THE NONUNIFORM CHARACTER OF THE POPULATION OF FISSILE NUCLEUS SPIN PROJECTIONS K IN THE SCISSION POINT AND ANISOTROPIES IN FISSION FRAGMENTS ANGULAR DISTRIBUTIONS FOR LOW-ENERGY AND HIGH-ENERGY FISSION

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The objective of the present study is to perform a global analysis of conditions of the appearance of anisotropies in the angular distributions of fragments originating from the spontaneous and induced fission of actinide nuclei with allowance for the results reported in the monograph by A. Bohr and Mottelson [1] and the results obtained within the quantum-mechanical fission theory that has been developed in recent years.

1. CONDITIONS OF THE APPEARANCE OF ANISOTROPIES IN ANGULAR DISTRIBUTIONS OF FISSION FRAGMENTS

The nuclear-fission process is determined by collective deformation motions leading to the transition of a fissile nucleus from the initial state to its scission point where it breaks up to fission fragments. In the case of induced fission, the initial state of a fissile nucleus is formed from the state of a compound system arising upon projectile capture by a target nucleus after the emission of prescission evaporated light particles whose number is determined by the excitation energy of this system. Angular distributions of nuclear-fission fragments are usually described [1] under the assumption that the axial symmetry of the fissile system being considered is conserved at all stages of the fission process and the assumption that the directions of fission-fragment emission are collinear to a high degree of precision with the symmetry axis of this system. In [2, 3], it was shown that, upon taking into account the quantum-mechanical uncertainty relation between the emission angle for a particle and its orbital angular momentum, the last assumption is valid in the fission of nuclei only in the case where the angular distribution of fission fragments in terms of the angle θ reckoned from the symmetry axis of the fissile nucleus has the character of a smeared delta function. This function is nonzero in the narrow angular range of $0 \le \theta \le \Delta\theta << 1$. It can be expressed in terms of a coherent superposition of spherical harmonics $Y_{L0}(\theta)$ for which the relative orbital angular momenta of fragments satisfy the condition $L \le L_m \approx 1/\Delta\theta >> 1$. The mechanism of pumping of large values of L_m was explained in [4, 5] by the appearance of transverse wriggling vibrations of the fissile nucleus [6] in the vicinity of the scission point for the separation of nuclei into fission fragments.

Using the concepts outlined above, relying on the generalized model [1], disregarding paritynonconservation effects, and allowing the possible interference between fission amplitudes for fissile-nucleus states characterized by different spins J and J' [2, 7], the general expression for the angular distribution of fragments originating from the fission of oriented nuclei in the laboratory frame is then given by

$$W(\Omega) = \sum_{JMJ'M'K\ge 0} A(J,J')P(|K|)\rho_{MM'}^{JJ'} \times$$
$$\times\left\{\left(D_{MK}^{J*}(\Omega)D_{M'K}^{J'}(\Omega)+(-1)^{J+J'+2K}D_{M-K}^{J*}(\Omega)D_{M'-K}^{J'}(\Omega)\right)\left(1-\delta_{K,0}\right)+\delta_{K,0}D_{M0}^{J*}(\Omega)D_{M'0}^{J'}(\Omega)\right\}, \tag{1}$$

where $A\left(J,J'\right)$ is a quantity that is proportional to the probability for the simultaneous appearance of fissile-nucleus states characterized by spins J and J', projections M (K) of the fissile-nucleus spin J onto the Z axis of the laboratory frame (symmetry axis of the nucleus); $D^{J}_{MK}\left(\Omega\right)$ is a generalized spherical harmonic depending on the solid angle Ω determining the direction of fission-fragment emission in the laboratory frame; $P\left(\left|K\right|\right)$ is the distribution of projections K in the vicinity of the scission point for the separation of the fissile nucleus into fission fragments, and $\rho^{JJ'}_{MM'}$ is the spin density matrix for the fissile nucleus. Expression (1) is obtained under the condition that intrinsic states of the fissile nucleus, which are determined by the superposition of the $K=\pm\left|K\right|\neq 0$ states, appear in the sum over $K\geq 0$ with a weight twice as high as the weight of the $K=0$ state. It arises as a generalization of the analogous expression from the monograph of A. Bohr and Mottelson [1] upon employing the results reported in [8, 9], where the spin density matrix $\rho^{JJ'}_{MM'}$ nondiagonal in the spins J and J' was constructed for the compound nucleus. Upon using the multiplication theorem for generalized spherical harmonics [1] and considering that the quantity $\left(2J+2K\right)$ is even for both integral and half-integer values of J, we can recast expression (1) into the form

$$W\left(\Omega\right)=\sum_{IJMJ'M'}\sqrt{\frac{4\pi}{2I+1}}Y_{IM-M'}\left(\Omega\right)\rho^{JJ'}_{MM'}\left(-1\right)^{J+M}C^{I(M-M')}_{JJ'-MM'}A\left(J,J'\right)\times$$
$$\times\left\{P\left(0\right)\left(-1\right)^{J}C^{I0}_{JJ'00}+\sum_{K>0}P\left(\left|K\right|\right)C^{I0}_{JJ'-KK}\left(-1\right)^{J+K}\left(1+\left(-1\right)^{I}\right)\right\}. \tag{2}$$

The first term in the braced expression at $K=0$ differs from zero for integral values of J (J') and even values of I. The second term in the expression (2) is nonzero at even values of I.

In the case of the fission of a nonoriented fissile nucleus, whose spin density matrix has the form

$$\rho^{JJ'}_{MM'}=\delta_{J,J'}\delta_{M,M'}\frac{1}{\left(2J+1\right)N_{J}}, \tag{3}$$

where N_J is the number of possible values of the spin J of the fissile nucleus, we can show that the angular distribution of fission fragments in (2) is isotropic, irrespective of the form of the distribution $P\left(\left|K\right|\right)$ of projections K. In order to prove this, we express the factor $\left(-1\right)^{J+M}$ appearing in expression (2) in terms of the Clebsch–Gordan coefficient $C^{00}_{JJ'-MM'}$ with allowance for the properties of this coefficient as

$$\left(-1\right)^{J+M}=C^{00}_{JJ'-MM'}\sqrt{2J+1}=\delta_{M,M'}\delta_{J,J'}C^{00}_{JJ-MM}\sqrt{2J+1}, \tag{4}$$

Upon summation over M and M' in (2) with allowance for the orthogonality of the coefficients C^{I0}_{JJ-MM} and C^{00}_{JJ-MM}, only the isotropic term with $I=0$ survives in the sum over I.

In the case where the distribution $P\left(\left|K\right|\right)$ of projections K of the spin of a fissile nucleus onto its symmetry axis is uniform at the scission point for this nucleus, in which case $P\left(\left|K\right|\right)=\text{Const}$, the angular distribution $W\left(\Omega\right)$ (2) similarly becomes isotropic, irrespective of the structure of the spin density matrix $\rho^{JJ'}_{MM'}$. for the fissile nucleus. Indeed, we can see that, if, in expression (2), one employs the representation in (4) for the quantity $\left(-1\right)^{J+K}$ with the substitution of K for M and performs summation over K with allowance for the orthogonality of the Clebsch–Gordan coefficients C^{I0}_{JJ-KK} and C^{00}_{JJ-KK}, then only the isotropic term with $I=0$ then survives in the angular distribution of fragments, $W(\Omega)$, as before. Thus, two conditions must hold for the emergence of an anisotropic angular distribution $W(\Omega)$ of fission fragments. These conditions require, first, the appearance of orientation for fissile nuclei and, second, the presence of a nonuniformity in the distribution $P(|K|)$ of projections K of the spin of a fissile nucleus in the vicinity of the scission point for this nucleus. We note that fulfillment of these conditions is also necessary for the appearance of the anisotropy in the angular distributions of binary, ternary, and quaternary fission products that is associated with P-odd, P-even, and T-odd asymmetries [8, 10–12].

2. POSSIBLE MEANS FOR ORIENTING FISSILE NUCLEI AND EFFECT OF THIS ORIENTATION ON ANGULAR DISTRIBUTIONS OF FISSION FRAGMENTS

Fulfillment of the first condition that is necessary for the appearance of anisotropies in angular distributions of fission fragments and that is associated with the emergence of fissile-nucleus orientation can be implemented in the case of the spontaneous fission of nuclei via the effect of applied magnetic and electric fields on the population of projections M of the fissilenucleus spin. But in spontaneous-fission reactions, three versions can be used individually or simultaneously to orient a fissile compound nucleus arising upon projectile capture by a target nucleus. The first takes advantage of the fact that a nonzero orbital angular momentum $\mathbf{L}$ of the relative motion of colliding particles is always orthogonal to the direction of their motion. The remaining two involve the orientation of the target-nucleus and projectile-particle spins ($\mathbf{I}$ and $\mathbf{S}$, respectively) under the effect of applied magnetic and electric fields.

We note that the appearance of a spin density matrix $\rho^{JJ'}_{MM'}$ that is nondiagonal in the spins J and J' of the fissile compound nucleus is possible only in the case of the nuclear-fission process induced by slow (cold, thermal, or resonance) neutrons. Upon the capture of such neutrons, resonance states with various energies and spins $\mathbf{J}$ determined by the composition of the vectors $\mathbf{I}$, $\mathbf{S}$, and $\mathbf{L}$ are formed in the first well of the deformation potential of the product compound nucleus. The interference between the fission amplitudes for these resonances may affect angular distributions of fission fragments. This influence is indeed substantial if incident neutrons have a high (if use is made, for example, of the time-of-flight procedure) resolution δE_n in energy; that is $\delta E_n << D^J$, where D^J is the energy spacing between neighboring resonances of spin J. But if the energy resolution is poor – that is, $\delta E_n >> D^J$ – a large group of resonances is excited in the compound nucleus, the random-phase approximation becoming then valid for the amplitudes of partial fission widths associated with these resonances, with the result that the interference in question disappears. A similar situation of the absence of the interference between the fission amplitudes of different resonance states excited in the compound nucleus arises in all cases of the threshold and above-threshold target fission induced by any projectile particles, with the exception of slow neutrons characterized by a high energy resolution. The spin density matrix $\rho^{JJ'}_{MM'}$ for the product compound nucleus is then diagonal in spins: $\rho^{JJ'}_{MM'} = \rho^{J}_{MM'}\delta_{J,J'}$. The situation where the spin density matrix is diagonal in spin also arises for the spontaneous fission of oriented nuclei in the ground state.

We now address the question of what types of fissile nucleus orientation affect angular distributions of nuclear-fission fragments in the case of a spin density matrix that is diagonal in their spins. In the coordinate frame where the Z axis is aligned with the axis of orientation of the fissile nucleus, this matrix is diagonal in M and M', and one can express it in terms of the nuclear-orientation parameters $p_Q(J)$ considered in [13] as

$$\rho^{J}_{MM'} = \delta_{M,M'} \sum_{Q} (-1)^{J-M} C^{Q0}_{JJM-M} C^{Q0}_{JJJ-J} p_Q(J). \tag{5}$$

Substituting (5) into (2) and performing summation over M with allowance for the orthogonality of the Clebsch–Gordan coefficients C^{Q0}_{JJM-M} and C^{I0}_{JJM-M}, we can find that $I = Q$. Since the quantity I in (2) is even, as was shown above, the quantity Q must also be even. This means that the angular distribution of fragments originating from the spontaneous and induced fission of nuclei is insensitive to the fissile-nucleus polarization associated with the nuclear-orientation parameters $p_Q(J)$ for even Q. On the contrary, it is determined by fissile-nucleus alignment associated with the orientation parameters for even Q. A similar situation is likely to appear for the spin-nondiagonal fissile-nucleus density matrix arising, as was shown above, in the fission of nuclei that is induced by slow neutrons characterized by a high energy resolution. The situation around the second condition that is necessary for the appearance of anisotropies in the angular distribution of fission fragments and which requires a non-uniformity of the distribution $P(|K|)$ of projections K is much more involved because of the possible changes in this distribution at various stages of the fission process. As will be shown below, the character of these changes is directly related to nuclear-fission dynamics caused by the interaction of collective deformation modes of motion of the fissile nucleus with its intrinsic nucleon modes of motion. In analyzing this interaction, it is necessary to take into account an experimental fact of paramount importance that characterizes its properties and which demonstrates the insensitivity of the observed kinetic energy of fission fragments to the excitation energy of the fissile nucleus [14]. This means that collective deformation-motion dynamics, which determines the kinetic energy of fission fragments, has a universal character conserved upon the transition from spontaneous to induced threshold and high-energy fission of nuclei and is independent of the excitation energy of the fissile nucleus.

3. CHARACTER OF CHANGES IN THE DISTRIBUTION OF PROJECTIONS K OF THE FISSILE-NUCLEUS SPIN IN THE PROCESSES OF INDUCED THRESHOLD AND SPONTANEOUS FISSION OF NUCLEI

Because of the conservation of axial symmetry of the fissile nucleus at all stages of its fission, Coriolis interaction that couples the collective rotation of the fissile nucleus to its intrinsic nucleon modes of motion and which mixes the wave functions with different K for this nucleus is, as was shown in [15–18], the only source of changes in the projection K of the spin of this nucleus. This interaction is weak for low-energy nuclear states having a low energy density, but it undergoes a dynamical enhancement for excited multiquasiparticle thermalized states characterized by a rather high energy density [19, 20]. Such an enhancement arises, for example, for low-spin neutron resonance states of a compound nucleus that are formed in the first well of the deformation potential upon slow-neutron capture by a target nucleus and whose excitation energy E^*_0 is close to the neutron binding energy B_n and to the heights B of the inner and outer fission barriers. In such states, the Coriolis interaction effect causes [19, 20] a uniform mixing of all possible values of the projection K, with the result that this projection no longer appears in the set of quantum numbers determining statistical properties of the Wigner ensemble [21] of the resonances in question. At the same time, the statistical distribution $P(|K|)$ of projections K for these resonances has a Gibbs character [1, 22]:

$$P(|K|) = A\exp\left\{-\frac{1}{T}\left(\frac{\hbar^2}{2\mathcal{J}_3}-\frac{\hbar^2}{2\mathcal{J}_\perp}\right)K^2\right\} = A\exp\left\{-\frac{K^2}{K_0^2}\right\}, \quad (6)$$

where T is the temperature and $\mathcal{J}_3$ and $\mathcal{J}_\perp$ are the rigid-body moments of inertia of the fissile nucleus for its rotation about, respectively, the symmetry axis of the nucleus and an axis orthogonal to it. In the case of thermalized state of low spin J, in which case the conditions $|K| \le J << K_0$, the distribution in (6) becomes perfectly uniform and independent of $|K|$.

We will now consider the evolution of the distribution $P(|K|)$ of projections K of the the spin of the fissile compound nucleus formed upon the capture of various projectiles by actinide target nuclei in the process of the threshold spontaneous fission of nuclei, the slow-neutron-induced fission of nuclei being a particular case of this process. The initial state of this compound nucleus may be considered as an excited thermalized equilibrium state characterized by an excitation energy E_0^* and by a rather high energy density, while the initial distribution $P_0(|K|)$ of projections K of spins J of these states may be thought to be uniform in the case of J values small in relation to K_0, which was defined in (6). Since, for the fission type under study, the compound-nucleus excitation energy E_0^* is close to the heights B of the inner and outer fission barriers, the wave function describing the mode of the collective deformation motion of the nucleus involved; undergoing mixing with the wave function for its excited thermalized multiquasiparticle state in the first well of the deformation potential; and governing the evolution of collective deformation parameters and, hence, the evolution of the shapes of this nucleus in the fission process up to its separation to fragments is determined by the energy $E_{coll} = E_0^*$. In this case, the collective deformation motion of the fissile nucleus through the inner and outer fission barriers has an adiabatic character, since the nucleon velocities in the nucleus being considered that are determined by this motion are much lower than the velocities of these nucleons in the self-consistent field of the nucleus in the vicinity of the Fermi surface. Therefore, we can introduce cold transition fission states whose spin and its projections are, respectively, J and K [1] and whose penetrability factors for traversing the above barriers are markedly different for different values of J and K. It is these penetrability factors that determine the distributions $P(J,|K|)$ of projections K for different spins J of the compound nucleus after it traverses the outer fission barrier. Here, it is of paramount importance that, as was shown in [23], Coriolis interaction in the second well of the deformation potential of the fissile nucleus proves to be rather weak, not leading to a sizable mixing of different projections K. A substantial change in the distribution $P(J,|K|)$, which is determined by transition fission states, may in principle arise only over a strongly nonadiabatic segment of the collective deformation motion of the fissile nucleus after it traverses the outer fission barrier, but before it undergoes separation to fission fragments. In this case, one can expect the transformation of a significant part of the kinetic energy of the collective deformation motion of the fissile nucleus to the excitation energy of its nucleon degrees of freedom, $\Delta E^* \approx 25$ MeV. If this energy had time to be thermalized before the separation of the fissile nucleus to fragments, its excited equilibrium states characterized by rather high temperature and energy density would be formed. For these states, the distribution $P\left(|K|\right)$ of spin projections *K* would have a Gibbs character [see (6)] and, in the case of low-spin states, would become uniform – that is, independent of *K*. But in this case, the angular distributions of fragments must be, as was shown above, perfectly isotropic, which is at odds with the observation of sizable anisotropies in such distributions for the fission of aligned target nuclei that is induced by unpolarized slow neutrons [24] and for the threshold photofission of nuclei [25]. In [20], it was therefore concluded that thermalization is not reached in fissile-nucleus states of excitation energy ΔE^* that appear in the vicinity of the scission point, and it was assumed there that nonequilibrium excited doorway states of the fissile nucleus that have a high excitation energy but a low energy density appear in this region. Coriolis interaction proves to be weak in these states, and the distribution $P\left(J,|K|\right)$ associated with the emergence of transition fission states and formed at the stages within which the fissile nucleus traverses the inner and outer fissions barriers survives for them. This makes it possible to explain a sizable anisotropy in the angular distributions of fragments originating from the induced threshold fission of nuclei. Since the initial state of an odd spontaneously fissile nucleus coincides with the ground state of this nucleus with spin J_0 in the first well of its deformation potential, the distribution $P_0\left(|K|\right)$ of spin projections *K* in this state is strongly nonuniform and different from zero only at the fixed value of $|K| = J_0$. Because of a deep-subbarrier character of spontaneous fission of nuclei with respect to the inner and outer fission barriers, the distribution in question will not change as the fissile nucleus traverses the fission barriers, surviving up to the stage preceding to the separation of the fissile nucleus to fission fragments. At this stage, its collective deformation motion becomes nonadiabatic, which leads to the transition of this nucleus, in just the same way as in the case of induced threshold fission considered above, to nonequilibrium "doorway" states of excitation energy $\Delta E^* \approx 25$ MeV, which have a low energy density and for which Coriolis interaction mixing different *K* states of the fissile nucleus, is small. Therefore, the angular distribution of fragments originating from the spontaneous fission of oriented nuclei is governed by the initial distribution $P_0\left(|K|\right) = \delta_{|K|,J_0}$. It would be of interest to measure angular distributions of fragments originating from the spontaneous fission of aligned nuclei and to obtain thereby direct arguments in support of the absence of mixing of spin projections *K* in the fission region where collective deformation motion has a strongly nonadiabatic character.

The developed methods have been used also for the analysis of the character of changes in the distributions of spin projections K for fissile nucleus states in the process of target-nucleus fission induced by fast light particles or multiply charged ions.

CONCLUSIONS

The above analysis of conditions necessary for the appearance of anisotropies in angular distributions of fragments originating from the spontaneous and induced fission of oriented fissile nuclei has confirmed once again that Coriolis interaction dynamically enhanced for excited thermalized states of the fissile nucleus between the total spin $\mathbf{J}$ of the nucleus and its intrinsic spin $\mathbf{I}$ plays a very important role in the evolution of the distribution $P\left(|K|\right)$ of projections *K* of the spin $\mathbf{I}$ onto the symmetry axis of the nucleus in the fission process. For basic types of spontaneous and induced low-and high-energy fission of nuclei, we have confirmed the conclusion that the excitation energy E^* of a fissile nucleus in the vicinity of the scission point for its separation to fission fragments features a component ΔE^{*} associated with the appearance of highly excited nonequilibrium "doorway" states of the nucleus that owe their existence to a nonadiabatic character of its collective deformation motion in the energy region indicated immediately above.

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