The Relation between the Isoscalar and Isovector Interaction Potential

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Abstract. The interaction potentials of (n,n), (p,p) and (p,n) scattering by different nuclei have been calculated in the framework of the semi-macroscopic approach, using isoscalar and isovector parameters of different types of potentials. The interaction potentials are used to calculate the differential scattering cross section of (n,n), (p,p) and (p,n) reactions by some nuclei at low energy region. The relation between the isoscalar and isovector potential is studied.

Keywords: Interaction potential, DWUCK4, elastic scattering, quasi-elastic scattering

1. Introduction

Among various models for the nucleon-nucleon interaction, the folding model has been widely used to generate the real part of the optical potential [1,2,3]. Antisymmetrization of the system has been taken into account to include the exchange terms [4]. Further investigations of the elastic and quasi-elastic scattering of neutrons and protons are required for better understanding the character of the nuclear interaction.

The (p,n) reactions have been studied in order to determine the isospin dependence of the nucleon-nucleus interactions. A description of these reactions in terms of the optical model was given by Lane [5]. He postulated a simple dependence of the nucleon-nucleus optical potential upon the isospin operators. The matrix elements resulting from this dependence are expressed in simple forms [6] for both of the (p,p), (n,n), and the (p,n) reactions.

In the present paper we have studied the (p,p), (n,n), and the (p,n) reactions in the framework of the folding model, in which the interaction potential is generated by folding the chosen potential with the spherically symmetrical densities of the nucleus. The nucleon-nucleon interactions are taken in the form of sums of direct and exchange terms derived from the DGN-potential [7], the DNG-potential [8], the HJ-potential [9], and the Reid-potential [10].

The angular distributions of the elastic scattering cross-sections by the nuclei ^{56}Fe and ^{64}Zn, are calculated by using the four different potentials.

2. The Model

The nuclear interaction between an incident nucleon and a target with non-zero isospin has an isospin dependent part. For the nuclear part of the nucleon-nucleus, optical potential expressed in terms of the isospin operators in the following simple term of Lane form [5]

$$U = U_o + \frac{4tT}{A} U_1 \qquad (2.1)$$

where t, T are the isospin of the particle and target nucleus respectively and A is the mass number of the target. The second term of equation (2.1) known as the Lane potential, contributes to both the elastic (p,p) and (n,n) scattering as well as to the charge exchange

(p,n) reaction. Knowledge of U_1 is of fundamental interest for studies of nuclear phenomena in which neutrons and protons are different (isovector modes). Many previous estimates of U_1 as the Distorted Wave Born Approximation (DWBA) analysis of (p,n) reactions exciting the isobaric analog state (IAS). However, these approaches are subject to serious uncertainties. For example, in the comparison of elastic nucleon scattering from different nuclei one must make assumptions [4] about the variation of nuclear geometry with A and ε.

It is in principle possible to avoid these uncertainties by extracting U_1 from a consistent study of the elastic proton and neutron scattering and the charge exchange (p,n) reaction on the same target nucleus, at the same energy. We recall here briefly the consistent isospin coupling scheme [3] for the elastic nucleon-nucleus scattering and charge exchange (p,n) reaction exciting.

The matrix elements resulting from equation (2.1) give the following relationships [6].

$$U_{pp} = U_0 - \varepsilon U_1 \tag{2.2}$$

$$U_{nn} = U_0 - \varepsilon U_1, \tag{2.3}$$

where $\varepsilon = \dfrac{(N-Z)}{A}$.

Similarly, the transition matrix element or (p,n) form factor for the charge exchange reaction is

$$U_{pn} = 2\left(\frac{\varepsilon}{A}\right)^{1/2} U_1 \tag{2.4}$$

Accordingly

$$U_{nn} - U_{pp} = 2\varepsilon U_1 = 2\left(\frac{\varepsilon}{A}\right)^{1/2} U_{pn} = (N-Z)^{1/2} U_{pn} \tag{2.5}$$

It is necessary, therefore, to add Coulomb corrections ΔU_c to the incident proton energy and ΔU_c to U_p to separate the main effects of the Coulomb field so that isospin is a good quantum number for the remainder of the nucleon optical potential, namely,

$$U_{pp} = U_0 - \varepsilon U_1 + \Delta U_C,$$

$$U_{nn} = U_0 + \varepsilon U_1$$

Then the Coulomb correction term must satisfy the from

$$\Delta U_C = U_{pp} - U_{nn} + 2\varepsilon U_1,$$

which can be written in the form

$$U_{nn} - U_{pp} + U_C = 2\varepsilon U_1 = 2\left(\frac{\varepsilon}{A}\right)^{1/2} U_{pn} = (N-Z)^{1/2} U_{pn} \tag{2.6}$$

Where ΔU_C is coulomb interaction potential, and can be determined by [7]

$$U_C = \frac{6e^2 Z}{5r_0 A^{1/3}} 0.3 = 0.4 \frac{Z}{A^{1/3}}$$

The present measurement of angular distributions of the (p,p), the (n,n), and the (p,n) elastic scattering cross sections were made by using the distorted-wave code DWUCK [8], and the optical potential in the folding model is

$$V(r) = V_{f0}(r) + iW(r)$$

for (n,n) and (p,p), and for (p,n) reaction.

$$V(r) = \frac{2(N-Z)^{1/2}}{A} V_{f1}(r) + iW(r) \quad (2.7)$$

Where V_f (r) is the central nuclear potential calculated by the folding procedure, including the exchange part of the potential. W(r) is the imaginary part of the potential. The final results for the angular distributions of scattering cross sections were obtained by varying the parameters of the imaginary part of the potential to obtain the best fit with the experimental values.

3. Method of Calculations

The folding model has been developed in the semi-macroscopic approach to assume the folded potential in the form [9].

$$U(r) = U_0(r) + I_{0,0}(r) \pm T\, U_1(r) + I_{1,0}(r) \quad (3.1)$$

The first bracket refers to the isoscalar potential and the second bracket refers to the isovector potential, in which U_0 (r) and U_1(r) are defined as

$$U_o(r) = \int \rho_o(r)\ V_{oD}(r)\ dr$$

$$U_1(r) = \int (\rho_p(r) - \rho_n(r))\, V_{1D}(r)\, dr$$

where ρ_o, ρ_p *and* ρ_n refers to the matter, protons and neutrons distribution in the target nucleus respectively. The exchange term in equation (3.1) is defined by

$$I_{k,o}(r) = V_{KE} \int F(s)\rho_{Ko}(r,s)\ \ J_o(K_o(r)s)s^2 ds, \quad (3.2)$$

$$K_0^2(r) = \frac{2m}{h^2}\ [E - U_o(r) + T_z U_1(r)]. \quad (3.3)$$

where V_{KE} is the exchange parameter, F(s) its radial dependence and has a Wood Saxon form, $J_o(K_o(r)$ is the spherical Bessel function. T_z= -1 for proton and +1 for neutron. The calculated form factor for each of the (n,n) and the (p,p), elastic scattering according to this approach showed a small difference between them, which is due to the difference between the proton and the neutron isospin. The folded potential form (equation 3.1) is different from the usual form [10]. It is to be mentioned that the generalized folded potential leads to more accurate results, but it's calculation is more complicated than that based on equation (3.1).

The direct parameters of the potentials are derived from the nucleon-nucleon interaction as follows [11].

$$V_{OD} = \frac{1}{16}(9V^{TO} + 3V^{SE} + 3V^{TE} + V^{SO}), \quad (3.4)$$

$$V_{1D} = \frac{1}{16}(3V^{TO} + 3V^{SE} - 3V^{TE} - V^{SO}), \quad (3.5)$$

where TO, SE, TE and SO refer to the triplet odd, the singlet even, the triplet even and the singlet odd states, respectively.

Similarly the exchange parameters are given by [12]

$$V_{OE} = \frac{1}{16}(-9V^{TO} + 3V^{SE} + 3V^{TE} - V^{SO}), \tag{3.6}$$

$$V_{1E} = \frac{1}{16}(-3V^{TO} + V^{SE} - 3V^{TE} - V^{SO}), \tag{3.7}$$

In our calculations we have used four different types of effective two-body interactions:

1-The first is derived from the DGN-potential [13] whose central term is given by

$$V(r) = {}^{ts}X\{V_{C1}\, e^{-r^2/a_1^2} - V_{C2} e^{-r^2/a_2^2}\}, \tag{3.8}$$

where V_{C1} = -56 MeV, α_1= 1.635 fm, V_{C2}=91 MeV, and α_2= 0.55 fm. The operator ${}^{ts}X$ depends on the spin and the isospin quantum numbers of the two nucleons as well as the exchange-force constants as follows:

$${}^{ts}X = C_w + (-1)^{s+t+1}C_M + (-1)^{S+1}C_B + (-1)^{t+1}C_H, \tag{3.9}$$

For the exchange constants we choose the following set of values which belong to the Serber-type forces:

$$C_W = C_M = -0.41 \text{ and } C_B = -C_H = -0.09$$

Accordingly, our parameters corresponding to the DGN-potential are given by:

$$V_{OD} = V_{OE} = -17.22\, e^{-r^2/(1.635)^2} + 27.98\, e^{-r^2/(0.55)^2} \quad V_{1D} = V_{1E} = -7.9\, e^{-r^2/(1.635)^2} + 13.42\, e^{r^2/(0.55)^2}$$

2-The second potential is derived from the DNG-potential [14] whose central term is given by

$$V(r) = {}^{ts}X V_C\, e^{-r2}/a^2, \tag{3.10}$$

where Vc = -38 MeV and α=1.910 *fm*. *Hence*,

$$V_{OD} = V_{OE} = -11.68\, e^{-r^2/a^2},$$

$$V_{ID} = V_{1E} = -5.6\, e^{-r^2/a^2}$$

3- The third potential is derived from the HJ-potential [15] and the results are

$$V_{OD} = V_{OE} = 3604\, e^{-4r}/(4r) - 1158\, e^{2.5r}/(2.5r) - 3.92\, e^{-.707r}/(.707r),$$

$$V_{1D} = V_{1E} = -1442\, e^{-4r}/(4r) + 476\, e^{-2.5r}/(2.5r) + 1.3\, e^{-.707r}/(.707r). \tag{3.11}$$

4- The fourth potential is taken from the Reid-potential [16] and the results are

$$V_{OD} = 5773e^{4r}/(4r) - 1461\, e^{2.5r}/(2.5r) - 5.885\, e^{.707r}/(.707r),$$

$$V_{OE} = 2405e^{-4r}/(4r) - 1113e^{-2.5r}/(2.5r) - 5.885e^{-.707r}/(.707r),$$

$$V_{1D} = V_{1E} = -3202\, e^{-4r}/(4r) + 1002\, e^{-2.5r}/(2.5r) + 1.308\, e^{-.707r}/(.707r). \tag{3.12}$$

The density distribution function for the nuclei ^{56}Fe, ^{64}Zn and ^{116}Sn are calculated according to the fermi distribution [17].

$$\rho(r) = \rho_o (1 + \omega r^2/c^2) / (1 + \exp(r - c)/z),$$

where ρ_o, c, z and ω are constants defined as the density parameters of the Fermi-model. They corresponding values for the nuclei ^{56}Fe, ^{64}Zn and ^{116}Sn are listed in table 1.

Table1: The values of the density parameters for the nuclei ^{56}Fe and ^{64}Zn.

Nucleus	ρ_o	C	Z	ω
^{56}Fe	0.1663	4.168	0.460	0.0
^{64}Zn	0.1580	4.178	0.572	0.0

Assuming that, the shapes of proton and neutron densities are identical, the normalization is set by equation $\int \rho(r)\, d^3r = A$ where A is the mass of the target nucleus.

4. Results and Conclusion.

The analysis of the protons and neutrons elastic scattering by using the semi-macroscopic approach shows a good description of the elastic scattering cross-sections for protons and neutrons as well as for the quasi-elastic scattering, for the different types of the used potentials.

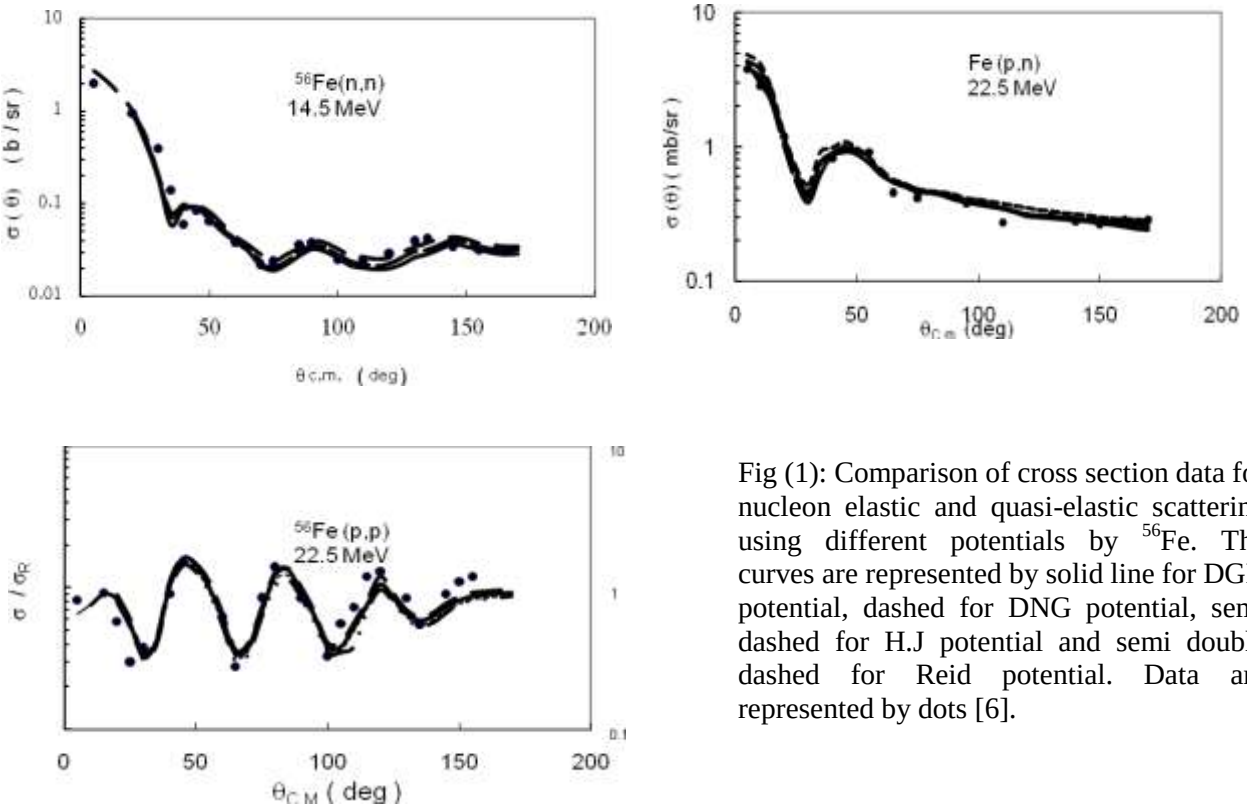

Fig (1): Comparison of cross section data for nucleon elastic and quasi-elastic scattering using different potentials by ^{56}Fe. The curves are represented by solid line for DGN potential, dashed for DNG potential, semi dashed for H.J potential and semi double dashed for Reid potential. Data are represented by dots [6].

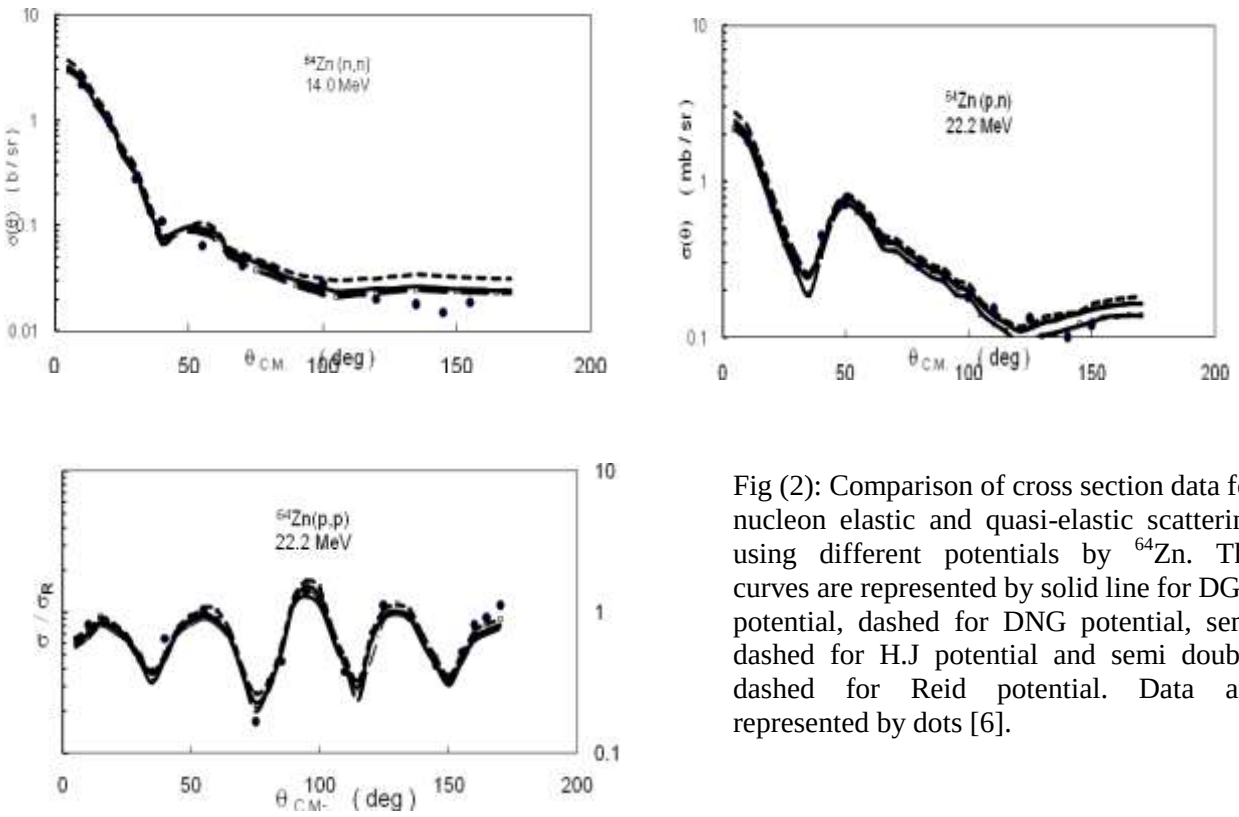

Fig (2): Comparison of cross section data for nucleon elastic and quasi-elastic scattering using different potentials by ^{64}Zn. The curves are represented by solid line for DGN potential, dashed for DNG potential, semi dashed for H.J potential and semi double dashed for Reid potential. Data are represented by dots [6].

Figures 1 and 2 show the cross section data for the nucleon elastic and quasi elastic scattering using different potentials for the nuclei ^{56}Fe and ^{64}Zn at energies of 14 and 22.2 MeV. The solid lines refer to the DGN-potential, the dashed for the DNG-potential, the semi dashed for the HJ-potential and the semi double dashed for the Reid-potential.

It is easy to notice from these figures that, all the used potentials give a good results for the scattering cross sections of each of the reactions (n,n), (p,p), and (p,n) although these potentials have different characteristic values. This is due to the fact that the calculations of the interaction cross sections depend also up on the imaginary potential.

The calculations were performed for U_{nn} at 14 MeV and for U_{pp} and U_{pn} at energy 22.2 MeV. The Coulomb force reduces the kinetic energy of protons by about 9 MeV, i.e proton and neutron interact with the atomic nucleus with the same energy. From the great similarity in the behavior of the differential elastic scattering of U_{nn} at 14 MeV and U_{pn} at 22.2 MeV, we can insure that the difference between the proton and the neutron for the same colliding energy is very small.

The Coulomb force plays an essential rule in studies of U_{nn} and U_{pp} and consequently U_{pn}. Accordingly, it is an objective to compare between the four used potentials from the point of view that they satisfy equation (2.6).

The Unn, Upp and Upn characteristics of the nuclei ^{56}Fe and ^{64}Zn for the four used potentials are presented in tables 2 and 3 respectively.

Table 2: $U_{nn,}U_{pp}$ and U_{pn} characteristics of ^{56}Fe

Form-factor	Potential	Depth (MeV)	Vol. Integral	R (fm)
U_{nn}	DGN	-65.69	22999	4.255
U_{nn}	DNG	-73.686	26684	4.394
U_{nn}	HJ	-65.998	24648	4.452
U_{nn}	Reid	-53.316	22898	5.091
U_{pp}	DGN	-64.74	22700	4.258
U_{pp}	DNG	-72.743	26342	4.344
U_{pp}	HJ	-65.181	24343	4.452
U_{pp}	Reid	-52.631	22604	5.091
U_{pn}	DGN	-1.109	389.2	4.230
U_{pn}	DNG	-1.011	367.5	4.388
U_{pn}	HJ	-1.910	702.9	4.375
U_{pn}	Reid	-2.975	1068.5	4.310

Table 3: $U_{nn,}U_{pp}$ and U_{pn} characteristics of nucleus ^{64}Zn

Form-factor	Potential	Depth (MeV)	Vol. Integral	R (fm)
U_{nn}	DGN	-64.728	26211	4.468
U_{nn}	DNG	-73.210	31304	4.578
U_{nn}	HJ	-66.866	27411	4.624
U_{nn}	Reid	-53.965	25838	5.229
U_{pp}	DGN	-63.868	25881	4.468
U_{pp}	DNG	-72.271	30903	4.578
U_{pp}	HJ	-66.009	27034	4.624
U_{pp}	Reid	-53.273	25507	5.229

Table 3(continuation): $U_{nn,}U_{pp}$ and U_{pn} characteristics of nucleus ^{64}Zn

Form-factor	Potential	Depth (MeV)	Vol. Integral	R (fm)
U_{pn}	DGN	-0.946	430.53	4.466
U_{pn}	DNG	-0.884	377.63	4.572
U_{pn}	HJ	1.682	729.69	4.551
U_{pn}	Reid	2.646	1119.8	4.490

To compare between the interaction potentials U_{nn}, U_{pp} and $U_{pn,}$ firstly, we find the difference (U_{nn}- U_{pp}) and compare it with the value $(N–Z)^{1/2}$ U_{pn} as indicated in equation (2.5). Secondly, we calculate that difference using coulomb correction to be in the form (U_{nn}–(U_{pp}+ U_C)) and finally again compare it with the value$(N–Z)^{1/2}$ U_{pn} as in equation (2.6). These data are summarized in table 4.

Table 4: The relation between U_{nn} ,U_{pp} and U_{pn}

Potential	Element	(U_{nn}– U_{pp})	U_{nn} – (U_{pp} + U_C)	(N–Z)* $^{1/2}U_{pn}$
DNG	^{56}Fe	0.95	3.668	2.218
DGN		0.943	3.761	2.022
HJ		0.817	3.525	3.820
Reid		0.685	3.403	5.950
DNG	^{64}Zn	0.860	3.859	1.892
DGN		0.939	3.939	1.768
HJ		0.857	3.857	3.364
Reid		0.692	3.692	5.292

From table 4 we found that the difference between U_{nn} and U_{pp} is a small value for all the used potentials and fare from (N–Z)$^{1/2}U_{pn}$value. Introducing the Coulomb correction to the proton incident energy we find that difference become more closely to the value $(N–Z)^{1/2}$ U_{pn} which indicate that the Coulomb force has an essential rule in satisfying the validity of equation (2.6). We notice also that the values of U_{nn}, U_{pp} and U_{pn} obtained by HJ potential gave results closer to satisfy equation (2.6) for the two elements in compared with the other three potentials. This indicates that the HJ potential give values of U_{nn}, U_{pp} and U_{pn} better than the other three potentials, for the elements ^{56}Fe and^{64}Zn.

On the other hand according to M3Y interaction potential [18] the calculations of U_{nn} and U_{pp} are performed as linear combination of isoscalar and isovector components of M3Y interaction potential, the difference between them is based on the construction of equation (2.6). For symmetric nuclei they are equals, but in that approach they have a small difference.

It is more interested to study the imaginary potentials obtained by iteration method for the case of (n,n) and (p,p) reactions which give best fitted values for the calculated cross-section with the experimental data. The question now are the imaginary potentials obtained in

the cases of (n,n), (p,p) and (p,n) reactions have the same relations as that of the real potentials. To answer on this question, we must study the imaginary potentials obtained in these reactions. Since the HJ potential is the most convenient potential in that approach, which gives the best results. Then it is more convenient to list the imaginary potentials for the three reactions that obtained by iteration method for the case of (n,n) and (p,p) reactions which gives best fitted values for the calculated cross-section with the experimental data, in the case of HJ potential. The imaginary potentials in that analysis by using HJ potential are listed in table (5).

Table 5: The imaginary potential for the (n,n),(p,p) and (p,n) reactions

Reaction	W_v ,MeV	r_v, fm	a_v, fm
^{56}Fe (n,n)	10	1.2	0.98
^{64}Zn (n,n)	8	1.2	0.98
^{56}Fe (p,p)	3.25	1.2	0.98
^{64}Zn (p.p)	2.32	1.2	0.98
^{56}Fe (p,n)	2.686	1.26	0.58
^{64}Zn (p,n)	2.472	1.26	0.58

From the values listed in table5 of the imaginary potentials W_V, for the three reactions, it is easy to find that the difference in W_V for (n,n) reaction and for (p,p) reaction is of order of $(N - Z)^{1/2}$ times W_V for (p,n) reaction, for both Fe and Zn elements. This shows that in this approach as the validity of equation (2.6) is applied on the real potentials, it can also be applied on the imaginary potential. Finally we can summarize our results in that, for non-symmetric nuclei the validity of equation (2.6) can be easily verified in our approach. The HJ potential is the more convenient potential in that analysis. The relation between U_{nn}, U_{pp} and U_{pn} is verified for the real potential as well as for the corresponding imaginary potentials.

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