NUCLEAR LEVEL DENSITY WITHIN MODIFIED GENERALIZED SUPERFLUID MODEL WITH VIBRATIONAL ENHANCEMENT

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Abstract

For nuclear level densities, an enhanced generalized superfluid model with different vibrational enhancement factors is studied. The ready-to-use tables for the asymptotic value of a-parameter of level density as well as for additional shift to excitation energy for 291 atomic nuclei are prepared using the chi-square fit of the theoretical values of neutron resonance spacing and cumulative number of low-energy levels to the experimental values. The systematics of these parameters as a function of mass number and neutron excess are obtained. The vibrational enhancement descriptions are additionally tested for a wide range of excitation energies. Simple methods of the vibrational enhancements lead to increasing the level density due to vibrational states by factor : $2 \div 3$ for the isotopes A: 100 with the excitation energies near the neutron binding energies.

1. Introduction

The nuclear level density (NLD) (ρ) describes excitation states of atomic nuclei. It is one of the most important quantities for calculations of nuclear reactions characteristics in the statistical models. The nuclear reaction calculations are as a rule time consuming and simple closed-form expressions are preferable in evaluation of the NLD.

If effects of vibrational states are included into consideration of excited level structure, the NLD of states with excitation energy U and spin J may be expressed as (see [1] and Refs. therein):

$$\rho(U,J) = \bar{\rho}(U,J) \cdot K_{vibr}(U), \qquad (1)$$

where $\bar{\rho}$ is the level density due to quasiparticle and rotational excitations, and K_{vibr} is the enhancement factor to account for the vibrational states.

Different expressions are used for calculations of the vibrational enhancement factor K_{vibr} (see, for example, [1-7]). The $\bar{\rho}$ is a function of some set of parameters that are fixed using Eq. (1) at given K_{vibr} to the experimental data on s-resonance spacing and cumulative numbers of low-lying discrete levels to the corresponding theoretical values. This procedure leads to different contributions of the vibrational states to the NLD.

Here, we use this method to obtain the NLD expressions with different phenomenological relationships for vibrational enhancement factor. For level density $\bar{\rho}$ of intrinsic and rotational states, we take corresponding components of the Empire Global Specific Model[5]. The asymptotic value $\tilde{a}$ of the a-parameter of level density and an additional shift $\tilde{\delta}_{shift}$ to excitation energy are considered as fitted parameters. The ready-to-use tables of these parameters for 291 atomic nuclei are prepared and the systematics of these parameters as a function of mass number and neutron excess are obtained. The NLD expressions with vibrational enhancement descriptions are additionally tested by comparison with the experimental data from Dubna [8], Oslo [9] and Obninsk [10].

2. Simple methods of vibrational enhancement calculations

The simple phenomenological methods for calculations of the vibrational enhancement factor are based on different phenomenological modifications of the expression for the partition function of the Boson gas.

We test the following prescriptions for vibrational enhancement. I. The Liquid drop parameterization of vibrational modes with the temperature damping in accordance with the EMPIRE 3.1 code (K_{EM}) [5]:

$$K_{vibr} \equiv K_{EM} = K_{LDM} \cdot (1-q_v) + q_v \ , \ K_{LDM} \equiv Z_{boz}(\{n(\omega_L)\}) = \exp\left[C_3 \, A^{2/3} \cdot T^{4/3} \right] \ , \qquad (2)$$

$$q_v = 1/[1+\exp\{(T_{1/2} - T)/DT\}] \ ,$$

where $T_{1/2}, DT$ - constants of damping with the increasing of temperature $T_{1/2} = 1$ (MeV), $DT = 0.1$ (MeV);

II. The Boson gas relationship with damped occupation numbers (K_{DN}) [1,2]:

$$K_{vibr} \equiv Z_{boz}(\{\bar{n}_L\}) = \exp\left(\bar{S} - \bar{U}/T \right) \equiv K_{DN}(T) \qquad (3)$$

$$\bar{S} = \sum_L (2L+1)\left[(1+\bar{n}_L)\ln(1+\bar{n}_L) - \bar{n}_L \ln \bar{n}_L \right], \ \bar{U} = \sum_L (2L+1)\hbar\omega_L \bar{n}_L,$$

$$\bar{n}_L = \frac{\exp\left[-\Gamma_L/(2\hbar\omega_L)\right]}{\exp(\hbar\omega_L/T) - 1}, \quad \Gamma_L = C \cdot \left[(\hbar\omega_L)^2 + 4\pi^2 T^2 \right],$$

where $\omega_L = E_L/\hbar$ is the characteristic frequency for vibrational mode of multipolarity L with energy E_L ; Γ_L is the damping width of this vibrational state; $C = 0.0075A^{1/3}$ (MeV^{-1});

III. The ratio of boson partition functions with averaged occupation numbers[6,7] without and with taking account of the change of the intrinsic state temperature due to presence of the vibrational modes (so called BAN and BANT approaches with K_{BAN} and K_{BANT}). The vibrational enhancement factor K_{BAN} at the temperature T has the form[6,7]:

$$K_{BAN}(T) = \prod_L \left(\frac{1 + < n(T, \omega_L) >}{1 + < n(T, \tilde{\omega}_L) >} \right)^{2L+1} \qquad (4)$$

with the boson occupation numbers averaged over the collective motion period

$$< n(T, \omega_L) >= \frac{\hbar\omega_L \left(1 - \exp(-2\pi\Gamma_L \hbar\omega_L)\right)}{2\pi\Gamma_L (\exp(\hbar\omega_L/T) - 1))}, \qquad (5)$$

where $\tilde{\omega}_L$ is a frequency of corresponding $1p1h$ state.

3. Results of calculations and conclusions

As mentioned above, we take the corresponding expressions of the Empire Global Specific

Model of the EMPIRE 3.1 code [5] for the level density due to quasiparticle and rotational excitations ($\bar{\rho}$ in the Eq.(1)). It is the properly parameterized Enhanced Generalized Superfluid Model(EGSM)[1]. This model describes all main peculiarities of the intrinsic and rotational excitations. Specifically, it includes the super-fluid model below critical excitation energy and the Fermi Gas model above; the rotational energy is subtracted from the intrinsic excitation energy and the rotational enhancement of the NLD is taken into account in non-adiabatic form.

The asymptotic value $\tilde{a}$ of the a-parameter of level density and additional shift to excitation energy $\tilde{\delta}_{shift}$ in $\bar{\rho}$ of the EGSM were considered as fitted parameters at given K_{vibr}. All other quantities for $\bar{\rho}$ (the shell effect energies, the moments of inertia, and so on) were adopted from the EMPIRE 3.1 code [5]. The quadrupole and octupole vibrations were included [4] with shell effects were used for their energies. The experimental data on s-resonance spacing at the neutron binding energies and cumulative numbers of low-lying discrete levels were taken from the RIPL-3 [1], and corresponding database of the EMPIRE 3.1 code [5] was used.

The fitting procedure was the same one as for the Empire Global Specific Model[5]. The parameters $\tilde{a}$ were obtained from fitting of theoretical values of s-resonance spacing to the experimental data. Then the additional shifts were found from fitting the theoretical values of cumulative numbers of low-lying discrete levels to the corresponding experimental values.

The ready-to-use tables of the values of $\tilde{a}$ and $\tilde{\delta}_{shift}$ were prepared for 291 atomic nuclei (experimental values) in the BAN and DN approaches for K_{vibr}. The following global systematics of experimental values are obtained with K_{BAN} and K_{DN} vibrational enhancements:

$$\begin{aligned} \tilde{a} &= \alpha_V A(1+\alpha_{VI} I^2) \quad + \alpha_S A^{2/3}(1+\alpha_{SI} I^2) \quad + \quad \alpha_C \frac{Z^2}{A^{1/3}}, \\ \tilde{\delta}_{shift} &= \delta_1(1+\delta_{1I} I^2) \quad + \quad \delta_2 A(1+\delta_{2I} I^2) + \delta_3 E_{2^+}, \end{aligned} \tag{6}$$

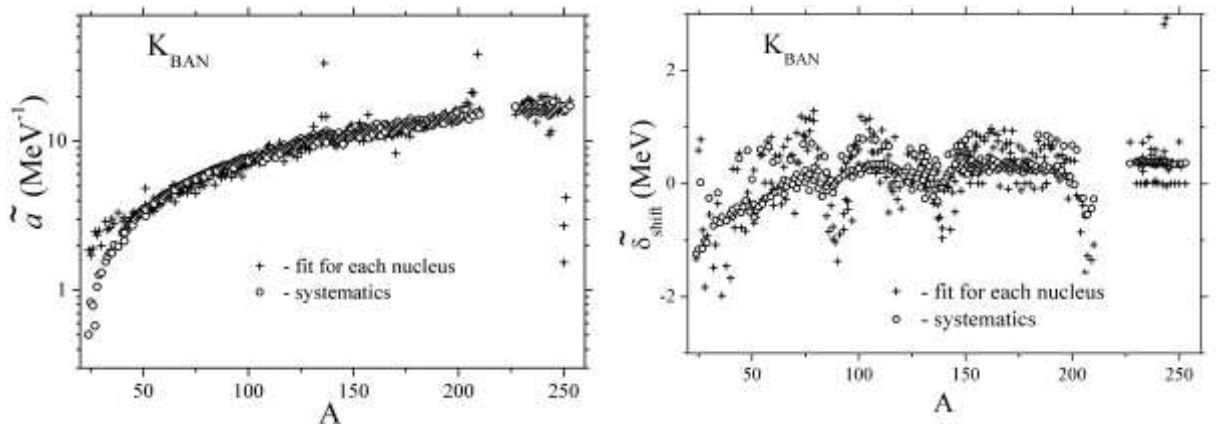
where A is the mass number, $I = (N-Z)/A$ neutron excess, and E_{2^+} the energy of the first 2^+ quadrupole state in MeV, $\tilde{a}$ in 1/MeV and $\tilde{\delta}_{shift}$ in MeV.

Table 1 gives the parameters of global systematics and their uncertainties with BAN and DN approaches for K_{vibr}.

Table 1. The coefficients of global systematics for $\tilde{a}$ and $\tilde{\delta}_{shift}$ with BAN and DN approaches for K_{vibr}

$\tilde{a}$	α_V	α_{VI}	α_S	α_{SI}	α_C
BAN	0.527*6*	4.50*6*	-1.17*2*	13.3*2*	-0.0447*6*
DN	0.800*4*	11.86*3*	-1.75*1*	32.8*1*	-0.0787*4*
$\tilde{\delta}_{shift}$	δ_1	δ_{1I}	δ_2	δ_{2I}	δ_3
BAN	1.37*2*	-11.7*5*	0.00000003*1*	7700*24000*	-0.697*9*
DN	0.94*4*	4.9*12*	0.001443*8*	-41.8*75*	-0.75*1*

Figure1 shows the comparison of the experimental values of $\tilde{a}$ ($\tilde{\delta}_{shift}$) with the global systematics (6). For the vibrational enhancement factor, the K_{BAN} is used.

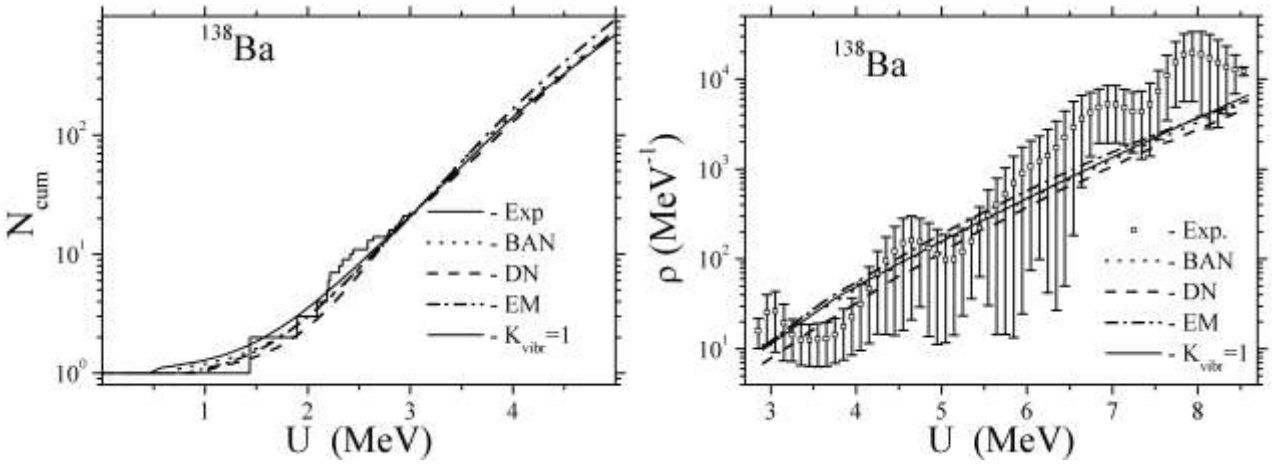


(*a*) (*b*)

Fig. 1. The comparison of the experimental values of $\tilde{a}$ (*a*) and $\tilde{\delta}_{shift}$ (*b*) with global systematics.

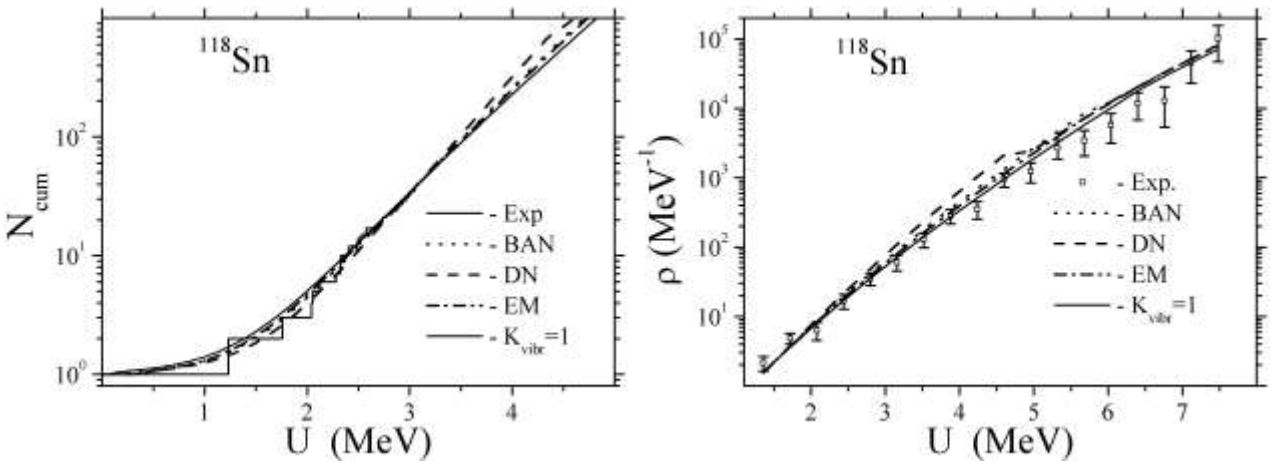
The values of vibrational enhancement $K_{EM}: 1 \div 2$, $K_{BAN}: 2 \div 3$ and $K_{DN}: 10 \div 30$ are obtained for the isotopes $A: 100$ at the excitations near the neutron binding energies. Note that the values of the factors K_{EM}, K_{BAN} are in line with the results of the microscopic quasiparticle-phonon model [11].

The NLD expressions are additionally tested for a wide range of excitation energies by comparison with the experimental data from Dubna [8], Oslo [9] and Obninsk [10]. The figures 2,3 demonstrate the dependence of cumulative number of levels and nuclear level density on excitation energy for the nuclei ^{138}Ba and ^{118}Sn. Experimental data are taken from Refs.[8] and [9] for ^{138}Ba and ^{118}Sn nuclei, respectively.



(*a*) (*b*)

Fig. 2. The dependence of cumulative number of levels (*a*) and nuclear level density (*b*) on excitation energy. Experimental data are from Ref.[8].



(*a*) (*b*)

Fig. 3. The dependence of cumulative number of levels (*a*) and nuclear level density (*b*) on excitation energy. Experimental data are from [9].

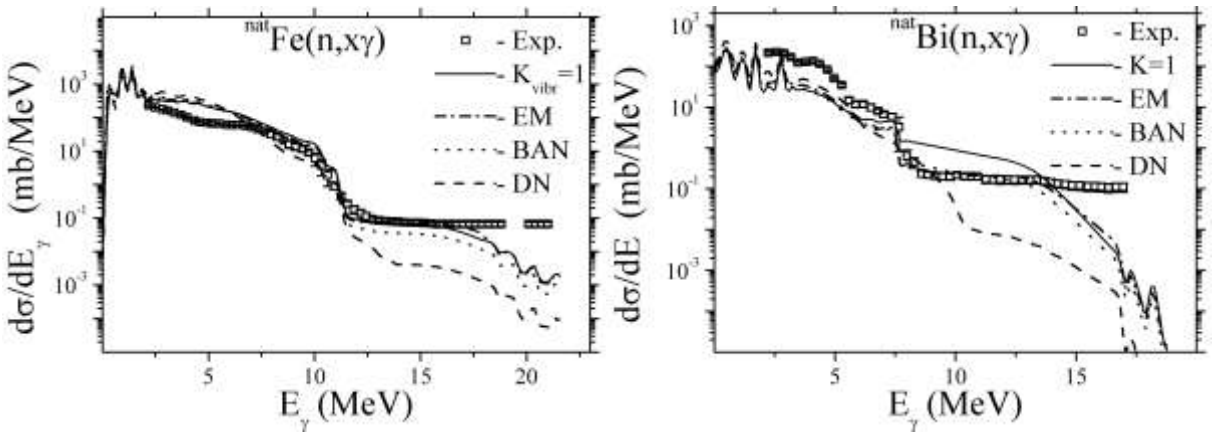
Table 2 gives the ratios $R_\alpha = \sum_{i=1}^{N_{nucl}} \chi_i^2(K_\alpha) / \sum_{i=1}^{N_{nucl}} \chi_i^2(K_{BAN})$ of the chi-square deviations $\chi_i^2(K_\alpha) = \sum_{j=1}^{n_i} (\rho_{theor,i}(U_j) - \rho_{\exp,i}(U_j))^2 / n_i$ of the theoretical NLD with different K_α from experimental data. The n_i is the number of experimental values of the NLD for the nucleus i and N_{nucl} is number of atomic nuclei.

Table 2. The comparison of the ratios $R_\alpha = \sum_{i=1}^{N_{nucl}} \chi_i^2(K_\alpha) / \sum_{i=1}^{N_{nucl}} \chi_i^2(K_{BAN})$.

Data	R_{EM}	R_{DN}	R_{BANT}
[8] (N_{nucl}=30)	1.00	0.90	0.90
[9] (N_{nucl}=49)	0.99	0.99	0.96
[10] (N_{nucl}=6)	1.55	7.73	0.83
average	1.18	3.21	0.90

It can be seen that the BAN and BANT approaches for vibrational enhancement lead, as a whole, to some better agreement with experimental data than other methods. The BAN approach is less complicated and looks like a simple method, which provides the good estimation of the vibrational enhancement.

Figure 4 presents gamma-ray spectra for ^{nat}Fe $(n, x\gamma)$ and ^{nat}Bi $(n, x\gamma)$ reactions at E_n =14.1 MeV. Theoretical spectra were calculated using modified EGSM with different K_{vibr}. The EMPIRE 3.1 code was used with K_{vibr} indicated on the figure. The other input parameters were by default. The calculations demonstrate rather strong dependence of the spectra on the choice form of the K_{vibr}.



(*a*) (*b*)

Fig. 4. The gamma-ray spectra for $^{nat}\mathrm{Fe}\,(n,x\gamma)$ (*a*) and $^{nat}\mathrm{Bi}\,(n,x\gamma)$ (*b*) reactions at E_n =14.1 MeV. Experimental data are from Ref.[12].

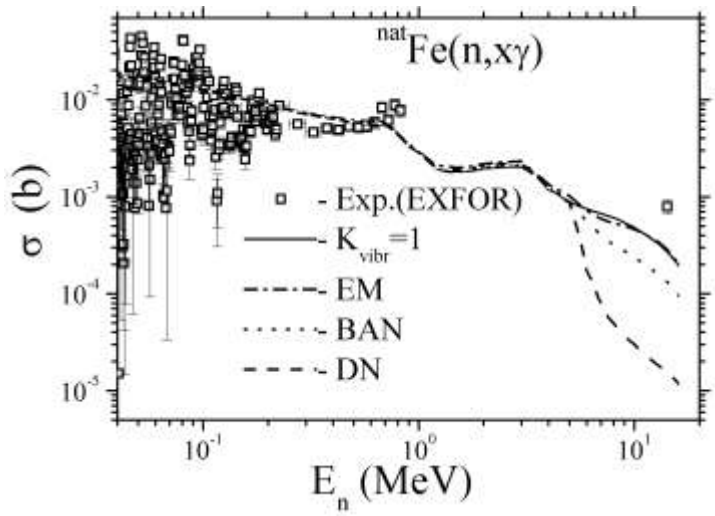


Fig. 5. The excitation functions of reaction $(n,x\gamma)$ on $^{nat}\mathrm{Fe}$.

The effect of the vibrational enhancement on the cross-sections is demonstrated on Fig. 5, where the excitation functions of the reaction $(n,x\gamma)$ on $^{nat}\mathrm{Fe}$ are demonstrated. The experimental data are taken from EXFOR database. One can see that in the range of the high energies increasing the vibrational enhancement (and the NLD) decreases the contribution of the statistical component to the cross-sections for the channels with gamma-ray emission due to growing amounts of the channels.

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