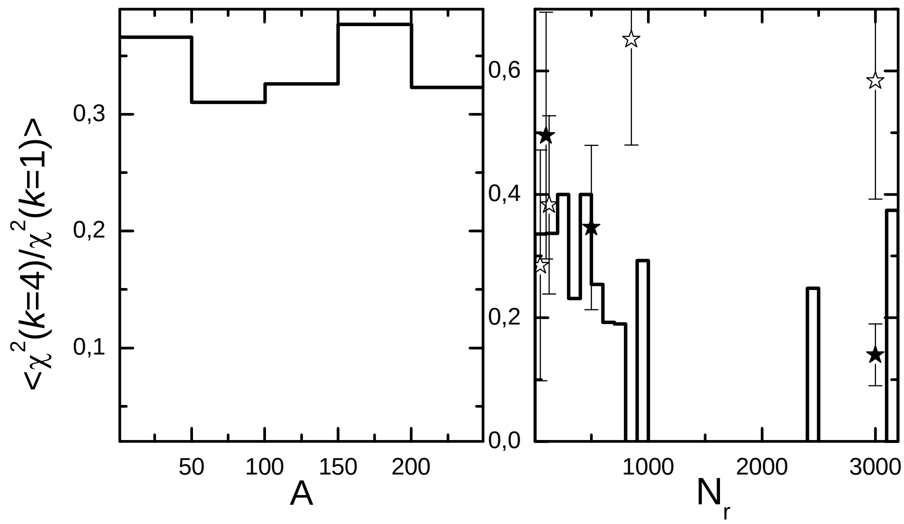

Fig. 2. Histograms are the ratio of average criterions χ^2 for two variants of analysis in function on nuclear mass *A* or number of resonances N_r. Points with errors – model (expected) value for independent neutron amplitude distributions (closed – the case *k*=1, open – the case *k*=4).

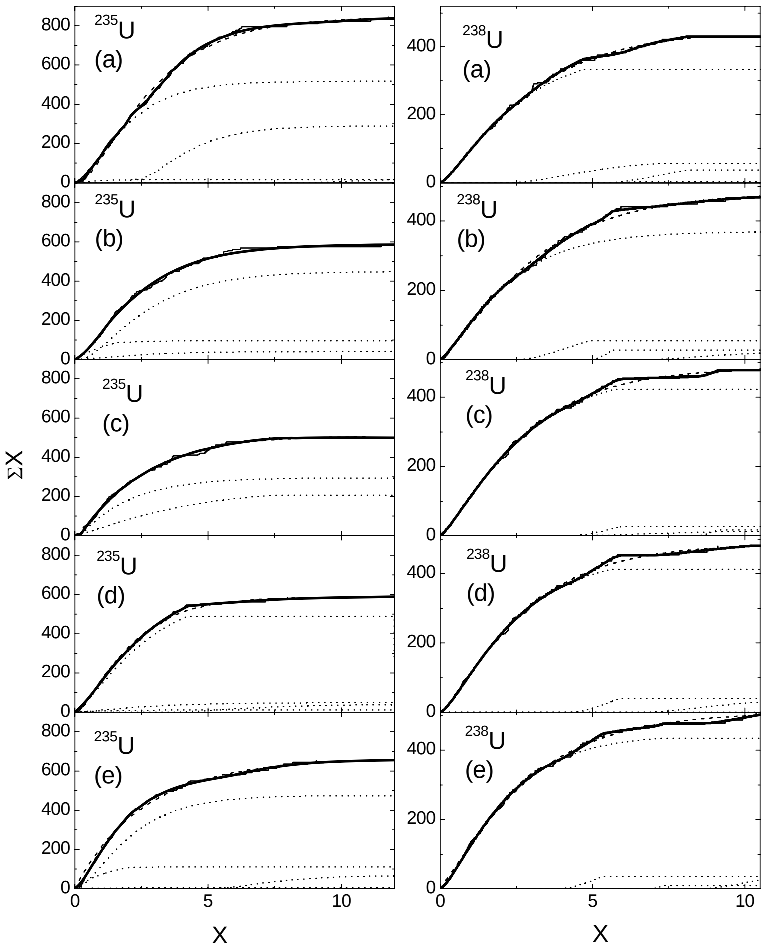

Fig. 3. Approximation of cumulative sums of the relative X values for five intervals of neutron energies of constant width in ^{235,238}U. Histogram – the experiment, dash line – approximation for k=1, thick line – for k=4, dot lines – the variant of decomposition of the best fit functions over partial functions.

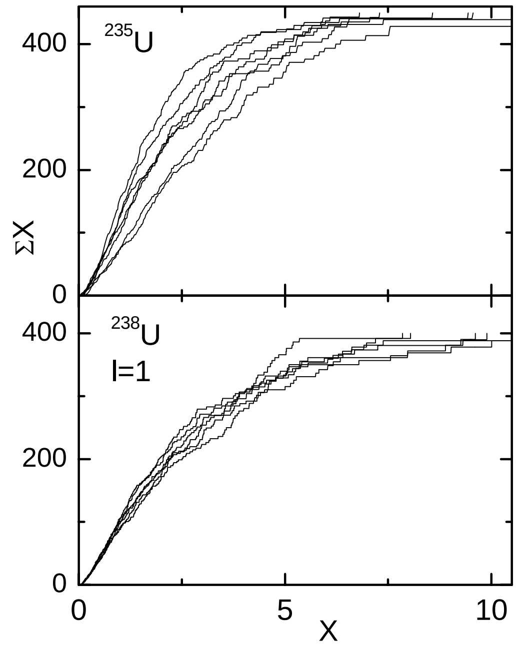

Fig. 4. Cumulative sums of the X values for the same number of resonances in each of 5 intervals of the E_n values of nuclei ^{235,238}U.

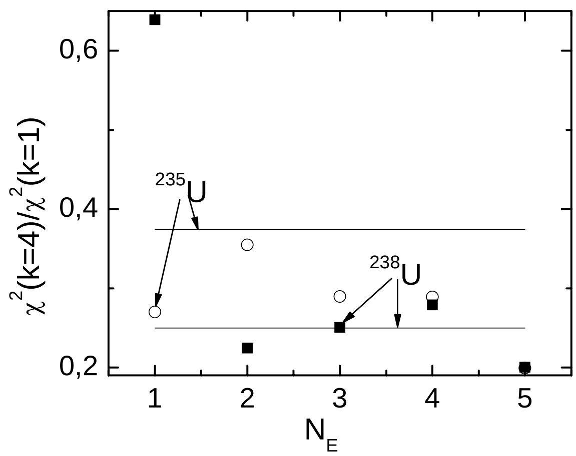

Fig. 5. Points – the ratio of criterions of quality of fitting for interval number N_E for two variants of analysis of the data of Fig. 3. Lines – the value for the total set of resonances [7].

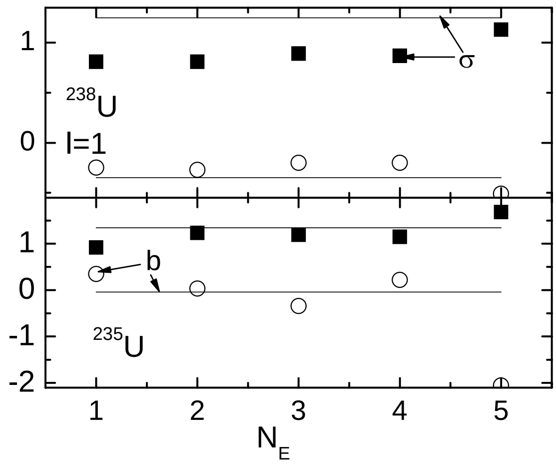

Fig. 6. The values of parameters b and σ for approximation of the data of Fig. 3 (variant k=1). The notations are analogous to Fig. 5.

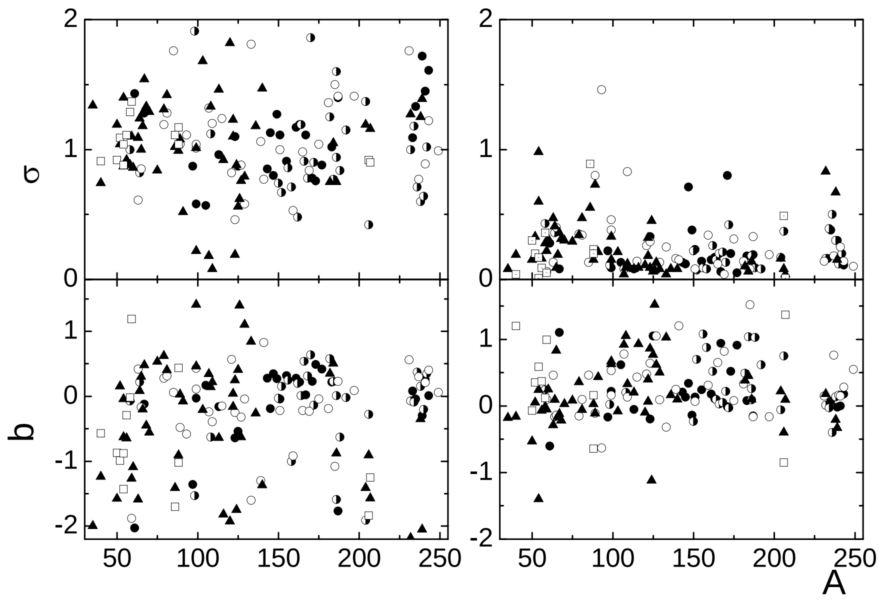

Fig. 7. Left column - the dependence of the best fit [7] values of parameters b and σ on nuclear mass A for variant k=1. Right column –the same but only for partial functions with maximal contribution in the total distribution of variant k=4.

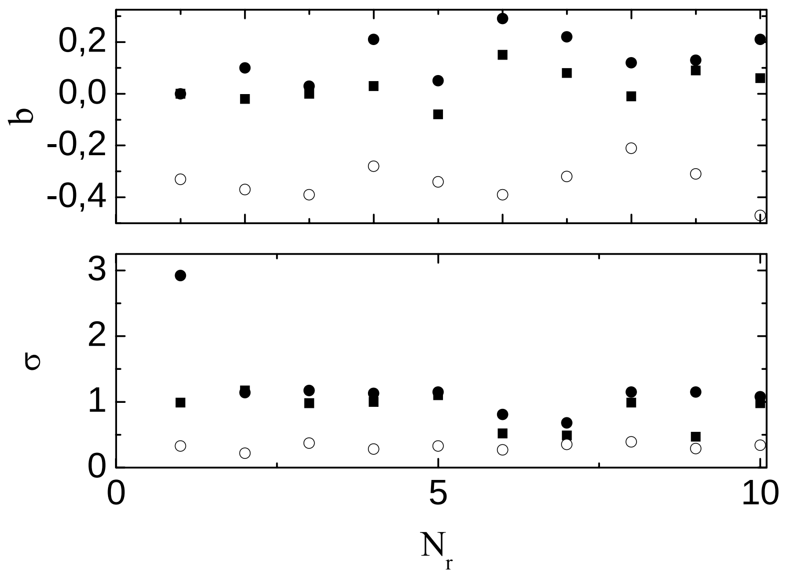

Fig. 8. The same, as in Fig. 7, for the best parameters of approximation of the model data. Closed points – modeling for k=1, b=0 and σ=1 for the sets N_r=4000 (squares) and with omission of 30% – (circles). Open points – the case k=4 for N_r=3000 and amplitude corresponding to the greatest contribution in cumulative sum.

EE- 27 0.37(21) EO- 28 0.41(22) OO- 34 0.34(16) L1= 53 0.30(16) L2 13 0.21(15)