On the validity of the potential-like dispersion law for neutrons in a matter moving with large acceleration

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Abstract

In recent experiments with ultracold neutrons (UCN) the change in energy at neutron transmission through the silicon sample moving with acceleration was detected. Obtained results were in a good agreement with theoretical prediction based on the assumption that dispersion law of neutron waves in accelerating matter is the same as in the fixed one.

In this communication we present some arguments testifying that above assumption is valid only in the case when acceleration does not exceed any critical value. We also discuss the possible experiment for the testing of the commonly accepted neutron dispersion law in matter in the case of giant acceleration.

It is known that the refraction results from the interference of the incoming and scattered waves. In the case of slow neutrons such scattering is isotropic and coherent scattering amplitude is as a rule a constant which assume to be equal –*b*, where *b* is the coherent scattering length. The scattered wave phase differs from the phase of the incoming wave by $\delta = -kb$, where *k* is the wave number of the initial wave. Just this phase shift gives rise that what we call as refraction is the prime cause of the refraction. And refraction index *n* defines by the Foldy equation [1,2]

$$n^2 = 1 - 4\pi\rho b / k^2 \ . \tag{1}$$

where ρ is the volume density of the atomic nuclei. The interaction of the long wavelength neutrons with matter may be described also with help of the effective potential

$$U_{eff} = \frac{\hbar^2}{2m}\frac{\rho b}{\pi}, \tag{2}$$

where *m* is the neutron mass. Equations (1) and (2) are completely equivalent and that is why the dispersion law (1) is often called the potential dispersion law.

The validity of (1) and (2) for the case of uniform motion is doubtless because for the essentially non relativistic problems which are typical for the neutron optics transition to system of reference where a matter is in rest leads only to the variation of the wave function phase.

It was predicted in ref [3,4] that the energy of neutrons passed through the layer of matter moving with acceleration must differ from the initial one. The theory based on the assumption of validity dispersion law (1) in an accelerating medium. The change of the energy was really detected in the experiments with UCNs [5,6]. The acceleration of sample was as large as $75 m/s^2$ and the energy changed at some parts of neV. The obtained experimental results were in reasonable agreement with the theoretical predictions.

Thus it was possible to conclude that assumption of the potential dispersion law validity in the accelerating matter was justified under indicated above conditions. Nevertheless it would be probably a mistake to make general conclusion that potential dispersion law in accelerating matter is valid always. The point is that the theory of dispersion which is in fact the theory of multiple wave scattering always operates with spherical waves. But in noninertial system of reference related with accelerating matter the concept of sphericity scattered waves is in error and the fact of waves asphericity may affect the condition of the interference [7].

Let us try to estimate qualitatively the value of acceleration for which it is possible to wait remarkable deviation from the dispersion law (2). Consider the scattered waves ot relatively small distances from the nuclei - scatterers where the asphericity of the waves is not yet large. Doing as the same way as in ref [8] where neutron optical problems in gravity were considered we introduce the specific “accelerative” refraction index arising in a noninertial system of references

$$n_w(x) = \sqrt{1-\frac{mWx}{E}} \approx 1-\frac{mWx}{2E}, \qquad \left(mWx << E\right), \tag{3}$$

where W is acceleration and E is energy. The phase of the wave propagates along axis X is obviously

$$\phi = k\left(1-{mWx}/{2E}\right)x\,. \tag{4}$$

Therefore at the distance x from the scatter the variation of phase due to acceleration is

$$\Delta\phi = k\left({mWx^2}/{2E}\right), \tag{5}$$

Obviously one can wait remarkable effects when shift of phase due to acceleration (5) appears to be the same order of value as phase shift due to scattering at the nuclei which as mentioned above is the physical cause of refraction. Putting in (5) $x = a$ where a is interatomic distance we can state now the condition of validity the potential like dispersion law in accelerating medium as

$$\frac{\mathrm{mWa}^2}{2\mathrm{E}} << \mathrm{b}, \qquad \mathrm{W} << \frac{2\mathrm{Eb}}{\mathrm{ma}^2} = \mathrm{W_c}. \qquad (6)$$

As that seen from eq. (6) the critical acceleration W_c is proportional to the neutron energy. Notice that as that follow from (4) the parameter characterizing asphericity of the wave front at $\mathrm{W} \approx \mathrm{W_c}$ is as assumed relatively small $\varepsilon = {}^{\mathrm{b}}\!/_{\mathrm{a}} \approx 10^{-5}$.

Let us estimate the value of the critical acceleration $W_c = \dfrac{2Eb}{ma^2}$ for the case of ultracold neutrons (UCN). Assuming E = 100 neV, $b \approx 5\times10^{-13}$cm, $a = 5\times10^{-8}$cm we arrive immediately to $W_c \approx 4\times10^7 cm/s^2$. Such value of acceleration is quite achievable in laboratory conditions. In [9] the reflection of cold neutrons from the vibrating mirror was observed. That mirror was set into vibration by the piezo drive. Vibration amplitude and frequency were in one of measurements 5.3 nm and 2.22MHz respectively. It is easy to find that the value of acceleration in this condition attained 10^8 cm/s^2. Similar experiments are

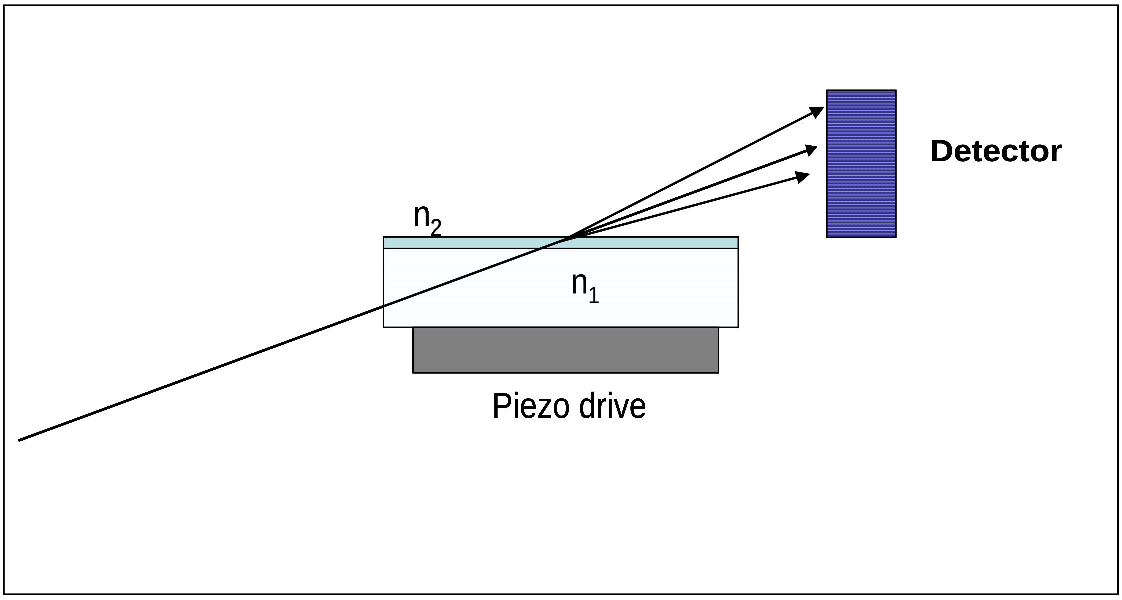

Figure 1. The beam of cold neutrons passes through the vibrating slab with deposed film with relatively high effective potential.

possible apparently in transmission geometry. The possible scheme of an experiment with cold neutrons displayed in figure below.

Results of such experiment may be compared with calculations based on the solution of the known quantum problem of interaction of particle with oscillating potential barrier. Depending on velocity component *v*, normal to the surface of the film, two different cases are possible. At $E_n = mv_n^2/2 > U$, the above barrier transmission will take place, analyzed in [10]. Elsewise the quantum tunneling through the oscillating barrier [11] will take place. The possibility to perform similar experiment with UCNs must be analyzed additionally.

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