THE MOST PROBABLE MEAN VALUES OF LEVEL DENSITY AND RADIATIVE STRENGTH FUNCTIONS OF ^{28}Al COMPOUND-STATE CASCADE GAMMA-DECAY

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Abstract

In this work the spectrum of random functions of level density and radiative strength functions of dipole *E1*- and *M1*-transitions was determined. Obtained functions reproduce with high precision the intensity of two-step cascades following radiative capture of thermal neutrons in ^{27}Al for given energy of primary transitions. The mean value of these functions for level density correctly enough reproduces density of intermediate levels corresponding to the observed energetically resolved cascades (including those firstly established in reaction $(n_{th},2\gamma)$). This fact gives the grounds to consider that the hypothesis on dependence of radiative strength functions of gamma-transitions in heated nucleus on density of excited levels (with the use practically realized up to now method of determination of cascade gamma-decay parameters) allows one to get their realistic estimation in any (including light) nuclei.

1. Introduction

The dynamics of the nucleus transition from a simple low-lying levels (e.g., quasiparticle or phonon structure) in the compound-state with a very complex and many component structure of wave function can be correctly described by theory only if there is full set of experimental information on the excited levels density (with given quantum numbers) and partial radiative widths values of any possible decay channels.

The quality of development of model descriptions of all parameters (for example, of neutron resonance gamma-decay) entirely depends on the degree of accuracy of experimental data. This means, it is necessary to minimize the values of full experimental error and, accordingly, the degree of distortion observed picture of this process (mostly probably at the use of certain assumptions and hypothesis about it). Ability to accurately solution of the problem substantially reduced by absence nuclear spectrometers with $FWHM << D_\lambda$ for all space D between the initial nucleus levels λ.

In this situation, the main problem for experimenters is fundamentally not removable connection between emission probability for reaction product and excited levels densities ρ. The sum of branching ratios B_r ($B_r = \Gamma_{\lambda i}/\Gamma_\lambda$) for partial $\Gamma_{\lambda i}$ and total widths Γ_λ (if there are no competing processes) is equal unit and does not depend on absolute values of levels density ρ and partial widths Γ. However, total gamma width of any level is equal to the sum of the partial width and therefore always depends on the density of the low-lying levels. So, one of the main tasks is to establish a quality and accurate description of dependence between the measured values (intensity of the emitted spectrum of particles observed in the reaction) and the excited levels density and partial radiative widths.

1.1. The current state of the experiment for determination of *Γ* and *ρ*

The only way to extract information about the properties of excited nuclei in this situation is measurement of spectra (sections) *S* of studied reactions products, and then determination functional dependency between *S* and parameters *Γ* and *ρ* ($S = \Psi(\Gamma, \rho)$). The experiment can be performed as registration of the reaction products spectra by single detector (one-step reaction [1]), or by coincidences between, for example, two detectors ("two-step" reaction [2,3]).

The first of those two variants is realized up to now for the analysis of spectra and cross-sections of evaporative nucleons [1, 4, 5] and full gamma spectra [6,7]; second variant – when measuring the spectra of two successively emitted photons [2,3] at the capture of a neutron by the nucleus. Comparison of *Γ* and *ρ* values obtained by analyzing data of one- and two-step reaction makes it possible to identify the main sources of experimental systematic errors and estimate their value for different methods of nucleus study.

a) Spectra of evaporative nucleons.

For determining the level density from evaporative nucleons spectra, it is necessary to theoretically calculate *Γ* value. This is done up to now [5] only by used a primitive optical nucleus model. Criterion of calculation validity based on acceptance of the coincidence between the calculated and experimental cross-section for analyzed reaction. But this does not take into account that experimentally measured cross-section (spectrum) is determined only by the absolute value of product *Γρ*, but not by the absolute values of the cofactors (*Γ* and *ρ*)! Because of these circumstances (as can be seen from the comparison of data [8,9] with two-step reaction analyses [2,3]), founded values of the level density in this technique are overestimated at least 5-10 times to results of type [5] on energy around the second gap threshold for nucleons Cooper pairs.

b) The first quanta spectra of cascade for levels with varying excitation energy.

The full gamma spectra measured for this purpose [7] does not depend on the absolute values of *Γ* and *ρ*. Also it very weakly depend on the form of functional dependence of searched parameters on gamma-quantum and excited level energy (sum cascade gamma transition energy at decay of any level is independent of *Γ* and *ρ*). Mean quadratic difference for full gamma-spectra forms calculated for various realistic representations of $\Gamma = f(E_\gamma)$ and $\rho = \varphi(E_{ex})$, does not exceed 30%, in the best case, [10].

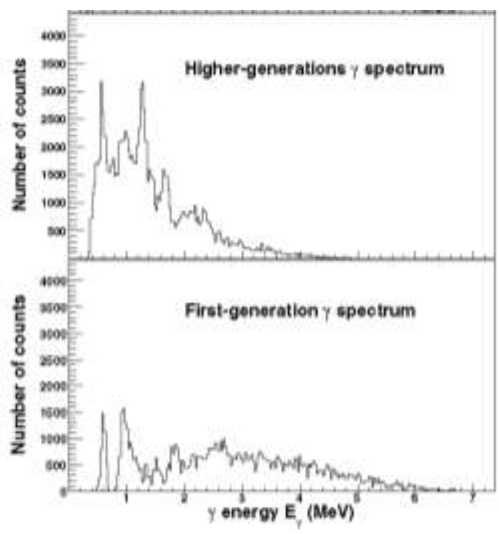

Fig. 1. Gamma-ray spectra in reaction ^{45}Sc(^{3}He,^{3}He'γ)^{45}Sc [11] for the first and next cascade quantum.

In Fig. 1 it is shown spectra of gamma rays following inelastic scattering of ^{3}He on ^{45}Sc isotope. The most intensive and energy-resolved peaks registering photons of the same energy, [11] as in the "first generation" and in "higher-generation spectrum" in most cases can only belong to "the last" cascade quanta. And they are presented in "first generation" spectra only because of error of the specific used techniques. Specifically, this is due to non-compliance basic condition technique [6] which the spectrum of gamma radiation for given excitation energy and quantum numbers at initialization in a beam of charged particles equated with the same spectrum witch excited of gamma transitions with the high-lying levels. As result, systematic error of the "first generation" spectra exceeds 100% in low energy photons region. And it decreases with an unknown rate at increasing E_γ. This fact is completely ignored in the evaluation [12] the errors of this approach [7]. So, as results obtained using scintillation detectors on method [7] values Γ and ρ in all publications of Oslo group have unknown but catastrophically errors.

c) Two-step cascade quanta.

As measured by the absolute intensity of cascades $I_{\gamma\gamma}=\Psi(\Gamma,\rho)$ for a limited number of their final levels of quality is defined by [13] inverse of the absolute value of level density and form of strength function $\Gamma=f(E_\gamma)$. The relationship of experimental values $I_{\gamma\gamma}$ with unknowns functions Γ and ρ for excitation energy of any interval defined in the experiment is always nonlinear. Therefore, from experimental data on the intensity of the cascades $I_{\gamma\gamma}$ can currently be defined only the range of probable values of Γ and ρ, precision reproducing the experimental values of $I_{\gamma\gamma}$. And it has a final, although a finite [14] size even at zero statistical errors of the experiment.

The listed above methods have also common sources of systematic errors.

First of all, there are the lack of suitable for the analysis of experiment, the modern model of nuclear excited levels for both nucleon or radiation channel. Such a model should explicitly take into account the coexistence and interaction of boson and fermion components of nuclear matter. Also, it is necessary to consider the dependence of the partial widths Γ their deposits in the wave function for both initial and final level when specified energy reaction product emitted.

All three of the above techniques, without exception, require additional, methodically independent experiment that could produce non-degenerate system of equations which connect the measured spectrum with the desired values of parameters Γ and ρ. And ensure the uniqueness of their definition.

The distorting effect of both listed factors, which is most significant for one-step reactions, is essentially reduced by conditions of two-step reactions experiment. Therefore, the above conclusion on necessity develops a modern model of properties of nuclear processes (that become more complicated at the increasing its excitation energy) requires the development of experiments that will follow only from data realized by two-step reaction, but not from the accumulated to now by one-step reaction results.

2. Status of the modern model of level density and partial gamma-widths

In modern theoretical views, for example, quasipartical-phonon model of nucleus, partial width is determined by coefficients of wave functions components for both decayed and excited level [15]. And their concrete values are specified by the degree of the different nuclear states fragmentation with a fixed number of quasipartical and phonons. That is directly determined by level density, because ρ value is definite by the strength of fragmentation of all possible states of the nucleus.

Currently used model of radiation strength functions of dipole gamma transitions for nucleus with the mass *A:*
$k = \Gamma_{\lambda i.} /(E_\gamma^3\ A^{2/3}\ D_\lambda)$. (1)
takes into account the dependence of partial radiative widths only from density $\rho_\lambda = D_\lambda^{-1}$ of decayed high-lying levels (especially the neutron resonances). But it does not take into account the possibility of their dependence on the density of the final levels with sufficiently high energy excitation of the heated nucleus. Modern two-step reaction (n_{th},2γ) experiment revealed the existence of such dependence [16,17].

Smooth functional form of evaporative nucleons energy spectra for composite ^{181}W nucleus for different initial excitation energies [18] may be reproduced only on the condition that partial width of a nucleon emission is strongly dependent on the energy of excitation of the residual nucleus and by use Strutinsky model for level density [19]. The set of this model parameters approximations for the masses $40 \leq A \leq 200$ is derived from level density obtained by (n_{th},2γ) reaction. In particular, in the second gap threshold Cooper pair of nucleons (at a minimum) partial width of nucleon emission raises many times in comparison with neighboring excitation energies of the residual nucleus. Also, the corresponding rate doesn't change (or very lightly varied) when the reaction (*p,n*) protons energy is changed [18]. This means, the product $\Gamma\rho$ conserved their form shape in almost any possible change in the level wave function of target nucleus (neutron resonance), during the decay of which appear (in the case of (*p, n*) reactions) evaporated neutron.

Therefore the form of dependence of $\Gamma\rho$ on energy and, accordingly, cross-section for a given reaction for fixed nucleus excitation energy and for different beam of charged particles energy can be reproduced as correct, even if calculation of values of Γ and ρ is wrong.

2.1. Principles of the proposed modified model of radiation strength functions

From the existence of a specified effect simply follows clear form of modification expressions (1) for radiation strength function of gamma transitions between arbitrary compound state λ and any low-lying level *i*:
$K_{mod}=k/D_i=\Gamma_{\lambda i}/(E_\gamma^3\ A^{2/3}\ D_\lambda)/D_i$, (2)
taking into account the average spacing D_i between the *i*.
In practice, in order to maintain continuity with the expression (1), it is advisable to use the following modification:
$K_{mod}=kD_{asim}/D_i=\Gamma_{\lambda i}/(E_\gamma^3\ A^{2/3}\ D_\lambda)D_{asim}/D_i$ (3)

Here D_{asim} is asymptotic spacing between levels of a heated pure fermion system (defined, for example, model noninteracted Fermi gas) and D_i is the expected space between them. Specific value D_i comes from coexistence and interaction in the quasiparticle and phonon types of excitations [20]. Bearing in mind that the fragmentation of any state of the nucleus is minimal in its initial energy and grows with its increasing [21] and almost experimentally observable fact coexistence [16, 17, 20] boson and fermion forms of nuclear matter, it is to be expected that $D_{asim} \leq D_i$ and $K \geq k$ for at least the bulk of gamma transitions.

3. The results of levels density and the radiation strength functions in the ^{28}Al nucleus

Two-step cascades in the target nucleus ^{27}Al at the thermal neutrons capture were measured in Rež, Czech Republic. Spectroscopic analysis data published in [22]. The intensity of these cascades to the 2 final levels was overridden by the data [23] and was used to determine Γ and ρ on methodology [2,20]. It is commonly believed that in light nuclei capture a neutron radiation reaction, for example, depends on the structure of excited level wave function much more strongly than in the heavy and average mass nuclei. Therefore, practical interest to determine the average parameters of cascade gamma decay of neutron resonance in this nucleus is small; but this can be important for estimating the reliability of the results of practical determination of radiation strength functions of gamma transitions between the levels of the heated nucleus in modified model (2,3).

Fig. 2 shows the half of measured cascades intensity to the ground state of the nucleus (for $E_\gamma<0.5\ B_n$). The other half is mirror [24] symmetric. Under the positions of the peaks in the spectrum it is uniquely identified by the energy of cascade photons and the intensity of the cascades. Their sequence is determined on the basis of clear conditions: primary gamma transitions have the same energy in various distributions of the type represented by fig. 2, and the cascade secondary transitions energies have been shifted to the difference of the final level energies [25].

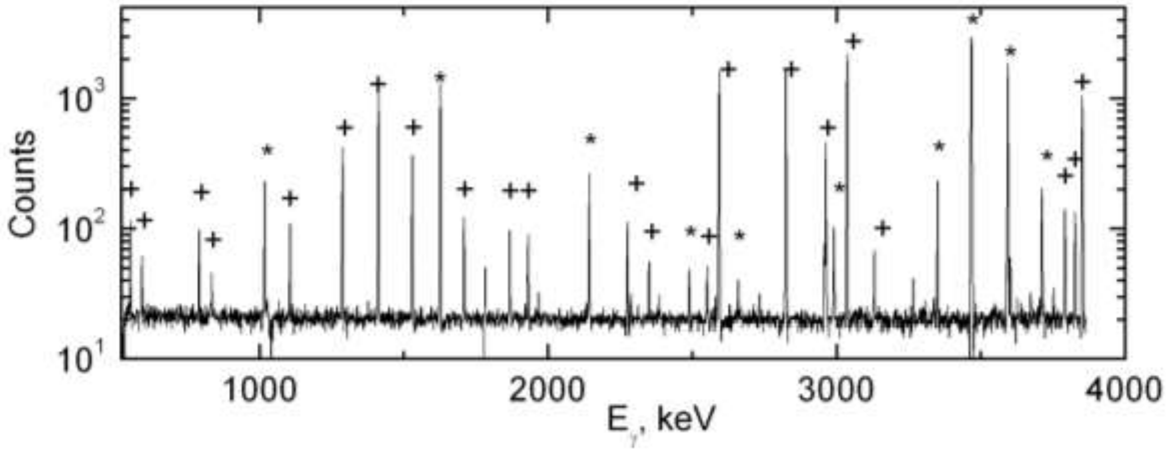

Fig. 2. Half the intensity distribution of two-step cascades on ground state. Crosses marked primary transition of ^{28}Al cascades, asterisks are secondary one's. Spectrum shifted up to 20 counts.

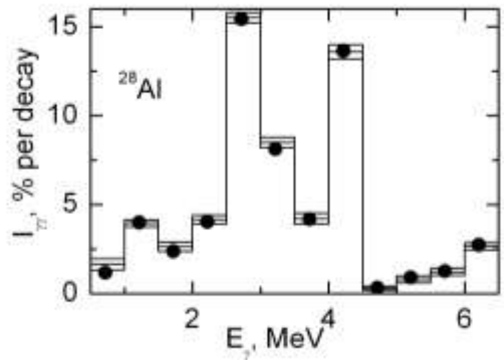

Fig. 3. Histogram – distribution of the intensity of the cascades to the 2 final level ^{28}Al in functions in their primary transition energy. Points are noted approximated value $I_{\gamma\gamma}$ for one of the variants to define the random radiative strength and level density functions.

Using defined condition, the distribution of the measured cascade intensity in the function of their primary transitions energy was calculated (fig. 3). It is done in a situation where resolved time of HPGe spectrometer at orders of magnitude larger than the lifetime of levels (i.e. FWHM $>>T_{1/2}$). The total number of observed by such manner levels in ^{28}Al reaches numbers ≈100; file [26] contains ≈ 45 identified levels up to now. This is due to the fact that under the experiment conditions in the spectrum two-step cascades of thermal neutron capture is always noticeably much more observed levels than in all methods of nuclear spectroscopy established to the present time. Level density in spin interval $1 \leq J \leq 4$ taking from the evaluated schemes is compared to similar data from the two-step cascades on fig. 4. Corresponding data together with the density of neutron resonances are always used for normalization of level density to retrieve both the one- and two-step reactions. Unfortunately, the independent analysis [27] of precision of values D_λ (does not use untested by experiment hypotheses about the form of neutron widths distribution and their parameters for each individual nucleus) showed that the possible systematical error may underestimate the density of neutron resonances, as much as possible, by an order of magnitude. A very significant difference of ^{28}Al low-lying levels density in their values region from few units up to ≈ 20 MeV^{-1} is a serious cause for skeptical estimation of today's experiment on the one hand and the possible serious methodical errors in determinate values of Γ and ρ. These parameters specify the basic properties of the nucleus, manifested in the nuclear reaction. From this circumstance, however, it is possible to conclude that the hypothesis (2) could be taken as a first approximation to model descriptions of radiative strength functions in any heated nucleus.

Each excitation energy interval of intermediate levels (two-step cascades, in particular) correspond to the unknown values of Γ and ρ. Practically, it should be taken into account the inequalities of the radiative strength functions with primary and secondary gamma transitions of one and the same energy and multipolarity. Principally degenerate system of nonlinear equations, even in this case allows one's to define a region of possible values of level density and the radiation strength functions. It was first shown in [2], that it is necessary to obtain a sufficiently large set of values of random functional dependencies of ρ and Γ in function of energy gamma-transition (excitation of nuclei). In stipulating that the difference between the average values of random and unknown (as defined) values always is minimal, the result can be considered as the value of the experiment [2,3]. With the corresponding increase in values of level density and the radiation strength functions errors.

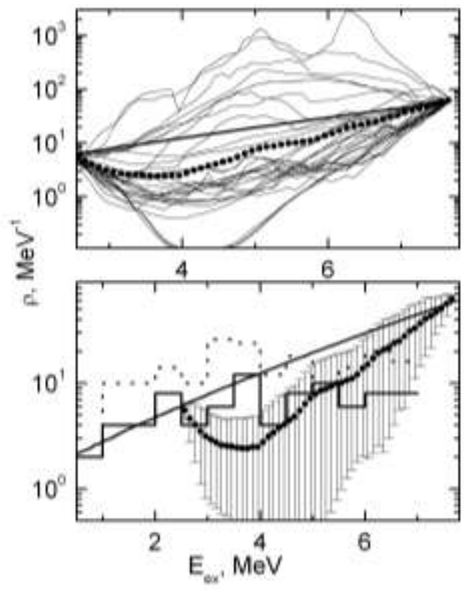

Fig. 4. A set of found level density random functions, reproducing the fig. 3 data with a very close and small χ^2 values. The point is the average value of the entire set of random functions; thick line - model value for ρ [28]. Solid histogram - density of ^{28}Al levels from [26], dotted points – from processing spectra [22] similar to those shown in Fig. 1.

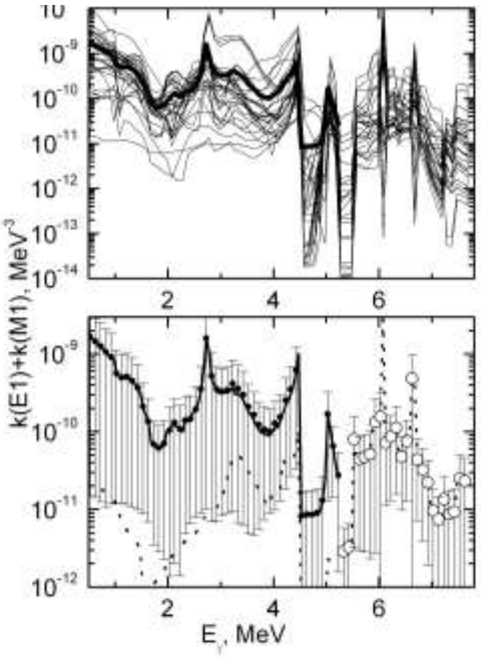

Fig. 5. The same as in Fig. 4, for the radiation strength functions in standard definition (1). Solid thick line is the average values of strength functions E1- and M1-transition; dotted – data only for M1-transition. Normalization is done on *s*-resonances value Γ_λ = 1.6 eV.

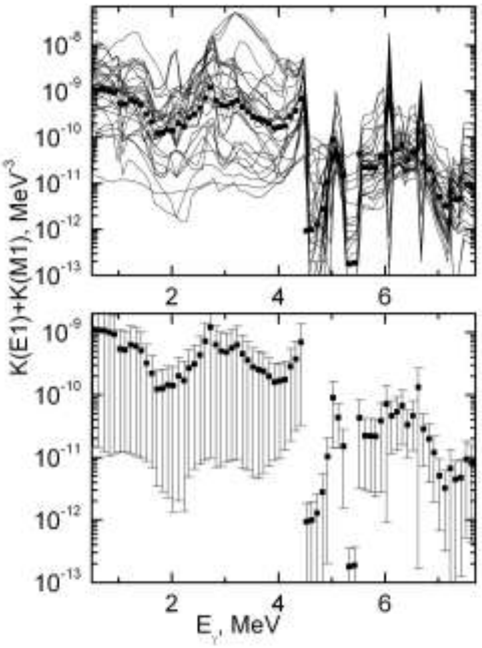

Fig. 6. The same as in Fig. 5, for radiation strength functions in version (3) strength functions for gamma transitions in the hot nucleus.

In the process of approximation, of course, it must include the maximum amounts of experimental data. The potential effectiveness of this technique is demonstrated in Fig. 7., where is presented two comprised variants the level density fixed by model [28] (left column) and the approximation close to fig. 4 data (but assuming equality of ρ for levels of positive and negative parity). Basic test result: potential performance of independent experiment to determine level density will make it possible to obtain precise information on radiation strength functions and vice versa.

4. The specificity and potential of (n, 2γ) reaction analysis

Search for any solution of degenerate nonlinear systems of equations can always result to a local maximum likelihood function. That is always a false solution. Good enough methods of identification and resolving this problem have been developed by mathematicians up to now. Data in fig. 3, which is the worst version of the experimental data at $I_{\gamma\gamma}$, has been used to determine Γ and ρ. Hence, the number of iterations required to achieve a minimum disagreement between experimental and approximation values for cascade gamma decay of neutron resonances of ^{27}Al nucleus usually exceeds 10^5 for each variant. Here, as in many other cases, it is appropriate that the first iteration includes in iteration process source data specific values of the strength functions for the most strengthful gamma-transition "spreaded" under the appropriate intervals of the primary gamma-transition energy.

Practical absence of negative-parity levels below 3.3 MeV is not allowed to define strength function of *E1*-transition in this interval of ^{28}Al excitation energies. It is also unachievable to obtain the precise approximation of the intensity distribution in the range of cascade primary energies from 0.5 to 1 MeV. Observed in Fig. 5 and 6 increase of strength functions in this interval without doubt is a consequence of the absence for secondary cascade transitions. The corresponding increase in the strength functions can be qualitatively explained only by present of the collective type primary transitions in the region of B_n by vibration enhancement of the level density. This is true if the threshold of breaking of next Cooper pair gets to the region of nuclear excitation near B_n.

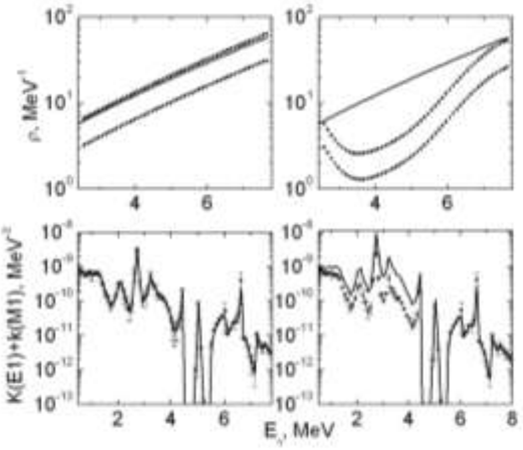

Fig. 7. Top row is fixed to the level density of ^{28}Al (points); lower – the best approximation of the radiative strength functions for fixed level density. Points with error bars – a standard presentation of radiation strength function, line - strength function in the equation (3).

5. Conclution

Finally, the results of the ^{28}Al two-step cascades intensity fit show that hypothesis (2) can ensure maximum precision, on average, a description of spectra and cross section of nuclear reactions in nuclei with any mass [20]. At the very least - for the gamma-ray. Unfortunately, even here the unsolved problems still remains the determination of the density of the low-lying levels and neutron resonances in a custom nucleus with acceptable and guaranteed precision (full error is no more than a few tens of percent). This is not allow to identify and correctly modally describe the interaction dynamics of nuclei superfluid and normal states in the transition from levels with a simple wave function to extremely complex compound-state.

From purely mathematical ideas to accomplish this task is indispensable for further experiments. This may be implementation of multi-step reactions. First of all to measure the intensity of three or more sequence cascade photons [29] in reactions of radiation capture of nucleons and light nuclei, as well as measuring the intensities of cascades, containing, in addition to photons, and nucleon products of nuclear reactions [8].

Another, and complements the first, the potential possibility to achieve the same goal is to develop models for strength functions and level density that have joint fitting parameters. First of all they should be thresholds gap Couper pair and mutually connected coefficients of vibration enhancement of levels density and collective - for radiative strength functions at decay of levels with large enough components of the phonon type in the structure of their wave functions. For example, the models of this type can be able to easy reproduce the intensity of cascades primary transition ≈ 0.5-1 MeV energy range (fig. 2) both for ^{28}Al and in a number of other nuclei. Under favorable conditions (a small number of parameters) it can be expected and rather uniquely determination of radiative strength functions and level density even without fixing one of this parameters.

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