REAL AND APPARENT ENHANCEMENTS OF THE FUNDAMENTAL SYMMETRY BREAKING EFFECTS

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Abstract– It is shown that not all the enhancements of the fundamental symmetry–breaking effects allow to increase the accuracy of their measurements. The relative error of the effect is suggested as criterion of its measurement accuracy rather than the effect's magnitude. It is shown that the kinematical or structural enhancements practically do not affect the magnitude of this error and thus do not increase the accuracy of the effect's measurements. Examples of some artificial normalizations are given which cause the non-physical enhancements of the effects.

1. ENHANCEMENT OF THE PARITY NON-CONSERVATION EFFECTS IN THE RADIATION TRANSITIONS BETWEEN THE BOUND STATES OF THE NUCLEI.

Consider the simplest case of the parity non-conservation in the radiation transitions between the nuclear excited states. The source of these effects is the weak interaction V_W leading to the fact that the wave function ψ_i of this state contains, besides the wave function of a definite parity ψ_1, the small admixture ψ_2 of the opposite parity state:

$$\psi_i = \psi_1 + c\psi_2 \tag{1}$$

The admixture coefficient in the first-order perturbation theory is:

$$c = \frac{<\psi_2 | V_W | \psi_1>}{| E_1 - E_2 |} \equiv \frac{v_p}{D} \tag{2}$$

Here v_p is the matrix element of the weak interaction which causes P-non-conservation, while D is the energy difference between the two mixing nuclear levels of the opposite parity.

The earliest systematic studies of the possible enhancements of parity non-conservation in radiation transitions between the compound states of the excited nuclei were carried in the review paper of I.S.Shapiro [1]. It was pointed that these effects are defined by the relation:

$$R = \frac{(M_a M_f)}{(M_a + M_f)^2} \tag{3}$$

Here M_a and M_f are the amplitudes of the parity-allowed and parity-forbidden transitions respectively. The amplitude M_f contains the admixture coefficient (2) . $|M_f| \ll |M_a|$ since the ratio of the weak to strong interaction constants is $F \approx 10^{-7}$, and therefore:

$$R \approx \frac{M_f}{M_a} = \alpha F \tag{4}$$

The factor α is defined by the structure of the states connected by the radiation transition and may contain the enhancement factors. The review [1] indicates 3 types of enhancement:
1) kinematical enhancement, 2) strutural enhancement and 3) dynamical enhancement. The kinematical enhancement appears when the allowed transition is the magnetic one with multipolarity L: $M_a = M(ML)$, while the forbidden one is the electric transition of the same

multipolarity: $M_f = F \cdot M(EL)$. Since in otherwise equal conditions the magnetic transition amplitude is suppressed with respect to the electric one by the factor v/c (v is the nucleon velocity in the nucleus and c is the light velocity), the enhancement factor in this case would be

$$\alpha \approx \frac{c}{v} \approx 10 \tag{5}$$

The structural enhancement appears when the allowed transition amplitude comes to be unusually small due to some suppression caused by the structure of the initial and final states.

One should point that both the kinematical and structural enhancements arise because of the decrease of the denominator in Eq. (4). Only the dynamical enhancement is caused by the increase of the numerator in (4). This enhancement appears for the transitions between the nuclear compound states since the admixture amplitude (2) of the opposite parity states caused by the weak interaction is inversely proportional to the energy distance D between the levels of the opposite parity. It is well known (see e.g. [2]) that for the complicated wave functions containing N basic components the level spacing decreases by approximately N times with respect to the simple single-particle states. However the matrix element v_p of the compound levels admixture due to the weak interaction also decreases (see e.g. [3,4]) by approximately the factor of $\sqrt{N}$. Therefore the total increase of the amplitude M_f in the numerator of (4) due to the dynamical enhancement is approximately $\sqrt{N} \approx 10^2 \div 10^3$. It is obvious that the effect (4) increases by the same factor.

However from the physical point of view we should be interested by such enhancements which allows to measure the symmetry breaking effects with the maximal precision, i.e. with the minimal relative error σ_R / R. Indeed, of the real value are not the record-breaking magnitudes of the effects but rather the most precise definition of the P-violating interaction constants which define the magnitude of the parity-forbidden amplitude M_f. It is believed usually that just the maximal value of the effect allows to measure these constants with the maximal precision. This however is easily proved to be wrong. Let us consider the magnitude (4) of the effect as the ratio of the numerator n to the denominator d which are normally distributed around their average values $\bar{n}$ and $\bar{d}$ with the variances σ_n and σ_d. Then the relative error in measuring the effect would be:

$$\frac{\sigma_R}{R} = \sqrt{\frac{\sigma_n^2}{\bar{n}^2} + \frac{\sigma_d^2}{\bar{d}^2} - 2\frac{\rho_{nd}\sigma_n\sigma_d}{\bar{n}\bar{d}}} \tag{6}$$

By neglecting the correlation factor ρ_{nd} and assuming that the absolute errors of the numerator and the denominator are equal:

$$\sigma_n = \sigma_d = \sigma \tag{7}$$

one can write the relative error in the form:

$$\frac{\sigma_R}{R} = \sqrt{\frac{\sigma^2}{\bar{n}^2} + \frac{\sigma^2}{\bar{d}^2}} \tag{8}$$

One can see from this expression that only the dynamical enhancement increasing the value of the numerator in (4) causes the decrease of the relative error in measuring the effect, that is the increase of the measurement precision. The kinematical and the structural enhancements do decrease the $\bar{d}$ value, but do not cause the decrease of the error in measuring the effect. Ex. (8) shows that these enhancements might lead only to the increase of this error, i.e. to the decreasing precision of the effect measurement. Since in the majority of cases $\bar{d} \gg \bar{n}$ even in the presence of the enhancements, the relative error (8) practically does not change in the cases of the kinematical and the structural enhancements of the effect. Since however the magnitude R of the effect increases, the magnitude of the its absolute error σ_R also increases to the same extend, so the precision of the effect measurement decreases.

2. THE ENHANCEMENT OF THE P-VIOLATION EFFECTS IN NEUTRON TRANSMISSION THROUGH THE UNROLARIZED TARGET

Thus if we chose among the various possibilities of the P-violation effects measurements in the radiation transitions those which allow us to measure the effect with maximal precision, one should compare not the magnitudes of the effects measured but rather the relative errors in heir measurements. Of course the maximal precision of the measurement would be received in the case of the minimal relative error.

The additional advantage in using the relative error criterion is the fact that it demonstrates the correct dependence of the measurement precision on the various experimental parameters (the flux intensity of the particles measured, the time interval which is necessary for reaching the given accuracy, the experiment geometry etc.). For demonstration consider the measurements of the P-odd correlation $(\hat{\boldsymbol{\sigma}}_n \cdot \boldsymbol{k}_n)$ between the neutron polarization vector $\boldsymbol{\sigma}_n$ and momentum $\boldsymbol{k}_n$ in polarized neutron transmission through the unpolarized target. The experimentally measured effect is defined by the relation:

$$P_{\exp} = \frac{N_+ - N_-}{N_+ + N_-} \tag{9}$$

where $N_\pm$ is the number of neutrons with helicity $\pm$ which pass through the target during the time of the measurement.

Let us analyze the dependence of this effect on the target thickness x. The magnitudes $N_\pm$ of the numbers of neutrons with given helicity, which pass through the target with thickness x, is given by the expression:

$$N_\pm(x) = N_0 \exp(-x\rho\sigma_{tot}^\pm) \ , \tag{10}$$

where N_0 is the number of neutrons incident on the target during all the time of the measurement, ρ is the target density (the number of nuclei per unit volume), while $\sigma_{tot}^\pm$ is the total cross section for the neutrons with given helicity on a given target. Let us present the total cross section in the form:

$$\sigma_{tot}^\pm = \sigma_{tot}^0 \pm \frac{\Delta_{tot}^P}{2} \ , \tag{11}$$

where σ_{tot}^0 is the total cross section for the unpolarized neutrons and

$$\Delta_{tot}^P = \sigma_{tot}^+ - \sigma_{tot}^- \tag{12}$$

is the total cross section difference for the neutrons of different helicities, which is proportional to the imaginary part of the neutron scattering amplitude caused by the weak interaction (see e.g. [5–7]). The effect magnitude (9) as a function of the target thickness would be:

$$P_{\exp}(x) = \tanh(x\rho\Delta_{tot}^P / 2) \tag{13}$$

Thus we see that the effect magnitude increases monotonically with the increasing target thickness and tends to unity for the infinite thickness. Because of the P-odd amplitude smallness of the neutron–nucleus interaction one can assume that for the reasonable target thickness $x\rho\Delta_{tot}^P / 2 \ll 1$. In this case:

$$P_{\exp}(x) \approx x\rho\Delta_{tot}^P / 2 \tag{14}$$

We see that in this approximation the effect is also proportional to the target thickness. But this does not mean that the maximal precision in measurement of the effect (and, therefore, of the P-odd amplitude) would be reached for the target with the maximal thickness. Indeed, let us estimate the relative error of the effect measurement, taking into account that the relative error in measuring the number of neutrons transmitted through the target is defined by the expression::

$$\frac{\sigma_N}{N_\pm} = \frac{1}{\sqrt{N_\pm(x)}} = \frac{1}{\sqrt{N_0}} \exp(x\rho\sigma_{tot}^{\pm}/2) \tag{15}$$

It follows from this expression that the relative error of measured value (9) is:

$$\frac{\sigma_N}{N_\pm} = \frac{1}{\sqrt{N_\pm(x)}} = \frac{1}{\sqrt{N_0}} \exp(x\rho\sigma_{tot}^{\pm}/2) \tag{16}$$

The minimal magnitude of this error is reached for

$$x\rho = \frac{2}{\sigma_{tot}^0} \tag{17}$$

Inserting this value into (14) we obtain for the effect magnitude

$$P_{\exp} \approx \frac{\Delta_{tot}^P}{\sigma_{tot}^0} \tag{18}$$

Just this value is measured in the experiment by choosing the target thickness of the order of the neutron mean free path in it. The effect's relative error for this choice of the target thickness is;

$$\frac{\sigma_{\exp}}{P_{\exp}} \approx \frac{e}{\sqrt{2N_0}} \frac{\sigma_{tot}^0}{\Delta_{tot}^P} \tag{19}$$

This formula also allows to estimate the exposition time necessary to measure the value Δ_{tot}^P with the desired accuracy for the given neutron flux of the experimental source.

The problems of enhancement of the symmetry–breaking effects for the transmission experiments (as well as for all experiments with polarized neutrons) are complicated by the fact that both the numerator and the denominator of the expression (18) depend on the incident neutron energy. Of major interest is the energy in the vicinity of the p-wave resonance. In this region (see [3,4]):

$$\Delta_{tot}^P(E) \approx \frac{4\pi}{k^2} \frac{v_p}{D} \frac{\sqrt{\Gamma_s^n \Gamma_p^n} \cdot \Gamma_p}{(E - E_p)^2 + \Gamma_p^2/4} \tag{20}$$

Here v_p is the matrix element of the compound nucleus s– and p–resonance mixing due to the weak interaction, D is the energy spacing of the two mixing resonances, $\Gamma_{s,p}^n$ are the partial neutron widths of these resonances, E_p and Γ_p are the p–resonance energy and total width. Expression (18) contains all the enhancement factors of the observed effect $P_{\exp}(E)$ which lead to the decrease of its relative error (19). Firstly, the ratio v_p/D contains the dynamical enhancement factor $\sqrt{N} \approx 10^3$ likewise in the case of bound states. Secondly, in transition from the non–resonance energies $|E - E_p| \approx |E - E_s| \approx D/2$ to the p-resonance energy the resonance enhancement of expression (20) takes place by a factor of approximately $(D/\Gamma_p)^2$. The physical origin of this enhancement comes from the fact that for resonance energy the neutron spends inside the weak interaction field of the nucleus much more time $(\tau_{res} \approx \hbar/\Gamma_p)$, than for the non–resonance energies when the neutron flies through the nucleus without the resonance delay. The first mentioning of the resonance enhancement was in reference [8] devoted to the possible effect of the T–invariance violation. However the authors of [8] were considering the energy region of overlapping resonances $D \le \Gamma$, where this enhancement is practically senseless. Th notion of the resonance enhancement was introduced independently for the case of the isolated resonances $D \gg \Gamma$ in refs. [3,9].

The total cross section σ^0_{tot} is a sum of contributions from s– and p–resonances as well as from the potential scattering and is (neglecting the interference terms):

$$\sigma^0_{tot}(E) \approx \frac{\pi}{k^2}\left[\frac{\Gamma^n_s\Gamma_s}{(E-E_s)^2+\Gamma^2_s/4}+4(kR)^2+\frac{\Gamma^n_p\Gamma_p}{(E-E_p)^2+\Gamma^2_p/4}\right] \quad (21)$$

Even in the vicinity of p–resonance the main contribution to (21) comes from potential scattering and the 'wings' of the s-resonance with smooth energy dependence. In the most well known case of ^{139}La target the p–resonance contribution to the cross section at $E = E_p = 0.75$ eV is about 20%. Just because of this the effect (18) demonstrates sharp resonance behavior in the vicinity of this energy (see e.g. Fig.4 of the review paper [4]). Using this fact the experimentalists prefer to present instead of the observed resonance curve $P_{\exp}(E)$ the auxiliary constant quantity $\tilde{P}$, which is obtained by normalizing the quantity (20) to the p–resonance contribution $\sigma_p(E)$ into the cross section (i.e. to the last term in the sum of (21)):

$$\tilde{P} = \frac{\Delta^P_{tot}(E)}{\sigma_p(E)} = 4\sqrt{\frac{\Gamma^n_s}{\Gamma^n_p}}\frac{v_p}{D} \quad (22)$$

This auxiliary quantity is related to $P_{\exp}(E)$ by the expression:

$$P_{\exp}(E) = \tilde{P}\frac{\sigma_p(E)}{\sigma^0_{tot}(E)} \quad (23)$$

Of course, there is no objection to the introduction of this quantity. However it is often confused with the observed $P_{\exp}(E)$, while the relation (23) is forgotten. In doing this they speak instead of the resonance enhancement about the kinematical enhancement , i.e. about the factor $\sqrt{\Gamma^n_s/\Gamma^n_p} \approx 1/kR$, which appear in ex. (22) (here *R* is the nuclear radius and $kR \sim 10^{-3}$). The initial statements about the kinematical enhancement of the P–violating effects in neutron transmission were done in refs. [10, 11]. Unfortunately, the substitution of the observed quantity $P_{\exp}(E)$ by the auxiliary constant (22) and the conviction that the extremely large P–violating effects in neutron transmission are caused by the kinematical enhancement are widely spread up to now, while the auxiliary quantities $\tilde{P}$ are always presented in literature as the observed effect values. Ex. (23) shows that quantities $\tilde{P}$ exceed the maximal observed effect values by a factor of 3–5 because even at the p–resonance energy σ_p is much smaller than σ^0_{tot}, which practically does not vary in the vicinity of the p–resonance. The same reasoning allows to put (23) into the approximate form:

$$P_{\exp}(E) = \frac{4\pi}{k^2\sigma^0_{tot}}\frac{v_p}{D}\frac{\sqrt{\Gamma^n_s\Gamma^n_p}\cdot\Gamma_p}{(E-E_p)^2+\Gamma^2_p/4} \quad (24)$$

assuming that within a good approximation σ^0_{tot} in the vicinity of p–resonance is a constant which does not depend on energy. We see that the kinematical enhancement of the auxiliary quantity (22) completely disappears in the really observed quantity (24) and is substituted by the hindrance factor of about $(kR) \sim 10^{-3}$, coming from the appearance of the factor $\sqrt{\Gamma^n_s\Gamma^n_p}$ in the numerator instead of the s–resonance partial neutron width Γ_s^n, which gives the main contribution to the total cross section (21) caused by the parity conserving strong interaction. One should point that this hindrance factor appears in all the experiments measuring P–violating effects in neutron transmission since P–violation leads to the fact that the neutron captured into the s–resonance would be emitted by the p–resonance and vice versa (see the diagrams of the review papers [9,4]). In observations of the P–odd correlations in the inelastic channels - for instance, in (n,γ) reaction - this hindrance factor disappears since the partial gamma widths of the s– and p–resonances do not differ practically. Moreover, even the structural enhancement effects $\sqrt{\Gamma_p^\gamma / \Gamma_s^\gamma}$ might appear in this correlations if $\Gamma_p^\gamma > \Gamma_s^\gamma$ (see [3, 4, 9]). This is just why the P–odd effects for the non-resonant polarized neutrons in the inelastic channels (n,γ) and (n, fis) exhibited quite noticeable values of $(\sim 10^{-4})$ and were measured [12-14] earlier than the effects in transmission [15].

The substitution of the observed $P_{\exp}(E)$ by the auxiliary quantity $\tilde{P}$ and the conviction that the large enhancement of the observed effects is caused by the kinematical factor leads to a lot of absurdities. While the observed quantity (18) is always smaller than unity and might be a measure of the weak interaction contribution into the nuclear processes, the normalization of the auxiliary quantity $\tilde{P}$ to the small part of the total cross section in principle allows to obtain the values exceeding unity to whatever factor one likes. As seen from (22), the "kinematical enhancement" causes the $\tilde{P}$ quantity tend to infinity for very small values of the p–resonance partial neutron widths Γ_p^n. Owning to this fact the author had to persuade the American experimentalists, who believed in this enhancement existence, to remove the suggestion to chose for the experiments the p–resonances with the minimal Γ_p^n values from their proposal of experiments in Los-Alamos. Since all the energy dependence of the quantity $\tilde{P}$ is contained in the kinematical enhancement factor and is proportional to $1/k$, it was necessary to explain to our experiments as well that measuring the effect near the thermal point energy (for $k \to 0$) would by no means increase its value. Of course for Γ_p^n tending to zero the observed effect value $P_{\exp}(E)$ would also tend to zero. In this case the contribution $\sigma_p(E)$ of the p–wave resonance into the total cross section tends to zero, while the relative error (19) would increases to infinity.

3. T-NONINVARIANCE EFFECTS IN POLARIZED NEUTRONS TRANSITION THROUGH THE POLARIZED TARGET

The above mentioned erroneous tendency to increase the P-odd effects by normalizing them to the small quantities was continued also in considering the P-violating T-noninvariance effects in the transmission of the polarized neutrons through the polarized target. The most odious example was presented in ref. [16]. It was suggested in this paper to measure the P- and T- violating quantity

$$\tilde{X} = \frac{N_{+-} - N_{-+}}{(N_{++} - N_{--}) - (N_{+-} - N_{-+})} \tag{25}$$

Here N_{+-} and N_{-+} are the numbers of neutrons transferred through the target, while the lower indices mean the neutron helicities before and after the transfer through the target. The normalizing quantity in the denominator of (25) depends on the angle θ between the target polarization vector and the neutron momentum and turns to be zero near the point $\theta = \pi/2$. It was suggested in [16] to carry measurements in the vicinity of this angle because of the effect enhancement tending to infinity. Of course the relative error of the denominator in (25), as well as the relative error of whole quantity (25) would tend to infinity near this angle. This is the striking example of the false effect enhancement caused by the choice of the improper normalization. Of course the measurement precision of the T - noninvariant difference $(N_{+-} - N_{-+})$ would not change if we divide it by zero.

The real enhancements of this effect and the proper experimental parameters were considered in refs. [5 - 7]. It was suggested to choose the effect normalization as:

$$T = \frac{N_{+-} - N_{-+}}{N_{+-} + N_{-+}} \tag{26}$$

The relative error analysis of this quantity has shown that it oscillates as a function of the target thickness and changes by several orders of magnitude on the interval of a few mm. This oscillation is connected with the pseudo-magnetic precession of the neutron spin [17] around the direction of the target polarization. In order to compensate this precession it was suggested to place the target into the static magnetic field, whose magnitude is chosen in such a way that Larmour precession in this field completely compensates the pseudo-magnetic one. In the presence of this compensation the relative error in measuring the quantity *T* has a minimum at $2 \div 2.5$ mean free path lengths with the energy dependence in the vicinity of the p–resonance:

$$T(E) = 2\sqrt{\frac{\Gamma_p^n}{\Gamma_s^n}}\frac{V_{TP}}{\Gamma_p}\frac{D(E-E_p)}{(E-E_p)^2+\Gamma_p^2/4} \tag{27}$$

One can see that the effect changes sign at the point of the resonance $E = E_p$ and reaches the maximal absolute value at $E = E_p \pm \Gamma_p / 2$:

$$T(E = E_p + \Gamma_p/2) \approx \sqrt{\frac{\Gamma_p^n}{\Gamma_s^n}}\frac{V_{TP}}{D}\left(\frac{D}{\Gamma_p}\right)^2 \tag{28}$$

Similar to the effect (24), the quantity *T* experiences the dynamical enhancement V_{TP}/D and the resonance enhancement $(D/\Gamma)^2$, which is partially compensated by the hindrance factor $\sqrt{\Gamma_p^n/\Gamma_s^n} \approx (kR)$ typical, as pointed earlier, for the neutron transmission experiments with P-violation.

It was shown in [5-7] that the minimal relative error of the quantity $T(E)$ in the above energy points (in the case of complete compensation of the pseudo-magnetic precession) is inversely proportional to the quantity (27). This means that the enhancement of the *T(E)* effect indeed allows to measure it with maximal precision.

4. SUMMARY

If one wants to find the optimal conditions for measuring the effects of fundamental symmetry breaking with maximal precision, one needs to compare the effect's relative errors rather than its magnitudes. Very often it turns out that the effects maximal magnitude does not mean at all that the error in its measuring would be minimal. This usually happens when the quantities, which normalize the effect, become smaller. The examples are given by the kinematical and the structural enhancement of the P-odd effects in radiation transitions. These enhancements increase the effect indeed but practically do not change its relative error. The more illustrative example is the "kinematical enhancement" of the P-odd effects in neutron transition, appearing when the "natural" normalization quantity of the total cross section is artificially substituted by the certainly smaller contribution into this cross section from the p-resonance. This substitution not only leaves the effects relative error unaltered, but also distorts the physical picture of the experimentally observed enhancement. Even more vivid example of the artificial effect enhancement is the choice of the normalizing quantity which vanishes and the suggestions to measure in the vicinity of this vanishing point. In this case the effect tends to infinity but so does also its relative error.

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