VERIFICATION OF THE WEAK EQUIVALENCE PRINCIPLE WITH LAUE DIFFRACTING NEUTRONS. RESOLUTION TEST AND SYSTEMATICS ANALYSIS

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Abstract

We propose a novel experiment to test the weak equivalence principle (WEP) for the Laue diffracting neutron. Our experiment is based on an essential magnification of an external affect on neutron diffracting by Laue for the Bragg angles close to the right one in couple with additional enhancement factor which exists due to the delay of the Laue diffracting neutron at such Bragg angles.This enhancement phenomena is proposed to be utilized for measuring the force which deviates from zero if WEP is violated. The accuracy of measuring inertial to gravitational neutron masses ratio for the introduced setup can reach 10^{-5}, that is more than one order superior to the best present-day result. In this paper we will present the results of the resolution test and also review possible systematic errors.

1 Introduction

For the macroscopic bodies weak equivalence principle is verified with unprecedented accuracy of $10^{-12} \div 10^{-13}$ by torsion-balance experiments. The best results were obtained by Adelberger [1] and Baeßler [2]. The situation for elementary particles is not so optimistic and the best result was attained for the neutron only with accuracy of $2 \cdot 10^{-4}$ by Schmiedmayer [3]. There are several ongoing projects trying to use neutron as a test mass in the weak equivalence principle verification. For instance, Alexander Frank and his group recently proposed an experiment aimed at testing WEP for ultra-cold neutrons [4]. It seems that ultra-cold and cold neutrons are the most suitable objects for investigations of gravitational interaction in the world of elementary particles.

Moreover if one considers a neutron undergoing Laue diffraction conditions (not a freely fliyng particle) inside the crystalline medium all of external impacts on it become appreciably magnified. In the case of diffraction for the Bragg angles close to the right one this magnification consists of two parts. One of them is a well known diffraction enhancement factor[1] or decreasing of diffracting neutron effective mass [5], another is additional enhancement which arises due to the time of diffracted neutron delay inside the crystal and is proportional to $tan^2(\theta_B)$ [6]. The total diffraction enhancement factor may be as large as 10^9 [7]. So, it becomes interesting to try to utilize this enhancement phenomenon for investigation of external affects acting on a diffracting neutron (for instance, gravity) [8].

[1]In response to a small change in the direction of motion of the incident particle (or photon) within the Bragg width (several seconds of arc), the direction of motion of the particle within the crystal changes by the Bragg angle.

In the present work we will discuss theoretical treatment of crystal-diffraction experiment aimed at testing WEP for the neutron, and we will show that dynamical diffraction predictions work well for the large ($150 \times 150 \times 220\,mm^3$) silicon crystal and also will discuss the possible resolution of the experimental setup and systematic effects.

2 Neutron Laue diffraction in the presence of an external force

Let us consider symmetrical Laue diffraction in a transparent crystal with the system of crystallographic planes described by the reciprocal lattice vector g normal to the planes (see Fig. 1), $\mathbf{g = 2\pi/d}$, $\mathbf{d}$ is the interplanar distance. Generally in this case one is using two-beam approximation of diffraction, when neutron wave function inside the crystal is formatted as a superposition of two Bloch waves $\psi(1)$ and $\psi(2)$ corresponding to two branches of dispersion surface [9]. There is also special theory which describes diffraction in deformed crystal [10]. The deformation of crystal means that the value and direction of vector g slightly differs for different points inside the crystal, i.e. g depends on the spatial coordinates Y and Z. As it follows from [10] we can implement effective "Kato forces", which are directed along g and their values are determined by the crystal deformation.

In terms of the "Kato force" neutron trajectories[2] in deformed crystal is determined by the equation

$$\frac{\partial^2 z}{\partial y^2} = \pm \frac{c}{m_0} f_k(y, z), \tag{1}$$

where $c \equiv \tan\theta_{\rm B}$ and $m_0 \equiv 2F_{\rm g}d/V$ is the so-called "Kato mass", $F_{\rm g}$ is the neutron structure amplitude and V is the unit cell volume. The sign $\pm$ in equation (1) corresponds to different Bloch waves.

It can be easily demonstrated that putting an undeformed perfect crystal in a force field affecting the neutron along the reciprocal lattice vector g, we will have the same result as for a deformed crystal. So, the theory which was developed for weakly deformed crystals also works well in the presence of any external field affecting the diffracting neutron in undeformed crystal. An external field affecting a diffracting neutron was first considered in [11].

It is also easy to show that a constant external force $\mathbf{F}_{\rm n}$ acting on a neutron along vector g (Z axis, see Fig. 1) is equivalent to a gradient of interplanar distance with the value [7]

$$\xi_{\rm f} = \frac{F_{\rm n}}{2E_{\rm n}}, \tag{2}$$

where $E_{\rm n}$ is the neutron energy.

As it was shown in [7] the "curvature" of the diffracting neutron trajectory in the crystal is magnified by the factor

$$K_{\rm e} = \pm \frac{c^2 g}{2m_0}, \tag{3}$$

in comparison with free neutron.

[2]By the neutron trajectory, we henceforth imply the "Kato trajectory", the tangent to which at each point determines the direction of the neutron flux density in the crystal[10].

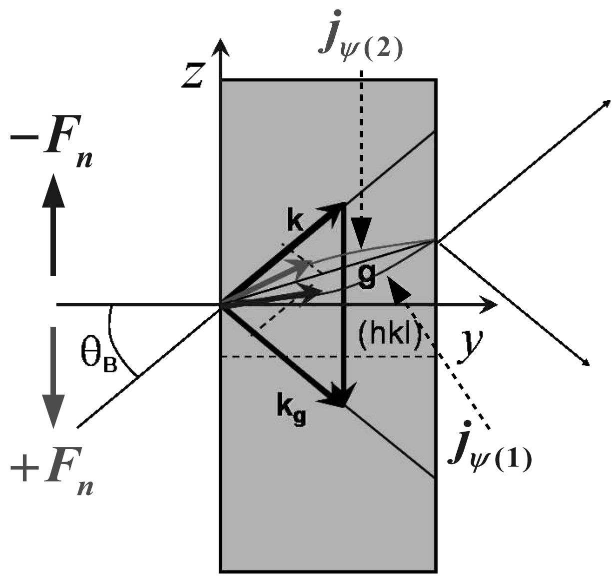

Figure 1: Scheme of Laue diffraction in a single crystal in the presence of external force acting along the reciprocal lattice vector. $j_{\psi(1)}$ and $j_{\psi(2)}$ are the so-called "Kato trajectories" for the different types of Bloch waves.

This factor depends on the Bragg angle as $c^2 \equiv \tan^2\theta_{\rm B}$, so for Bragg angles $\theta_{\rm B} \approx (84{-}88)^0$ influence of deformation can be intensified by a factor $\sim 100-1000$ as compared with a Bragg angle of $\sim 45^0$.

The numerical calculation of the factor $K_{\rm e}$ for (220) silicon crystallographic planes [7] gives

$$K_e^{(220)} = \pm 0.85 \cdot 10^8, \tag{4}$$

for a Bragg angle $\theta_{\rm B} = 87^0$ (c=20).

Therefore, a 10 cm long crystal is equivalent to $\sim$ 1 km of free flight. The diffraction enhancement of the angular deflection of a neutron trajectory inside a crystal is well known, see for instance [5], but we have to note that such an effect can be considerably magnified by an additional gain factor proportional to $\tan^2\theta_{\rm B}$ for Bragg angles close to $\pi/2$ [7]. The observed effects give us a chance to build a device with unprecedented sensitivity to external force acting on a neutron.

Principle scheme of the setup is based on two-crystal scheme of diffraction as it is shown in Fig. 2. The necessary high collimation of the beam was provided by the first crystal with slits placed on inlet and outlet surfaces, for details see [13]. An external force which is parallel to the reciprocal lattice vector curves the neutron trajectories inside the crystals. This results in a shift of the neutron beam along the outlet surface of the second crystal with a value [12, 13]:

$$\Delta_{\rm S} = \frac{\pi\, c^2 L^2}{m_0 d\, E_{\rm n}} F_{\rm n}. \tag{5}$$

The resolution of the external force, i.e. magnitude of force when the neutron beam shift $\Delta_{\rm S}$ is equal to the slit size δ_s, is equal to [12, 13]:

$$W_{\rm F} = \frac{m_0 d\, E_{\rm n}}{\pi\, c^2 L^2} \delta_s \tag{6}$$

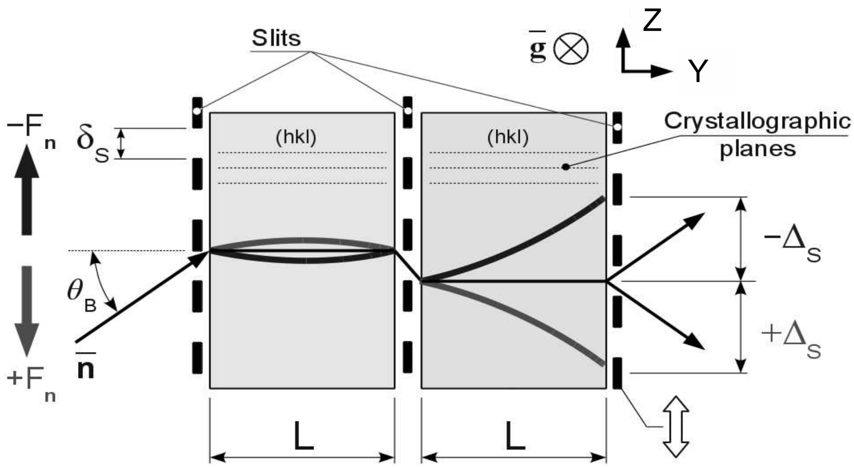

Figure 2: Two-crystal scheme of Laue diffraction. Collimating slits are situated at the inlet and outlet crystal faces.

One of the applications can be connected with the measurement of inertial to gravitational neutron mass ratio (In fact, direct test of WEP). The idea of this experiment was discussed in [8, 13]. The detailed analysis of this experiment can be found in [12, 14]. Our Earth is moving at a stationary orbit around the Sun, it means that the gravitational force which is proportional to the gravitational mass is in balance with the centrifugal force which is proportional to the inertial mass. If this is not so for free neutrons, then in the coordinate system connected with the Earth a free neutron will feel a non zero force[3]

$$F_{\mathrm{n}} = \frac{(m_i - m_G) \cdot GM_S}{R_S^2} \approx \Delta_{Gi} \cdot 6 \cdot 10^{-4} m_G g \tag{7}$$

where m_G and m_i are the neutron gravitational and inertial masses, G is the gravitational constant, M_S is the mass of the Sun, g is a free fall acceleration, R_S is the distance to the Sun, $\Delta_{\mathrm{Gi}} \equiv (m_i - m_G)/m_G$. The presence of this external perturbing force must lead to the bending of the neutron trajectory in the crystal. Moreover, this force will oscillate in the laboratory coordinate system with one day period due to the Earth spinning motion. Hence bending of the neutron trajectory which will lead to the beam spot shift also depends on the period of the day.

3 Analysis of the possible systematic errors

By the systematic effects we first of all mean influence of non-inertial forces (tidal forces, Coriolis force) which exist on the surface of the Earth and occur as well in our laboratory system of coordinates (LSC).

We made theoretical estimations of such impact on the moving neutron in the lab coordinates [14]. It has already been mentioned above that force, which deviates from zero if WEP is violated (7), oscillates with one day period. In [14] we calculated the amplitude of

[3] The idea of this experiment is an analogue to the well known Eõtvõs experiment for the equivalence principle test [15].

this oscillations and it's equal to $\Delta_{Gi} \cdot 5,94 \cdot 10^{-3} m/s^2$, where $\Delta_{Gi} = (m_i/m_G - 1)$ – is the parameter that characterize equality of inertial m_i and gravitational m_G masses of the test object. At the same time it's well known fact that tidal forces have the half day period. So, it becomes easy to eliminate spurious effect from tidal forces.

Coriolis force doesn't depend on time, but acts only on the moving object (in our LSC it's a diffracting neutron). So, we should control the position of the working crystal with external device which would be at rest in LSC (for details see [12], [14]). Moreover there is an easy way to measure the influence of Coriolis force acting on a neutron in our experimental scheme. One should just make measurements with two antiparallel crystal positions with the same diffraction angle and hence obtain unlike signs of Coriolis force. The experiment of such type were performed by Raum et.al. [16].

4 Experimental setup resolution test

As a first step we performed a test experiment aimed at investigation of dynamical diffraction for large silicon crystal and for the Bragg angles close to the right one [17]. In the case of Laue diffraction at Bragg angles close to $\pi/2$, the effect known as the Borrmann effect (anomalous absorption) [18] and associated with different absorption of neutron waves in a crystal that correspond to different branches of the dispersion surface begins to play a significant role (see [9, 19]). Moreover we note that the presence of the anomalous absorption effect (that is, a substantially weaker absorption of one of the neutron waves) makes it possible to perform experiments at large values of the working crystal thickness (in excess of 200 mm) and large diffraction angles (of up to 88°).

The experiment was performed in the second horizontal beam of the WWR-M reactor of the Petersburg Nuclear Physics Institute (PNPI, Gatchina). We studied neutron Laue diffraction at Bragg angles close to $\pi/2$ at the system of (220) crystallographic planes (the interplanar spacing is $d = 1.92 \cdot 10^{-8} cm$ of a silicon single crystal characterized by dimensions of $150 \times 150 \times 220\, mm^3$). The single crystal used has a transverse cut at a half-thickness for ensuring investigations according to the two-crystal scheme (see Fig. 2). The collimation of the neutron beam is implemented with the aid of two slits at the inlet surface and at the cut of the crystal. A movable slit at the outlet face makes it possible to scan the spatial displacement of the beam (see Fig. 2). It is planned to use this in our experiment aimed at testing the weak equivalence principle for the neutron. The working crystal was placed in a thermostat in order to minimize the crystal deformation effect that stems from the possible appearance of temperature gradients and which may lead to variations in the interplanar spacing over the crystal volume. The thermostat used maintains the crystal temperature to a precision of about 0.01 K/day. The respective gradient of the interplanar spacing in the crystal does not exceed $10^{-12} cm^{-1}$ [12], having a negligible effect on the intensity of the diffracted beam. The theoretical calculations that take into account the anomalous absorption effect predict a sizable intensity up to the Bragg angle value of 88^0, and our experiment confirmed this. For more details see [17].

Subsequently we made an experimental test of the two-crystal setup possible resolution. In fact, as it been already mentioned, by resolution we mean the magnitude of external force which shifts the neutron beam by the outlet slit size. Measurements were made for the fixed Bragg angle value of 78^0 and slits size was 15 mm wide. On the Fig. 3 one can see

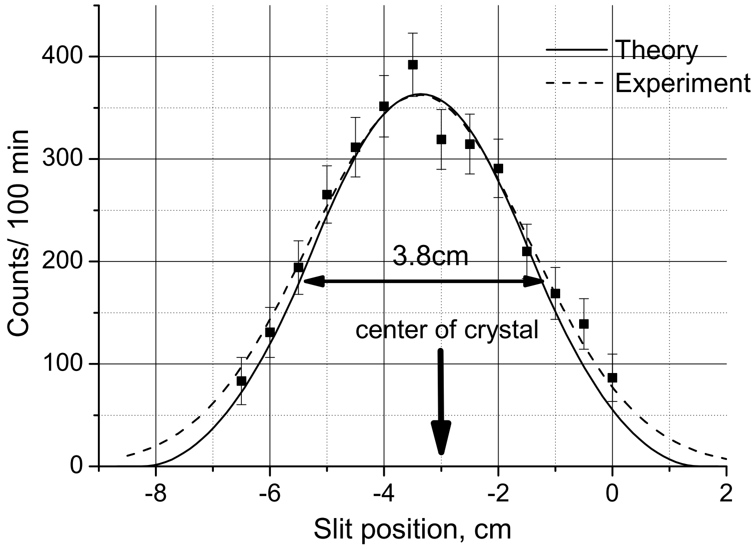

Figure 3: Measured and predicted intensity distribution as a function of the crystal outlet surface slit position.

experimental intensity distribution as a function of the outlet surface slit position relatively to crystal. Theoretical dependence (solid line) has been calculated for such geometry and normalized to neutron beam flux. From Fig. 3 one can see that without a presence of external force neutron spot at the outlet surface is localized at the center of the crystal (along Z axis, see Fig. 2). This fact coincides with dynamical diffraction predictions for the two-crystal scheme [12]. The resolution value was calculated from experimental data and was equal to

$$W_F^{exp} \approx 4 \cdot 10^{-11}\, eV/cm.$$

The main problem during the experiment was a low luminosity of the setup and because of that we couldn't make a resolution test for the higher Bragg angles and more narrow slits. In the full-scale experiment we plan to work at the Bragg angle $\theta_B = 86^0$ and with the slits size value $\delta_S = 2mm$. Consequently the resolution should be improved in 75 times

$$W_F \approx 5 \cdot 10^{-13}\, eV/cm \approx 5 \cdot 10^{-4} m_n g,$$

where m_n is a neutron rest mass and g is a free fall acceleration.

5 Conclusions

Effect of neutrons anomalous absorption (Borrmann effect) together with the phenomena that consist in the diffraction enhancement and the "slowing down" in a crystal of a neutron undergoing Laue diffraction at Bragg angles close to $\pi/2$, opens new prospects for creating an experimental setup for studying small external forces acting on a neutron. Owing to anomalous absorption we were able to reach diffraction angles of $86^0 \div 88^0$. In this case sensitivity of our method to the applied force at such values of the angle is approximately 300 times higher than that at Bragg angles around 45^0.

Observed dynamical diffraction effects discussed in this paper give a chance to measure any small external force acting on a neutron with unprecedented sensitivity. Preliminary estimations and test experiment [17] showed us that the possible resolution of the experimental setup can reach the magnitude

$$W_F \cong 5 \cdot 10^{-13}\, eV/cm.$$

This extremely high resolution provides us the possible sensitivity to the external force on the level of $10^{-17}\, eV/cm$ for the available silicon crystal and cold neutron beam flux. For instance, in measuring of m_i/m_G ratio we plan to attain the accuracy $\sigma(m_i/m_G) \sim 10^{-5}$. This is more than one order of magnitude superior to the best present-day value for neutron [3].

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