SELF-SHIELDING AND MULTIPLE SCATTERING CORRECTIONS TO MEASURED NEUTRON CAPTURE YIELD IN THE UNRESOLVED RESONANCE REGION

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ABSTRACT: A numerical procedure for calculation a coefficient to account the self-shielding and multiple scattering corrections to measured neutron absorption yield is proposed in the Unresolved Resonance region (URR). It uses a periodical resonant cross sections structure, which being modulated by multi-level Reich-Moore approximation of the Wigner's R- matrix theory. In this scheme the characteristic function model is applicable for even-even nuclei and for one open neutron channel under the threshold for neutron inelastic scattering. In a common case the multi-channel R-matrix elements must be constructed by Monte-Carlo method and using the statistical law distributions of level spacing and resonance widths for each compound system J^{π}. Some results are shown for nuclei ^{197}Au and ^{241}Pu. Integral formulae, expressed through self-indication functions for neutron for neutron absorption, are also proposed. Example is given with using the experimental data for self-indication functions for neutron capture of ^{238}U in the computational task.

1. INTRODUCTION

The reaction cross section $\Sigma_{\gamma}(E)$ is derived from measured absorption (capture) yield $Y(E)$. Analysing neutron absorption data, performed on a sample with a finite thickness *(d)* and averaged over an incident energy interval ΔE, is necessary to account for a reduction of the incident neutron flux $J(E)$ n/sec due to the neutron scattering and (or) neutron absorption in the target:

$$Y(d,E)=C(d,E)d\Sigma_{\gamma}(E), \qquad C(d,E)=C_1(d,E)+C_2(d,E) \tag{1}$$

In general this is a classical problem of neutron transport in a flat sample when self-shielding and multiple scattering effects cause the neutron flux reduction. The coefficient C_1 accounts for the first collision of incident neutrons with the sample nuclei (self-shielding effect), and the coefficient C_2 accounts for the effect on the multiple scattering of the slowing dawn neutrons in the target. The first effect decreases the number of neutrons in the incident neutron flux $J(E)$ and tends to lower capture yield, and the second one increases the capture yield. Both effects increase in magnitude with sample thickness (d).

A procedure for calculation C_1 and C_2 in the URR has been proposed, based on a statistical model for a periodical resonant cross sections structure [1]. It is applicable for a slap sample geometry and also using any simplifications in the theory of neutron resonance capture in reactor calculations [2,3]. The procedure has been developed and applied for even-even nuclei and for one open neutron channel under the threshold for neutron inelastic scattering [4].

In this paper the application of the numerical procedure for nuclei with a few open neutron (and fission) channels and under the threshold for neutron inelastic scattering is given and some results are shown (^{197}Au and ^{241}Pu).
Integral formulae for calculation the correction coefficients through transmission functions are also proposed, which functions give complete and comprehensive information for the neutron cross sections structure in the URR and reflect the physical phenomena occurring in the sample. The correction coefficient, calculated through the experimentally derived transmission function for ^{238}U is compared to results, obtained by the conventional procedure.

2. CORRECTION COEFFICIENT TO NEUTRON CAPTURE YIELD

The correction coefficient to capture yield is defined as a ratio of the capture yield in a sample of finite thickness (d) to the corresponding one in an infinitely thin sample ($\Sigma d << 1$), exposed to the same incident neutron flux $J(E)$:

$$C(d,E) = \frac{\int Y(d,E)dE}{\int_{\Delta E} Y(E)dE} = C_1(d,E) + C_2(d,E) \tag{2}$$

$$C_1(d,E) = \int \frac{\Sigma_\gamma(E)}{\Sigma(E)} \left(1 - exp[\Sigma(E)d]\right)dE \Big/ d\int \Sigma_\gamma(E)dE \tag{3}$$

$$C_2(d,E) = \int P(E)\frac{\Sigma_\gamma(E)}{\Sigma(E)} dE \int \frac{\Sigma_s(E')}{\Sigma(E')} F(E')W(E' \to E)dE' \Big/ d\int \Sigma_\gamma(E)dE \tag{4}$$

Here: $\Sigma_\gamma(E), \Sigma_s(E), \Sigma(E)$ are cross sections respectively for neutron capture, neutron elastic scattering and total neutron cross section, $1 - exp[-\Sigma(E)d]$ - first neutron collision probability in the sample surface at energy E, $P(E)$ - collision probability of slowing down neutron in a volume sample at energy E, $\int \frac{\Sigma_s(E')}{\Sigma(E')} F'(E')W(E \to E')dE'$ - neutron flux of slowing down neutrons in the target, $W(E \to E')$ - probability a neutron with energy E, after its scattering get (slowed down) into interval dE' (energy distribution function of slowing dawn neutrons).

The coefficient C_1 is considered in terms of self-shielding effect and *it can be exactly accounted using appropriate nuclear models for the resonance cross sections description*. For simplifying the computation task of C_2 some schemes from the theory of neutron resonance capture in a heterogeneous reactor medium are borrowed and the neutron processes are considered at resonance energies (E_λ) of the sample nucleus.

A/. Narrow Resonance Approximation (NRA)

The flux of slowing down neutrons at energy $E \equiv E_\lambda$ is determined by the neutron flux above resonance energy $(E > E_\lambda)$, where $\Sigma(E) \equiv \Sigma_p = constant$, and also assuming $J(E) \approx constant$.
Then the coefficient for multiple scattering corrections becomes;

$$C_2(d,E)=\frac{1-e^{-d\Sigma_p}}{1-P(d\Sigma_p)}\int_{\Delta E_\lambda}\frac{\Sigma_\gamma(E)}{\Sigma(E)}P[d\Sigma(E)]dE\Big/ d\int_{\Delta E_\lambda}\Sigma_\gamma(E)dE\,, \qquad (5)$$

B/. Flat Flux Approximation (FFA)

The collision probability of slowing down neutron $P[(d\Sigma(E)]$ in Eq.5 is approximately computed, assuming the neutrons are spread uniformly and isotropic in the volume sample. In certain circumstances, as $\Sigma_p d \le 0.1$ and $\frac{R}{d} \ge 10$, the collision probability approximates to this for an infinite sample surface (Σ_p is the potential cross section, R – sample radius):

$$P[d\Sigma(E)] \approx 1-\frac{1}{x}[0.5 - E_3\{x\}], \qquad (x = d\Sigma(E)), \qquad (6)$$

Here $E_3\{x\} = \int_1^\infty \frac{e^{-xt}}{t^3}dt$ is the exponential integral.

3. UNRESOLVED RESONANCE REGION – formulae and results

In considerations of the above paragraph, (E) denotes the energy of the incident neutron as well as the slowed down neutron to energy E in the volume sample. Therefore the above formulae are applicable for analyzing the experimental data obtained with a broad experimental resolution and these are interpreted as values averaged over many resonances.

The measured data in URR are obtained as averages in energy interval much wider than average resonance levels spacing ($\Delta E >> \overline{D}$), and the average resonance parameters are used in the computational task with the statistical laws for distributions of resonance parameters. If there several independent compound nucleus systems $(\nu = J^\pi)$, the coefficient $C_2(d,E)$ is calculated as [1]:

$$C_2(d,E) = \sum_\nu < \overline{\Sigma_\gamma^\nu}(E) > C_2^\nu(d,E) / < \Sigma_\gamma(E) >, \quad < \Sigma_\gamma(E) > = \sum_\nu < \overline{\Sigma_\gamma^\nu}(E) >, \qquad (7)$$

Brackets “< >” denote averaged capture cross section in energy interval ΔE and “--” denote average over statistical distributions of the resonance parameters for a compound system J^π.

The characteristic function model was applied in the computational procedure for even-even nuclei with one open neutron channel under the threshold for neutron inelastic scattering [4]. Now this procedure is also applicable on nuclei with a few open neutron channels and fission channels. In this case a periodical resonant cross section structure is modulated in a common way as it was previously described [5].

Results for the correction coefficient $C(d,E)$ for nuclei ^{197}Au are given in Figure 1 in comparison with those from SESH [6]. (SESH results are taken from Ref.7). Calculations with two different sample thicknesses are performed using the resonance parameters from ENDF/B-VII.1 [8] for *s*-wave (energy averaged values in the Resolved Resonance Region - RRR) and from RIPL 2 library [9] for *p*-wave in energy range from 5 keV (upper limit of RRR) up to E= 77.35 keV (first stage for inelastic scattering).

Results of ^{241}Pu for a sample shaped with a thickness of *d*=0.05cm (2.49e-3 n/barn) and radius *R*=4 cm are shown in Figure 2. The average resonance parameters for *s*- and *p*-wave from ENDF/B-VII.1 are used. (In this case $\Sigma_{abs} = \Sigma_\gamma + \Sigma_{fis}$).

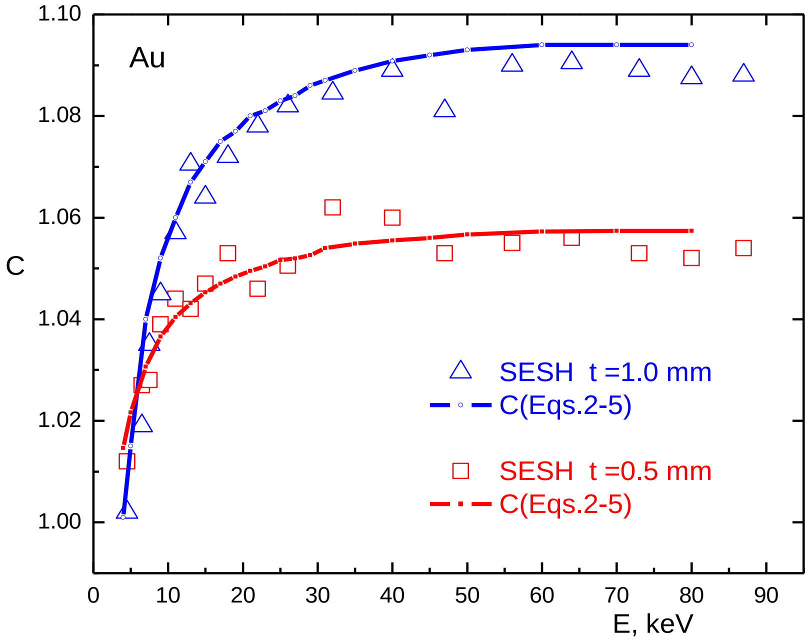

Figure 1. Coefficient $C(d,E)$ of ^{197}Au compared to these from SESH.

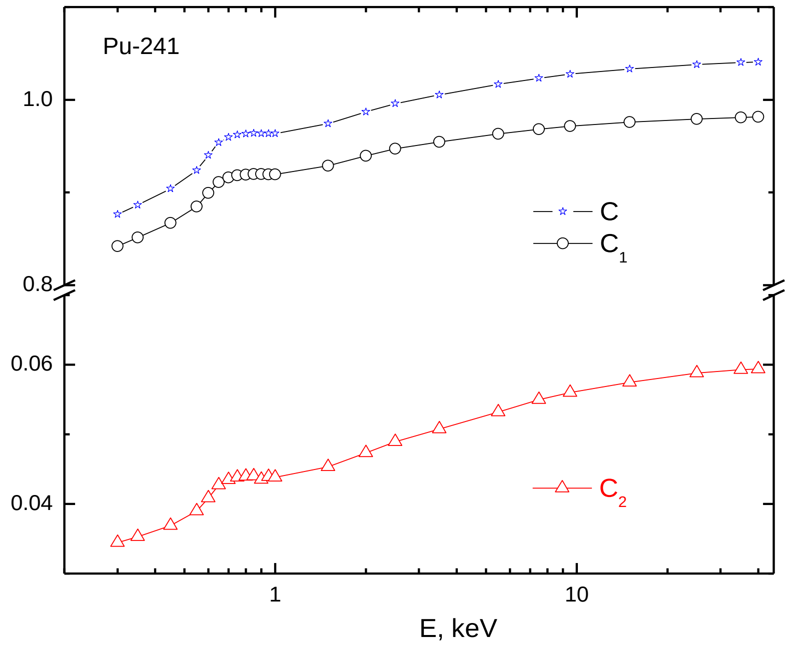

Figure 2. Coefficients C, C_1, C_2 of ^{241}Pu.

4. INTEGRAL FORMULAE FOR $C_2(d,E)$

In frame of the assumptions of (*FFA*) and (*NRA*) a possible description of $C_2(d,E)$ by the coefficient for first collision $C_1(d,E)$ (self-shielding effect) is done. The probability $P[\bar{l}\Sigma(E)]$ can be expressed as [1];

$$P_0(x) = \int_1^\infty \frac{dt}{t^3} \int_0^t (1 - e^{-xt'})dt', \quad (x = \Sigma d). \tag{8}$$

By its substitution in Eq.(6) one can obtain:

$$C_2(d,E) = f_0 \int_1^\infty \frac{dt}{t^3} \int_0^t t' C_1(dt')dt'' = f_0 \frac{1}{2}\left[\frac{1}{d^2} \int_0^d tdtC_1(t) + \int_d^\infty \frac{dt}{t} C_1(t) \right] \tag{9}$$

Where: $f_0 = \dfrac{1 - e^{-d\Sigma_p}}{1 - P(d\Sigma_p)}$

$$C_1(t,E) = \frac{1}{t}\int_0^t T_\gamma(t')dt', \qquad T_\gamma(t') = \int \Sigma_\gamma e^{-\Sigma t'} dE \Big/ \int \Sigma_\gamma dE, \tag{10}$$

Finally $C_2(d,E)$ in Eq.10 be converted into:

$$C_2(d,E) = f_0 \left[C_1(d,E) + \frac{1}{2}\int_d^\infty \frac{dt}{t} T_\gamma(t) - \frac{1}{2d^2}\int_0^d tT_\gamma(t)dt \right] \tag{11}$$

Table 1. $C(d,E)$ of ^{232}Th (sample with *d*=0.5mm and *R*=4cm). Comparison between results obtained with conventional formulae (Eqs. 2-5) to those with integral formulae (Eqs. 10,11).

E, keV	$C(d,E)$ *(Eqs.2-5)*	$C(d,E)$ *(Eqs.10,11)*	(a-b)/a %
4.5	1.017	1.017	0.0
6.5	1.024	1.024	0.0
8.5	1.028	1.027	0.1
10.5	1.0307	1.0289	0.17
15.5	1.036	1.032	0.38
17.5	1.037	1.032	0.48
21	1.0372	1.0324	0.46
25	1.0385	1.0328	0.54
29	1.039	1.033	0.56
32.5	1.039	1.033	0.57
42.5	1.0398	1.0335	0.58
47.5	1.0399	1.0336	0.6

Values for $C_1(d,E)$ and $C_2(d,E)$ obtained by transmission functions, reflect correctly all physics phenomena in the sample, as these functions give complete and comprehensive information about neutron cross sections structure in the URR [10].

In Table 1 the coefficient values of ^{232}Th obtained with integral formulae (Eqs. 10, 11) and these by the conventional procedure (Eqs.2-5) are compared, by using the evaluated average resonance parameters from Ref.5.

In Figure 3 the results of coefficient *C(d,E)* for ^{238}U are shown in (4-5) KeV. The results obtained by Eqs. 2-5 are compared with these calculated by integral formulae (Eqs.10,11). The self-indication functions $T_{\gamma}(d)$ in integral formulae are calculated in two ways; with the evaluated average resonance parameters in the URR and by using the experimental data for $T_{\gamma}(d)$[11].

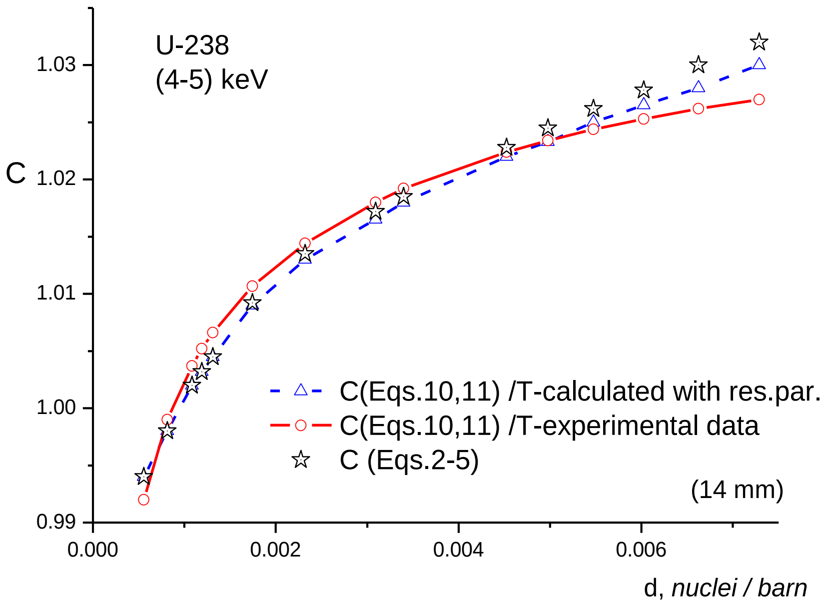

Figure 3. *C(d,E)*, calculated with Eqs.10,11 for different thick ^{238}U sample in (4-5) KeV**;** Δ - Eqs.10,11, self-indication functions $T_{\gamma}(d)$ are calculated with eval. aver. res. par., O - Eqs.10,11, experimental data for self-indication function $T_{\gamma}(d)$ [11], ∗ - Eqs. 2-5.

5. CONCLUSION

An analytical approach for self-shielding and multiple scattering corrections to measured capture (absorption) yield is proposed in the URR. The resonance Doppler broadening and resonance interference are accounted for in the applied numerical procedure, based on the periodical resonant cross sections structure, being modulated by multi-level RM approximation of the Wigner's R- matrix theory. The procedure is applicable for nuclei with few open neutron and fission channels under the threshold of inelastic neutron scattering.

Integral formulae for calculation the correction coefficients to absorption yield are proposed also. The coefficient is computed through self-indication functions for neutron absorption, which give complete and comprehensive information for the neutron cross sections structure in the URR and reflect the physical phenomena in the sample.

The possibility to obtain the correction coefficient to measured neutron capture yield on the base only of experimental data for self-indication function for neutron capture is demonstrated. Thus it is possible to assess the reduction coefficient *C(d,E)* without using the evaluated average resonance parameters in the URR .

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