STATISTICAL MODEL ANALYSIS OF (n,α) AND (n,p) CROSS SECTIONS AVERAGED OVER THE FISSION NEUTRON SPECTRUM

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I. Introduction

The study of (n,α) and (n,p) reactions cross sections for fast neutrons is important to estimate radiation damage caused by helium and hydrogen production, nuclear heating and transmutations in the structural materials of fission and fusion reactors. On the other hand, systematical analysis of neutron cross sections is perhaps useful to study nuclear reaction mechanisms. In addition, it is often necessary, in practice, to evaluate the neutron cross sections of the nuclides for which no experimental data are available, using the systematical analysis results.

Analysis of the experimental (n,α) and (n,p) cross sections was carried out in 1963 by Levkovsky [1] and a systematic dependence of the cross sections on the relative neutron excess $(N-Z)/A$ of the target isotopes was observed in the 14–15 MeV neutrons. This systematic behavior of the cross sections is termed in the literature as the isotopic effect. Several formulae were proposed to describe the isotopic effect for 14–15 MeV neutrons (see for example [1-8]).

We observed a similar isotopic dependence of the (n,α) and (n,p) cross sections in the wide energy range of 6 to 20 MeV [9]. Moreover, the statistical and exciton models and PWBA were suggested [10] to explain the dependence of the cross sections on the relative neutron excess parameter.

In this paper we have used the statistical model based on the Weisskopf and Ewing theory [11] to carry out a systematical analysis of known experimental (n,α) [12,13] and (n,p) [14] cross sections averaged over the fission neutron spectrum of ^{235}U by thermal neutrons. The Weisskopf Ewing evaporation model provides a convenient and simple calculation way of the cross sections to continuum states which take place in this energy range. As our principal aim is a obtaining of the average systematic behavior of the cross sections for medium and heavy weight nuclei in the energy region of fission neutron spectrum, we did not use more detailed theory of Hauser and Feshbach [15] in which is employed the optical potential depending on the individual properties of the target nuclei.

II. Statistical model formulae

The direct and pre-equilibrium mechanisms are neglected for fission neutrons and the compound mechanism can be considered, only. Then, the (n,x) reaction cross section can be written as follows:

$$\sigma(n,x) = \sigma_c(n)G(x) \tag{1}$$

where $\sigma_c(n) = \pi(R+\lambda/2\pi)^2$ is the compound nucleus formation cross section; R is the target nucleus radius; λ is the wavelength of the incident neutrons; The probability of the compound nucleus decay into channel x $(x = p, n, \alpha, \dots)$ is expressed as

$$G(x) = \frac{\Gamma_x}{\Gamma} = \frac{\Gamma_x}{\sum_i \Gamma_i}\ , \tag{2}$$

where Γ_x, Γ_i and Γ are the partial and total level widths, respectively.

In the framework of Weisskopf-Ewing theory using the principle of detailed balance we can determine the x–particle width Γ_x as following:

$$\Gamma_x = \frac{2S_x + 1}{\pi^2 \hbar^2 \rho_c(E_c)} M_x \int_{V_x}^{E_x^{max}} E_x \sigma_x(E_x)\, \rho_y(U_x) dE_x \tag{3}$$

Here: S_x, M_x, E_x and V_x are the spin, mass, energy and the Coulomb potential for the outgoing x–particle, respectively; $\rho_c(E_c)$ and $\rho_y(U_x)$ are the level densities of the compound and residual nuclei, respectively; U_x is the excitation energy of the residual nuclei; $\sigma_c(E_x)$ is the inverse reaction cross section which is determined in the semi-classical approximation as follows:

$$\sigma_c(E_x) = \begin{cases} \pi R^2 \left(1 - \dfrac{V_x}{E_x}\right) & \text{for} \quad E_x > V_x \\ 0 & \text{for} \quad E_x < V_x \end{cases} \tag{4}$$

If we use the nuclear entropy and constant temperature approximation can get from (3) and (4) the following formula for the x–particle width:

$$\Gamma_x = \frac{2S_x + 1}{\pi \hbar^2} M_x R^2 \int_{V_x}^{E_x^{max}} E_x \left(1 - \frac{V_x}{E_x}\right) e^{-\frac{B_x + \delta_x + E_x}{\Theta}} dE_x \tag{5}$$

where B_x and δ_x are the binding energy and the odd-even effect parameter for the outgoing x–particle, respectively; $\Theta = \mathrm{kT}$ is the nuclear thermodynamic temperature; k is the Boltzmann constant.

Then, neglecting the γ–emission, from (1), (2) and (5) we get [10] following expression for (n,*x*) cross section:

$$\sigma(n,x) = \sigma_c(n) \frac{(2S_x + 1) M_x e^{-\frac{B_x + \delta_x + V_x}{\theta}} \left\{1 - \frac{W_{nx}}{\theta} e^{-\frac{W_{nx}}{\theta}} - e^{-\frac{W_{nx}}{\theta}}\right\}}{\sum_i (2S_i + 1) M_i e^{-\frac{B_i + \delta_i + V_i}{\theta}} \left\{1 - \frac{W_{ni}}{\theta} e^{-\frac{W_{ni}}{\theta}} - e^{-\frac{W_{ni}}{\theta}}\right\}}, \tag{6}$$

where $W_{nx} = E_n + Q_{nx} - V_x$ and $W_{ni} = E_n + Q_{ni} - V_i$; Q_{nx} and Q_{ni} are the reaction energy; E_n is the incident neutron energy. For fast neutrons total level width can be approximately taken as $\Gamma \approx \Gamma_n$.

Also, the odd-even effect parameters were neglected. In the energy relations can be used the following assumptions:

$$(E_n + Q_{nx} - V_x) \gg \theta \text{ and } (E_n + Q_{ni} - V_i) \gg \theta. \tag{7}$$

So, the fast neutron induced (n,*x*) reaction cross section is determined from (6) as follows

$$\sigma(n,x) = \sigma_c(n) \frac{(2S_x + 1)}{(2S_n + 1)} \frac{M_x}{M_n} e^{\frac{Q_{nx} - V_x}{\Theta}} \tag{8}$$

If we use a formula for nuclear thermodynamic temperature as in [16] and the Fermi gas model formula for level density parameter [17] can get the following expression:

$$\Theta = \sqrt{\frac{13.5\left(E_n + Q_{np}\right)}{A}} \tag{9}$$

In the case of medium mass and heavy nuclei ($Z \gg 1$) using the Weizsacker formula [18] for binding energy we get a formula for fast neutron induced (n,p) reaction cross section from Eqs. (8) and (9) as following

$$\sigma(n,p) = C_p \pi (R + \lambda/2\pi)^2 e^{-K_p \frac{N-Z+1}{A}} \tag{10}$$

where

$$C_p = \exp\left(Z A^{1/6} \frac{2\gamma - 1}{\sqrt{13.5(E_n + Q_{np})}} \right) \tag{11}$$

and

$$K_p = 4\xi \sqrt{\frac{A}{13.5(E_n + Q_{np})}} \tag{12}$$

Also, using the same calculation method we can get following formula for (n,α) cross section

$$\sigma(n,\alpha) = C_\alpha \pi (R + \lambda/2\pi)^2 e^{-K_\alpha \frac{N-Z+0.5}{A}}, \tag{13}$$

where

$$C_\alpha = 2\exp \sqrt{\frac{A}{13.5(E_n + Q_{n\alpha})}} \left(-3\alpha + \gamma\left(\frac{4Z}{A}\right) + \varepsilon_\alpha - 2.058 \frac{Z}{A^{1/3}} \right) \tag{14}$$

and

$$K_\alpha = 2\xi \sqrt{\frac{A}{13.5(E_n + Q_{n\alpha})}} . \tag{15}$$

Here: Z, N and A are the proton, neutron and mass numbers of the target nucleus, respectively; α, γ, and ξ are the Weizsacker formula constants; ε_α is the internal binding energy of α-particle. The parameters K_i and C_i (i=p or α) in formulae (11), (12), (14) and (15) can be fitted and determined as the constants at each energy points for all isotopes.

The values of the fitted parameters K_α and K_p for different neutron energies [19,20] are given in Table.1.

Table 1. The parameters K_α and K_p for different neutron energy

E_n (MeV)	K_α	K_p
6	62.1	75.3
8	59.0	62.8
10	53.5	52.1
13	48.3	38.9
14.5	37.5	37.4
16	35.0	33.6
18	32.5	22.4
20	25.3	17.9

III. The effective neutron energy

The effective neutron energy is important for theoretical analysis of averaged over the continuum neutron spectrum cross sections. Average energy for fission neutrons is, usually, around 2 MeV [21]. At the same time, the threshold energy of (n,α) and (n,p) reactions for most of isotopes lies in the region of $E_{th}\approx 2–5$ MeV [22]. So, the effective average energy of the incident neutrons for fission spectrum in the case of (n,α) and (n,p) reactions should be different from 2 MeV. To clarify this statement the excitation function of the ^{58}Ni(n,α)^{55}Fe and ^{58}Ni(n,p)^{58}Co reactions [22] are displayed in Figs.1 and 2, as example. Fission neutron spectrum of 235U [21] is, also, shown in Figs.1 and 2. Similar pictures can be obtained for other isotopes.

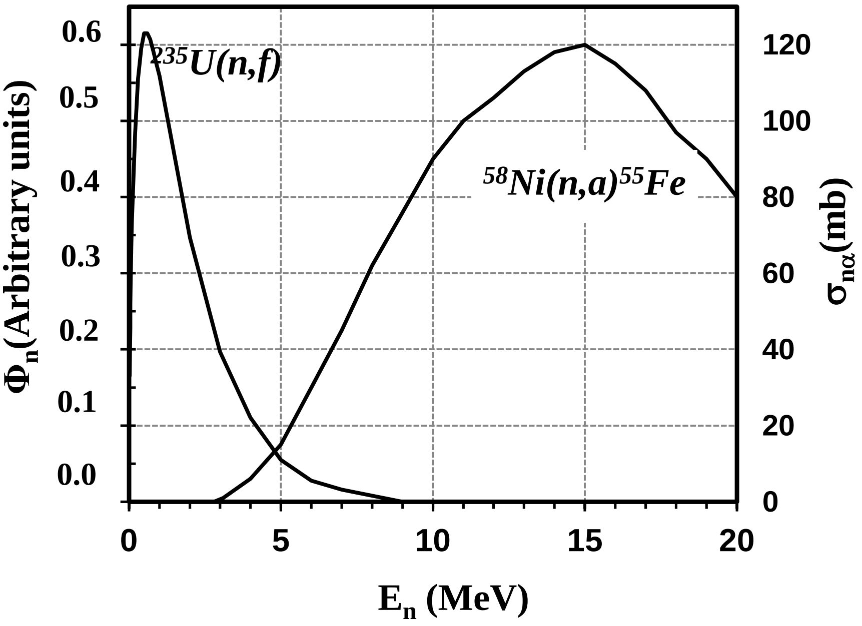

Fig.1. Fission neutron spectrum of ^{235}U and excitation function of the ^{58}Ni(n,α)^{55}Fe reaction.

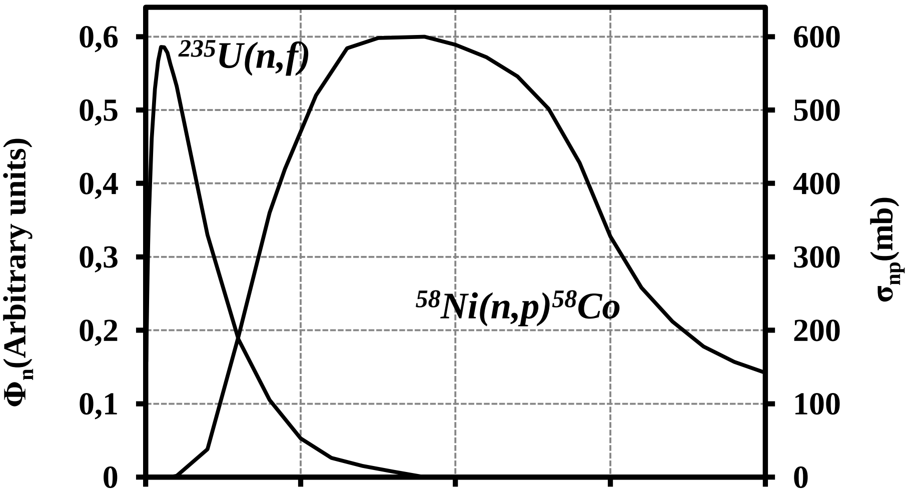

Fig.2. Fission neutron spectrum of ^{235}U and excitation function of the ^{58}Ni(n,p)^{58}Co reaction.

An average (n,α) or (n,p) cross section is determined by overlap of two curves for neutron energy spectrum and excitation function:

$$\langle\sigma(n,x)\rangle = \frac{\int \sigma_{nx}(E_n)\phi(E_n)dE_n}{\int \phi(E_n)dE_n}. \qquad (16)$$

Here: $\phi(E_n)$ is the neutron spectrum and x is the outgoing particle (x=α or p).

The average effective neutron energy for the fission neutron induced (n,α) and (n,p) reactions was got to be around 5 MeV, when we have used the averaged cross sections calculated by expression (16) and took into account the systematical regularity of parameters K_p and K_α for other incident neutron energies (see Table.1, Figs.3 and 4).

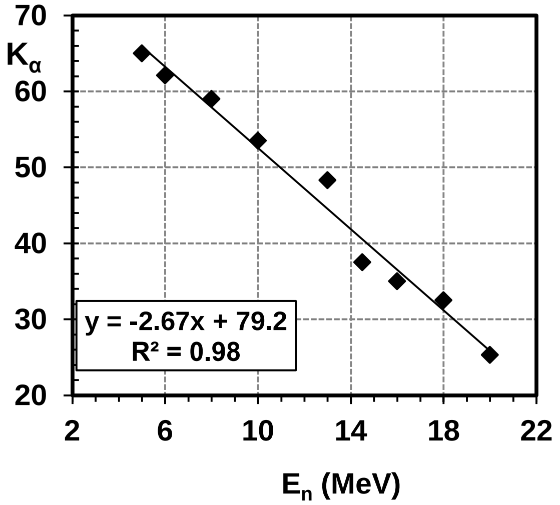

Fig.3. Energy dependence for parameter K_α.

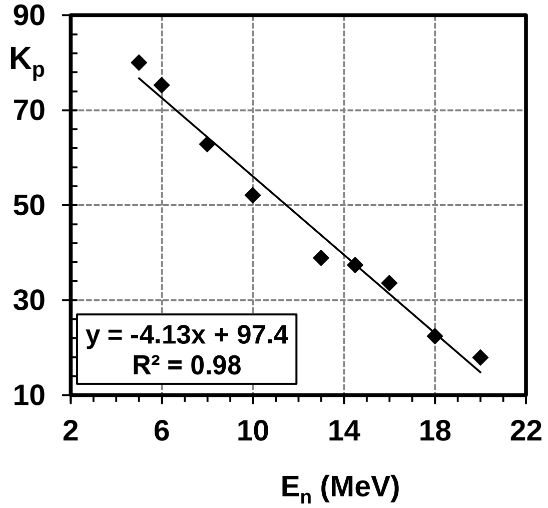

Fig.4. Energy dependence for parameter K_p.

IV. Systematical analysis of the averaged (n,p) and (n,α) cross sections

The theoretical (n,p) cross sections calculated by formula (10) with fitted parameter $K_p = 80$ and $C_p = 2.8$ for average neutron energy $\langle E\rangle \approx 5$ MeV of fission spectrum are compared with known experimental values taken from [14] in Fig.5.

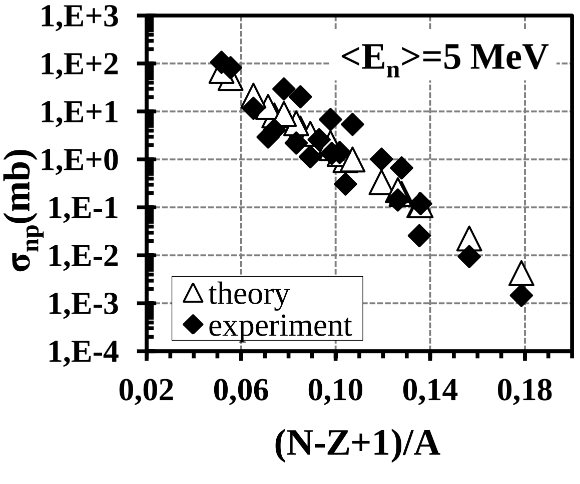

Fig.5. Theoretical and experimental (n,p) cross sections.

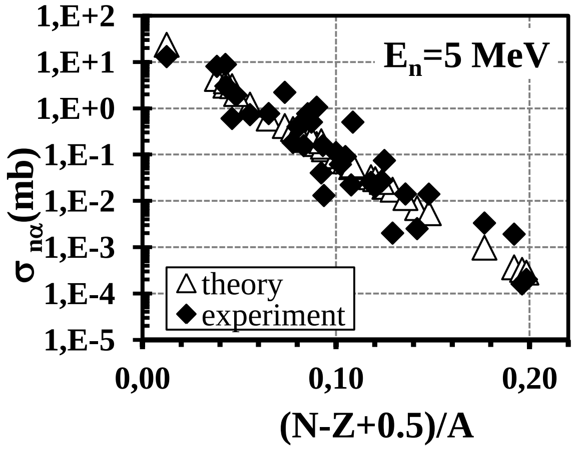

Fig.6. Theoretical and experimental (n,α) cross sections.

It can be seen that the statistical model formula (10) with average energy $\langle E\rangle \approx 5$ MeV for fission neutrons is satisfactorily describes the systematical dependence of known experimental (n,p) cross sections on the relative neutron excess parameter $(N - Z + 1)/A$.

Also, almost the same result was obtained for the averaged over the fission neutron spectrum (n,α) cross sections (see Fig.6). In this case the fitting parameters K_α and C_α were found to be 65 and 0.04, respectively, at average neutron energy $\langle E \rangle \approx 5$ MeV.

V. Conclusion

1. Known experimental (n,p) and (n,α) cross sections averaged over the fission neutron spectrum of ^{235}U were analyzed using the statistical model based on the Weisskopf-Ewing theory and certain systematical regularity was observed.
2. It was shown that the experimental of (n,p) and (n,α) cross sections for fission neutrons are satisfactorily described by the statistical model.
3. The average effective neutron energy for (n,p) and (n,α) reactions induced by fission neutrons of ^{235}U was found to be around 5 MeV.

VI. References

1. V.N.Levkovsky, *Sov. Phys. JETP*. **45**, pp. 305-311, (1963)
2. H.L.Pai and D.G.Andrews, *Can. J. Phys*. **56**, pp. 944-949, (1978)
3. G.Eder, G.Winkler and P.Hille, *Z. Phys.* **253**, pp. 335-349, (1972)
4. S.Ait-Tahar, *J. Phys.G.* **13**, pp. 121-125, (1987)
5. K.Gul, *INDC (PAK)-009.* IAEA, Vienna, (1995)
6. Kh.T.Osman and F.Habbani, *INDC (SUD)-002.* IAEA, Vienna, (1997)
7. E Tel *et al., J. Phys.G.* **29**, pp. 2169-2177, (2003)
8. A.Yu.Konobeyev, U.Fischer, C.H.M.Broeders, *Appl.Radiat.Isot*. **67**, pp. 357-361, (2009)
9. G.Khuukhenkhuu *et al., Proc. Intern. Conf. Nuclear Data for Science and Technology.* Trieste, Italy, pp. 934-936, (1997)
10. G.Khuukhenkhuu, G.Unenbat, M.Odsuren *et al., JINR Communications*. E3-2007-25, E3-2007-26, E3-2007-27. Dubna, (2007)
11. V.F.Weisskopf and D.H.Ewing, *Phys. Rev*. **57**, pp. 472-485, (1940)
12. A.Calamand, In Book: Handbook on Nuclear Activation Cross Sections. IAEA, *Tech. Rep. Ser,* N**156**, pp. 273-324, (1974)
13. E.M.Gryntakis, *J.Radional. Chem.* **52**, pp. 219-261, (1979)
14. [14] O.Horibe *et al., Proc. Intern. Conf. 50 Years with Nuclear Fission.* Washington, D.C, USA, pp. 923-930, (1989)
15. W.Hauser and H.Feshbach, *Phys.Rev*. **87**, pp. 366-373, (1952)
16. J.M.Blatt, V.F.Weisskopf, *Theoretical Nuclear Physics.* John Wiley and Sons, New York, 1952.
17. A.Bohr, B.R.Mottelson, *Nuclear Structure*. Vol.I, W.A.Benjamin, Inc. New York-Amsterdam, 1969.
18. C.F.v. Weizsacker, *Z.Phys.* ***A*96**, pp. 431-458, (1935)
19. G.Khuukhenkhuu *et al., Proc. ISINN-18. JINR,* Dubna, pp. 150-152, (2011)
20. G.Khuukhenkhuu and M.Odsuren, *AIP Conf. Proc*. **1109**, pp. 61-65, (2009)
21. B.I. Starostov *et al.,Yadernye Konstanty*. **2**, pp. 3-44, (1980)
22. V.N.Manokhin *et al.,* In book: Handbook On Nuclear Activation Data. IAEA, Vienna, *Tech. Rep. Ser,* **N273**, pp. 305-411, (1987)