II. Statistical model formulae

The direct and pre-equilibrium mechanisms are neglected for fission neutrons and the compound mechanism can be considered, only. Then, the (n,x) reaction cross section can be written as follows:

\[ \sigma(n,x) = \sigma_c(n)G(x) \]  

where \( \sigma_c(n) = \pi(R^{2}/\lambda)^2 \) is the compound nucleus formation cross section; \( R \) is the target nucleus radius; \( \lambda \) is the wavelength of the incident neutrons. The probability of the compound nucleus decay into channel \( x \) (\( x = p,n,\alpha,... \)) is expressed as

\[ G(x) = \frac{\Gamma_x}{\Gamma} = \frac{\Gamma_x}{\sum_i \Gamma_i}, \]
where $\Gamma_x$, $\Gamma_i$ and $\Gamma$ are the partial and total level widths, respectively.

In the framework of Weisskopf-Ewing theory using the principle of detailed balance we can determine the $x$–particle width $\Gamma_x$ as following:

$$\Gamma_x = \frac{2S_x + 1}{\pi^2 \hbar^2 \rho_c(E_x)} \int_{E_x}^{E_x^{\text{max}}} E_x \sigma_x(E_x) \rho_x(U_x) dE_x$$  \hspace{1cm} (3)

Here: $S_x$, $M_x$, $E_x$ and $V_x$ are the spin, mass, energy and the Coulomb potential for the outgoing $x$–particle, respectively; $\rho_c(E_x)$ and $\rho_x(U_x)$ are the level densities of the compound and residual nuclei, respectively; $U_x$ is the excitation energy of the residual nuclei; $\sigma_x(E_x)$ is the inverse reaction cross section which is determined in the semi-classical approximation as follows:

$$\sigma_x(E_x) = \begin{cases} \pi R^2 \left( 1 - \frac{V_x}{E_x} \right) & \text{for } \ E_x > V_x \\ 0 & \text{for } \ E_x < V_x \end{cases} \hspace{1cm} (4)$$

If we use the nuclear entropy and constant temperature approximation can get from (3) and (4) the following formula for the $x$–particle width:

$$\Gamma_x = \frac{2S_x + 1}{\pi \hbar^2} M_x R^2 \int_{V_x}^{E_x^{\text{max}}} E_x \left( 1 - \frac{V_x}{E_x} \right) e^{-\frac{B_x + \delta_x + E_x}{\Theta}} dE_x$$ \hspace{1cm} (5)

where $B_x$ and $\delta_x$ are the binding energy and the odd-even effect parameter for the outgoing $x$–particle, respectively; $\Theta = kT$ is the nuclear thermodynamic temperature; $k$ is the Boltzmann constant.

Then, neglecting the $\gamma$–emission, from (1), (2) and (5) we get [10] following expression for $(n,x)$ cross section:

$$\sigma(n,x) = \sigma_c(n) \frac{(2S_x + 1)M_x e^{-\frac{B_x + \delta_x + V_x}{\Theta}} \left( 1 - \frac{W_{nx}}{\Theta} e^{-\frac{W_{nx}}{\Theta}} - e^{-\frac{W_{nx}}{\Theta}} \right)}{\Sigma_l(2S_l + 1)M_l e^{-\frac{B_l + \delta_l + V_l}{\Theta}} \left( 1 - \frac{W_{nl}}{\Theta} e^{-\frac{W_{nl}}{\Theta}} - e^{-\frac{W_{nl}}{\Theta}} \right)}$$ \hspace{1cm} (6)

where $W_{nx} = E_n + Q_{nx} - V_x$ and $W_{nl} = E_n + Q_{ni} - V_i$; $Q_{nx}$ and $Q_{ni}$ are the reaction energy; $E_n$ is the incident neutron energy. For fast neutrons total level width can be approximately taken as $\Gamma \approx \Gamma_n$.

Also, the odd-even effect parameters were neglected. In the energy relations can be used the following assumptions:

$$(E_n + Q_{nx} - V_x) \gg \Theta \text{ and } (E_n + Q_{ni} - V_i) \gg \Theta.$$ \hspace{1cm} (7)

So, the fast neutron induced $(n,x)$ reaction cross section is determined from (6) as follows

$$\sigma(n,x) = \sigma_c(n) \frac{(2S_x + 1)M_x e^{-\frac{Q_{nx} - V_x}{\Theta}}}{(2S_n + 1)M_n}$$ \hspace{1cm} (8)

If we use a formula for nuclear thermodynamic temperature as in [16] and the Fermi gas model formula for level density parameter [17] can get the following expression:

$$\Theta = \sqrt{\frac{13.5(E_n + Q_{np})}{A}}$$ \hspace{1cm} (9)
In the case of medium mass and heavy nuclei $(Z \gg 1)$ using the Weizsacker formula [18] for binding energy we get a formula for fast neutron induced $(n,p)$ reaction cross section from Eqs. (8) and (9) as following

$$\sigma(n, p) = C_p \pi (R + \lambda / 2\pi)^2 e^{-K_p \frac{N-Z+1}{A}}$$

where

$$C_p = \exp \left( Z A^{1/6} \sqrt{\frac{2\gamma - 1}{13.5(E_n + Q_{np})}} \right)$$

and

$$K_p = 4\xi \frac{A}{\sqrt{13.5(E_n + Q_{np})}}$$

Also, using the same calculation method we can get following formula for $(n,\alpha)$ cross section

$$\sigma(n, \alpha) = C_\alpha \pi (R + \lambda / 2\pi)^2 e^{-K_\alpha \frac{N-Z+0.5}{A}}$$

where

$$C_\alpha = 2\exp \sqrt{\frac{A}{13.5(E_n + Q_{n\alpha})}} \left( -3 \alpha + \gamma \left( \frac{4Z}{A} \right) + \varepsilon_\alpha - 2.058 \frac{Z}{A^{1/3}} \right)$$

and

$$K_\alpha = 2\xi \frac{A}{\sqrt{13.5(E_n + Q_{n\alpha})}}$$

Here: $Z$, $N$ and $A$ are the proton, neutron and mass numbers of the target nucleus, respectively; $\alpha$, $\gamma$, and $\xi$ are the Weizsacker formula constants; $\varepsilon_\alpha$ is the internal binding energy of $\alpha$-particle. The parameters $K_i$ and $C_i$ ($i=p$ or $\alpha$) in formulae (11), (12), (14) and (15) can be fitted and determined as the constants at each energy points for all isotopes.

The values of the fitted parameters $K_\alpha$ and $K_p$ for different neutron energies [19,20] are given in Table 1.

<table>
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<tr>
<th>$E_n$ (MeV)</th>
<th>$K_\alpha$</th>
<th>$K_p$</th>
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<tr>
<td>6</td>
<td>62.1</td>
<td>75.3</td>
</tr>
<tr>
<td>8</td>
<td>59.0</td>
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</tr>
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<td>10</td>
<td>53.5</td>
<td>52.1</td>
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<td>14.5</td>
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</tr>
<tr>
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<tr>
<td>20</td>
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</tr>
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</table>

### III. The effective neutron energy

The effective neutron energy is important for theoretical analysis of averaged over the continuum neutron spectrum cross sections. Average energy for fission neutrons is, usually, around 2 MeV [21]. At the same time, the threshold energy of $(n,\alpha)$ and $(n,p)$ reactions for most of isotopes lies in the region of $E_{th} \approx 2–5$ MeV [22]. So, the effective average energy of
the incident neutrons for fission spectrum in the case of (n,α) and (n,p) reactions should be different from 2 MeV. To clarify this statement the excitation function of the $^{58}$Ni(n,α)$^{55}$Fe and $^{58}$Ni(n,p)$^{58}$Co reactions [22] are displayed in Figs.1 and 2, as example. Fission neutron spectrum of 235U [21] is, also, shown in Figs.1 and 2. Similar pictures can be obtained for other isotopes.

Fig.1. Fission neutron spectrum of $^{235}$U and excitation function of the $^{58}$Ni(n,α)$^{55}$Fe reaction.

Fig.2. Fission neutron spectrum of $^{235}$U and excitation function of the $^{58}$Ni(n,p)$^{58}$Co reaction.

An average (n,α) or (n,p) cross section is determined by overlap of two curves for neutron energy spectrum and excitation function:
Here: \( \phi(E_n) \) is the neutron spectrum and \( x \) is the outgoing particle (\( x = \alpha \) or \( p \)).

The average effective neutron energy for the fission neutron induced \((n,\alpha)\) and \((n,p)\) reactions was got to be around 5 MeV, when we have used the averaged cross sections calculated by expression (16) and took into account the systematical regularity of parameters \( K_p \) and \( K_\alpha \) for other incident neutron energies (see Table.1, Figs.3 and 4).

![Fig.3. Energy dependence for parameter \( K_\alpha \).](image)

![Fig.4. Energy dependence for parameter \( K_p \).](image)

IV. Systematical analysis of the averaged \((n,p)\) and \((n,\alpha)\) cross sections

The theoretical \((n,p)\) cross sections calculated by formula (10) with fitted parameter \( K_p = 80 \) and \( C_p = 2.8 \) for average neutron energy \( \langle E \rangle \approx 5 \text{ MeV} \) of fission spectrum are compared with known experimental values taken from [14] in Fig.5.

![Fig.5. Theoretical and experimental \((n,p)\) cross sections.](image)

![Fig.6. Theoretical and experimental \((n,\alpha)\) cross sections.](image)

It can be seen that the statistical model formula (10) with average energy \( \langle E \rangle \approx 5 \text{ MeV} \) for fission neutrons is satisfactorily describes the systematical dependence of known experimental \((n,p)\) cross sections on the relative neutron excess parameter \((N - Z + 1)/A\).
Also, almost the same result was obtained for the averaged over the fission neutron spectrum (n,α) cross sections (see Fig.6). In this case the fitting parameters $K_\alpha$ and $C_\alpha$ were found to be 65 and 0.04, respectively, at average neutron energy $\langle E \rangle \approx 5$ MeV.

V. Conclusion
1. Known experimental (n,p) and (n,α) cross sections averaged over the fission neutron spectrum of $^{235}$U were analyzed using the statistical model based on the Weisskopf-Ewing theory and certain systematical regularity was observed.
2. It was shown that the experimental of (n,p) and (n,α) cross sections for fission neutrons are satisfactorily described by the statistical model.
3. The average effective neutron energy for (n,p) and (n,α) reactions induced by fission neutrons of $^{235}$U was found to be around 5 MeV.

VI. References