STATISTICAL MODEL ANALYSIS OF (n,α) REACTION CROSS SECTIONS FOR FAST NEUTRONS

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INTRODUCTION

Systematical study of (n,α) reaction cross sections is of interest for both nuclear energy applications and the understanding of basic nuclear physics problems: on the one hand is important to estimate helium production, nuclear heating and transmutations in the structural materials of fission and fusion reactors; and on the other hand is useful to clarify nuclear reaction mechanisms. In addition, it is often necessary in practice to evaluate the neutron cross sections of the nuclides, for which no experimental data are available.

Slow neutron cross sections are perceptibly varied in connection with narrow resonance structure of the compound states. Therefore, it is difficult to obtain a systematical regularity in the slow neutron cross sections. But, in the case of fast neutrons excited level spacings of the heavy and medium mass compound nuclei are very small and resonance states are not resolved. So, even for quasi-monoenergetic fast neutrons the individual properties of the excited nuclei are averaged over the many states and it is became possible to find some systematical behavior in the cross sections. In 1963-1973 Levkovsky observed [1,2] certain systematical dependence of (n,p) and (n,α) cross sections on the asymmetry parameter neutron and proton numbers (N-Z)/A at neutron energy of 14.5 MeV. Also, he suggested empirical formulae to describe the systematical regularity which in literature is termed as the isotopic effect. In addition, several formulae were proposed [3-11] to explain the isotopic effect in the (n,p) and (n,α) cross sections around neutron energy of 14-15 MeV, only.

In 1993-1994 we observed [12,13] a similar dependence for (n,p) and (n,α) cross sections in the energy range of 8 to 16 MeV. Moreover, the statistical model was suggested [14,15] to describe the isotopic effect in the (n,p) and (n,α) cross sections.

In this paper we have used the statistical model based on the Weisskopf and Ewing theory [16] to carry out a systematical analysis of known experimental (n,α) cross sections. We did not use more detailed Hauser-Feshbach theory because this one is employed the optical potential which depends on the individual properties of the target nuclei.

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1. STATISTICAL MODEL FORMULAE

In order to deduce a formula for (n,α) reaction cross section we can use the statistical model based upon Bohr's postulate of compound mechanism in which nuclear reactions proceed in two stages:

$$\sigma(n,\alpha)=\sigma_c(n)\cdot G(\alpha). \quad (1)$$

Here:

$$\sigma_c(n)=\pi(R+\lambdabar_n)^2 \quad (2)$$

is the compound nucleus formation cross section, where R is the target nucleus radius and $\lambdabar_n$ is the wavelength of the incident neutrons divided by 2π. The α-decay probability of the compound nucleus is expressed as

$$G(\alpha)=\frac{\Gamma_\alpha}{\Gamma}=\frac{\Gamma_\alpha}{\sum_i \Gamma_i}\ , \tag{3}$$

where Γ_α and Γ are the alpha and total level widths. In the framework of Weisskopf-Ewing theory using the principle of detailed balance we can determine the alpha width Γ_α as following:

$$\Gamma_\alpha=\frac{2S_\alpha+1}{\pi^2\hbar^2\rho_c(E_c)}M_\alpha\int_{V_\alpha}^{E_\alpha^{max}}E_\alpha\sigma_c(E_\alpha)\rho_y(U_\alpha)dE_\alpha\ . \tag{4}$$

Here: S_α, M_α, E_α and V_α are the spin, mass, energy and the Coulomb potential for the outgoing α-particle, respectively; $\rho_c(E_c)$ and $\rho_y(U_\alpha)$ are the level densities of the compound and residual nuclei, respectively; U_α is the excitation energy of the residual nuclei; $\sigma_c(E_\alpha)$ is the inverse reaction cross section which is determined in the semi-classical approximation as follows:

$$\sigma_c(E_\alpha)=\begin{cases}\pi R^2\left(1-\dfrac{V_\alpha}{E_\alpha}\right) & for\ E_\alpha>V_\alpha\\ 0 & for\ E_\alpha<V_\alpha\end{cases} \tag{5}$$

If we use the nuclear entropy and constant temperature approximation can get from (4) and (5) the following formula for the α-width:

$$\Gamma_\alpha=\frac{2S_\alpha+1}{\pi\hbar^2}M_\alpha R^2\int_{V_\alpha}^{E_\alpha^{max}}E_\alpha\left(1-\frac{V_\alpha}{E_\alpha}\right)e^{-\frac{B_\alpha+\delta_\alpha+E_\alpha}{\theta}}dE_\alpha\ . \tag{6}$$

Here: B_α is the binding energy of α-particle for daughter nucleus; δ_α is the odd-even effect parameter for Weizsacker's formula [17]; θ is the thermodynamical temperature: $\theta=kT$, where k is the Boltzmann constant. Similar formulae can be written for all partial level widths Γ_i.

Then, neglecting the γ-emission, from (1), (3) and (6) we get [14,15] following expression for (n,α) cross section:

$$\sigma(n,\alpha)=\sigma_c(n)\frac{(2S_\alpha+1)M_\alpha e^{-\frac{B_\alpha+\delta_\alpha+V_\alpha}{\theta}}\left\{1-\frac{W_{n\alpha}}{\theta}e^{-\frac{W_{n\alpha}}{\theta}}-e^{-\frac{W_{n\alpha}}{\theta}}\right\}}{\sum_i(2S_i+1)M_i e^{-\frac{B_i+\delta_i+V_i}{\theta}}\left\{1-\frac{W_{ni}}{\theta}e^{-\frac{W_{ni}}{\theta}}-e^{-\frac{W_{ni}}{\theta}}\right\}}\ , \tag{7}$$

where $W_{n\alpha}=E_n+Q_{n\alpha}-V_\alpha$ and $W_{ni}=E_n+Q_{ni}-V_i$.

For fast neutrons total level width can be approximately taken as $\Gamma\approx\Gamma_n$. Also, the odd-even effect parameters were neglected. In the energy relations can be used the following assumptions: $(E_n+Q_{n\alpha}-V_\alpha)>>\theta$ and $(E_n+Q_{ni}-V_i)>>\theta$. (8)

So, from (7) the fast neutron induced (n,α) reaction cross section is determined as follows:

$$\sigma(n,\alpha)=2\pi(R+\lambdabar_n)^2e^{\frac{Q_{n\alpha}-V_\alpha}{\theta}}\ . \tag{9}$$

Similar formula was obtained by Cuzzocrea *et al.* [18].

The Coulomb potential of α-particle can be written [19] in the following form:

$$V_\alpha = 2.058 \frac{Z-2}{(A-3)^{1/3} + 4^{1/3}} MeV \ . \qquad (10)$$

Weizsacker’s formula for binding energy is used to calculate the (n,α) reaction energy:

$$Q_{n\alpha} = -3\alpha + \beta\left(A^{2/3} - (A-3)^{2/3}\right) + \gamma\left(\frac{Z^2}{A^{1/3}} - \frac{(Z-2)^2}{(A-3)^{1/3}}\right) + \xi\left(\frac{(A-2Z)^2}{A} - \frac{(A-2Z+1)^2}{(A-3)}\right) \pm \left(\frac{\delta_f}{(A-3)^{3/4}} - \frac{\delta_i}{A^{3/4}}\right) + \varepsilon_\alpha \qquad (11)$$

Here: $\varepsilon_\alpha = 28.2$ MeV is the internal binding energy of α-particle; $\alpha = 15.7$ MeV, $\beta = 17.8$ MeV, $\gamma = 0.71$ MeV, ξ=23.7 MeV and $|\delta|$=34 MeV (or 0) are the Weizsacker's constants.

Then, if we neglect the odd-even effect parameter $\Delta = \delta_f - \delta_i$, from (9), (10) and (11) the (n,α) cross section can be written as following

$$\sigma(n,\alpha) = C\pi\left(R + ƛ_n\right)^2 e^{-K\frac{N-Z+0.5}{A}} , \qquad (12)$$

where *N*, *Z* and *A* are the neutron, proton and mass numbers of the target nuclei, respectively;

$$C = 2\exp\frac{1}{\theta}\left(-3\alpha + \beta\left[A^{2/3} - (A-3)^{2/3}\right] + \gamma\left(\frac{Z^2}{A^{1/3}} - \frac{(Z-2)^2}{(A-3)^{1/3}}\right) + \varepsilon_\alpha - V_\alpha\right); \qquad (13)$$

$$\text{and} \quad K = \frac{2\xi}{\theta} . \qquad (14)$$

If we use Fermi gas model for level density parameter [20] the nuclear thermodynamic temperature [21] is expressed as follows

$$\theta = \sqrt{\frac{U_\alpha^{max}}{a}} = \sqrt{\frac{13.5(E_n + Q_{n\alpha})}{A-3}} \qquad (15)$$

The parameters *K* and *C* in formula (12) can be determined by two methods. First, they can be found by fitting of theoretical cross sections to experimental data as constant parameters at each energy point for all isotopes. Second, K and C parameters can be immediately obtained from the formulae (13), (14) and (15).

2. SYSTEMATICS OF (n,α) CROSS SECTIONS

The systematics of known experimental (n,α) cross sections by using the formula (12) at neutron energies of E_n=6, 8, 10, 13, 14.5, 16, 18 and 20 MeV is shown in Fis.1. The values of fitted parameters *C* and *K* are given in Fig.1, also. The Fig.1 shows that theoretical line with the fitted parameters *C* and *K* is in agreement with known experimental reduced (n,α) cross sections. The values of the fitted parameters *C* and *K* for different neutron energies are given in Table.1. It is seen that the parameter *K* is linear in the neutron energy (Fig.2) and at the same time the parameter *C* is slowly increased and after that decreased with maximum value at around 13 MeV

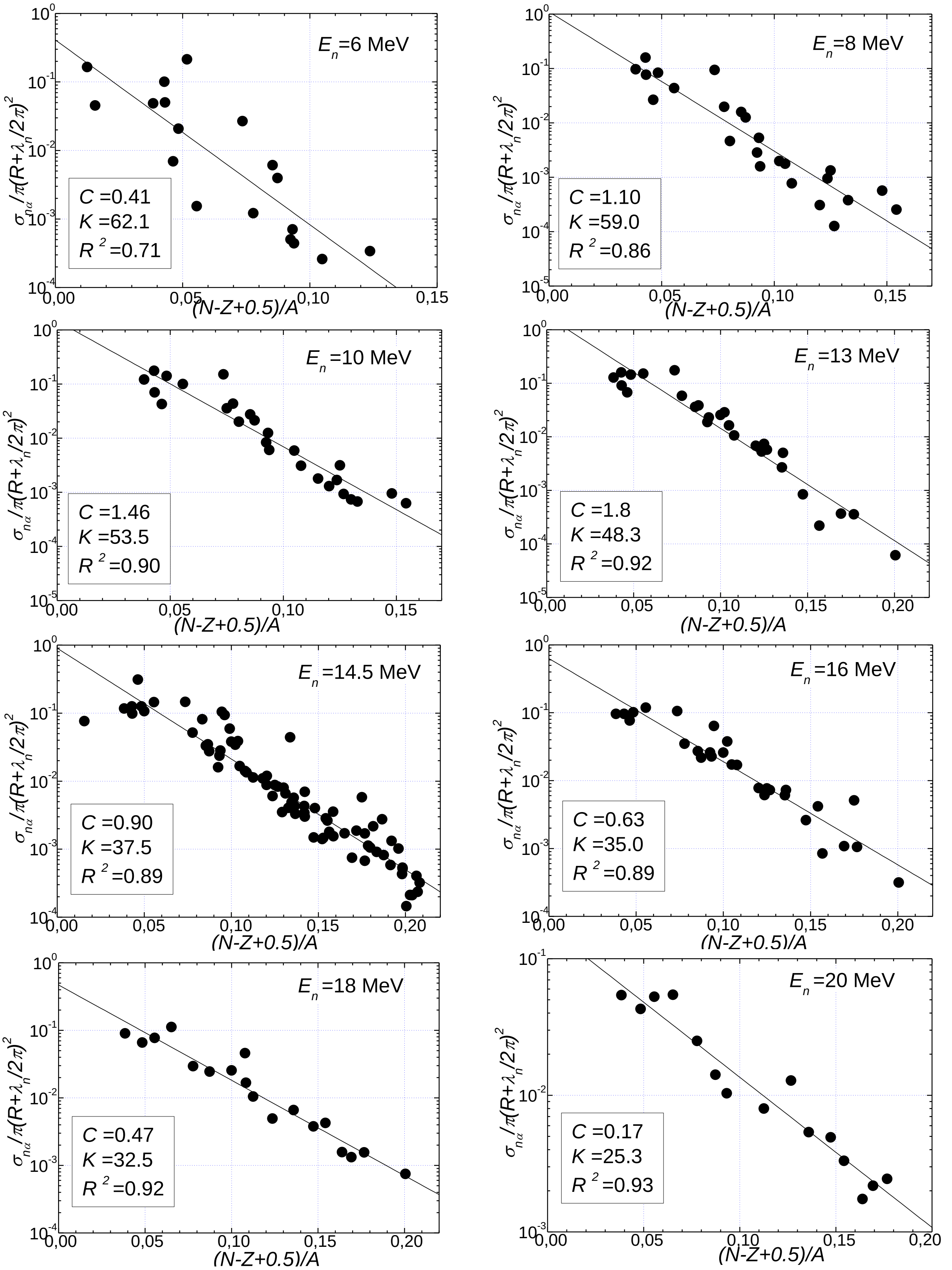

Fig.1. The dependence of reduced (n,α) cross sections on the asymmetry parameter of neutron and proton numbers *(N-Z+0.5)/A* for neutron energies of E_n=6, 8, 10, 13, 14.5, 16, 18 and 20 MeV.

Table.1. The parameters K and C for different neutron energy

E_n (MeV)	K	C
6	62.1	0.41
8	59.0	1.10
10	53.5	1.46
13	48.3	1.80
14.5	37.5	0.90
16	35.0	0.63
18	32.5	0.47
20	25.3	0.17

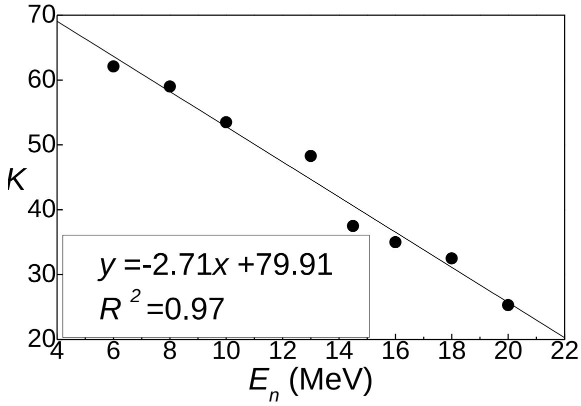

Fig 2. The energy dependence of the parameter *K*.

3. THE COMPARISON OF THEORETICAL AND EXPERIMENTAL (n,α) CROSS SECTIONS

The comparisons of the absolute values for theoretical (n,α) cross sections calculated by statistical model with known experimental data at neutron energies of 6 to 20 MeV are shown in Fig.3. It was observed that statistical model formulae (12), (13), (14), and (15) give overestimated values for the (n,α) cross sections at all energy points. These results, possibly, are caused by the α-particle clusterization effect [22-24].

4. THE (n,α) CROSS SECTION AND α-CLUSTERIZATION FACTOR

The α-clusterization factor which is taken into account α-particle formation probability on the surface of nuclei was not considered in the formulae (7) and (12). So, the formula (7) is correct for neutron induced nucleon emission reactions [25]. As to (n,α) reaction the α-clusterization effect should be considered in the cross section. In order to evaluate the α-particle formation factor or reduced α-width Bethe suggested [26] to use the reduced neutron width:

$$\gamma_n^2 \approx \gamma_\alpha^2 . \tag{16}$$

Yu.P.Popov *et al.* investigated this hypothesis by using the experimental data of (n,α) reaction for resonance neutrons [22-24] and found a following relation for the reduced average neutron- and alpha-widths:

$$W_{n\alpha} = \frac{\langle \gamma_n^2 \rangle}{\langle \gamma_\alpha^2 \rangle} \approx 2.5 - 8.0 \tag{17}$$

If we use average value of $W_{n\alpha} \approx 4.5$ and assume $\langle \gamma_n^2 \rangle \approx \langle \gamma_p^2 \rangle$, then can write the following relation for the reduced average proton- and alpha-widths:

$$W_{p\alpha} = \frac{\langle \gamma_p^2 \rangle}{\langle \gamma_\alpha^2 \rangle} \approx 4.5 \tag{18}$$

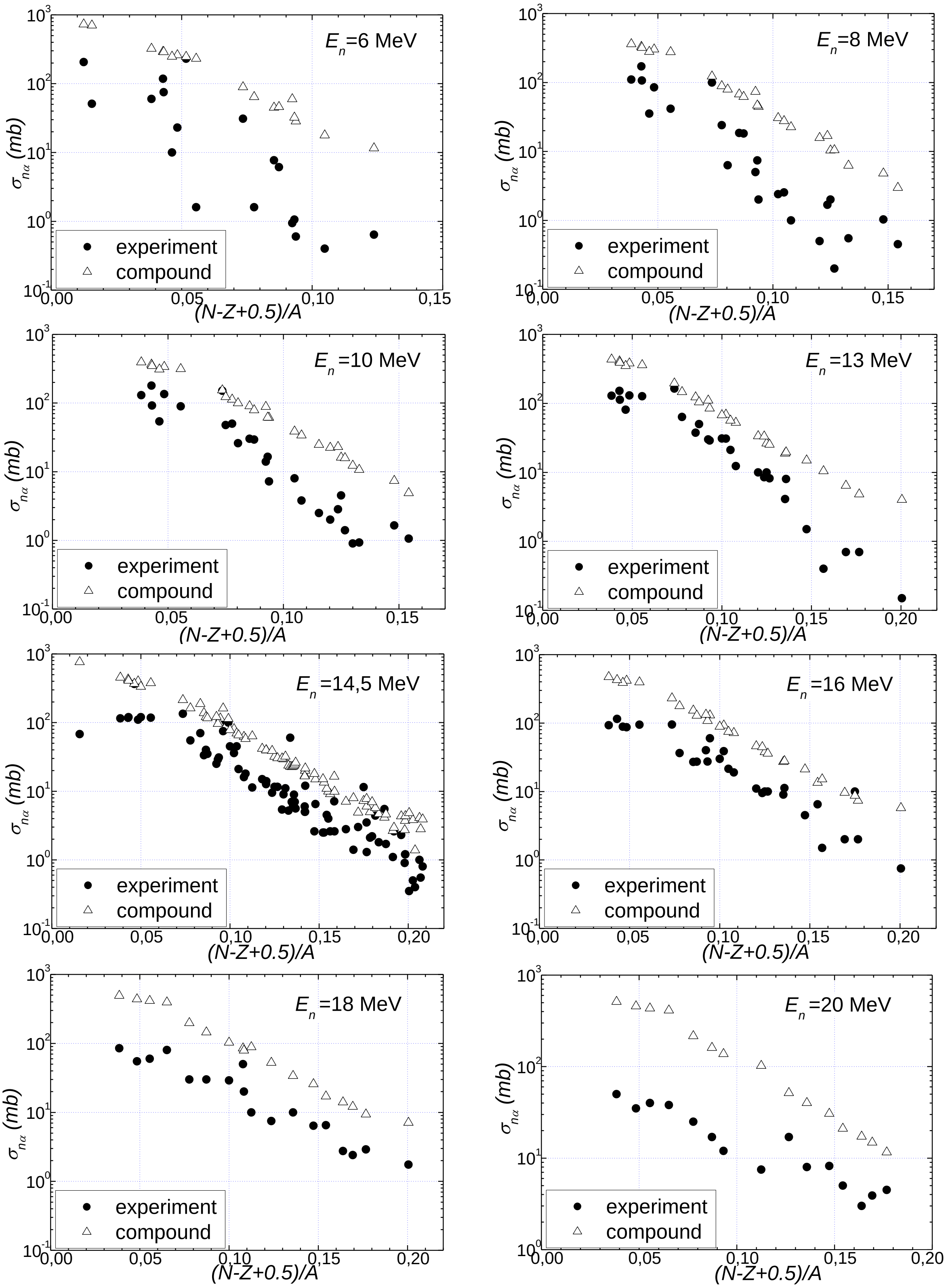

Fig.3. The (n,α) cross sections calculated by statistical model and experimental data at neutron energies of 6, 8, 10, 13, 14.5, 16, 18 and 20 MeV

So, theoretical (n,α) cross sections (7) and (12) should be divided by factor of $W_{p\alpha}$=4.5 to compare with experimental data. In addition, it should be noted that similar formula without clusterization factor for (n,p) reaction gives results which are satisfactorily in agreement with experimental (n,p) cross section [27].

The comparison of experimental data and theoretical (n,α) cross sections divided by α-clusterization factor for different neutron energy is given in Fig.4. It is seen that theoretical and experimental (n,α) cross sections are in agreement in the energy range of 6 to 18 MeV. In the case of E_n=20 MeV we obtained α-clusterization factor $W_{p\alpha}$=12 by normalization of theoretical cross section to experimental data for $(N\text{-}Z+0.5)/A \leq 0.10$. Also, Fig. 4 shows that the pre-equilibrium and direct reaction mechanisms should be considered at neutron energy of E_n=20 MeV for the asymmetry parameter of $(N\text{-}Z+0.5) / \geq 0.10$. In addition, we can conclude that α-clusterization factor for (n,α) reaction depends on the neutron energy and the asymmetry parameter of proton and neutron numbers $(N\text{-}Z+0.5)/A$ is essential for the nuclear reaction mechanisms.

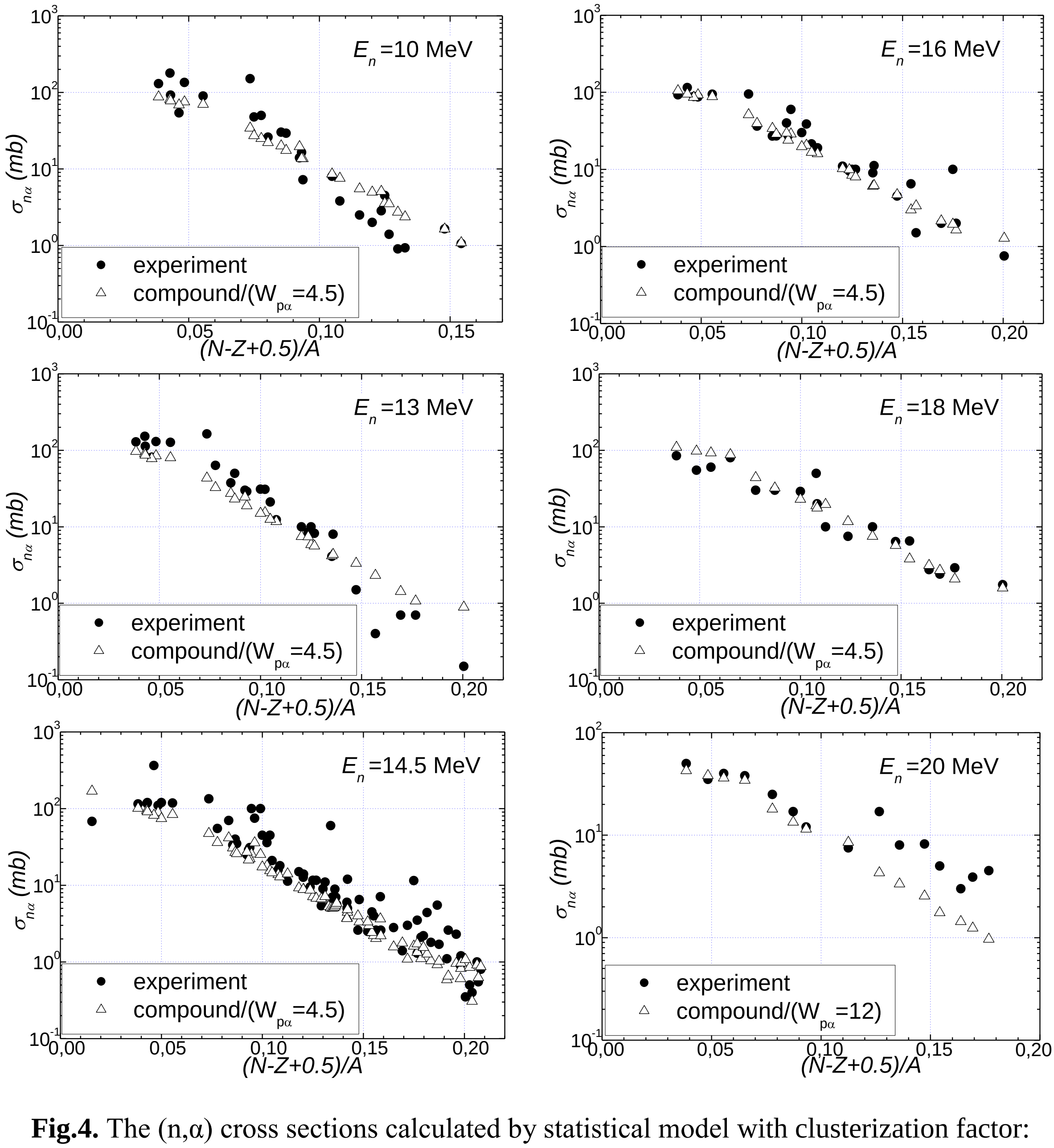

Fig.4. The (n,α) cross sections calculated by statistical model with clusterization factor: $W_{p\alpha}$=4.5or 12 in comparison with experimental data at E_n=10, 13, 14.5, 16, 18, 20 MeV

CONCLUSIONS

1. The statistical model based on the Weisskopf and Ewing theory was used for systematical analysis of the fast neutron induced (n,α) reaction cross sections. It was found that the reduced (n,α) cross sections depend on the asymmetry parameter of neutron and proton numbers for the target nuclei at neutron energy of 6 to 20 MeV.
2. The comparison of the theoretical and experimental (n,α) cross sections shows that statistical model formula gives overestimated values at all energy points of neutrons.
3. The discrepancy between the theoretical and experimental (n,α) cross sections was explained by the α-clusterization effect on the surface of nuclei. It was shown that the α-clusterization factor for (n,α) reaction depends on the neutron energy.

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