STATISTICAL MODELING METHOD FOR ”ROMASHKA”- TYPE 4Π DETECTOR SYSTEM

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Abstract: At Frank Laboratory of Neutron Physics (*FLNP*) of the Joint Institute for Nuclear Research (*JINR*), a new versatile multidetector “Romashka”-type gamma-ray spectrometer was built and tested. Up to 44 *NaI(Tl)* hexagonal detectors can be arranged in 2 cylindrical arrays with varying diameter and distance between them. This system is supposed to be used for investigation of the neutron induced inelastic, capture and fission reactions in a number of important for fundamental physics and nuclear industry nuclei.

The aim of the present work is to study the response function of the system to a cascade of gamma-rays, when the basic configuration, consisted of 24 *NaI(Tl)* detectors, is used. This is done by the method of statistical modeling, which has already been successfully applied to a 12 *NaI(Tl)* detector system.

In addition to the methodological importance of the obtained results, they stitch the mathematical model, describing the gamma-ray response of the detectors, to the simple, but interesting in Monte-Carlo treatment model of mixing the gamma-ray cascades, striking the multidetector system.

The detectors’ gamma-sensitivity data are a good starting point for further phenomenological analysis of the measured, by such detector arrays, gamma-ray multiplicity and spectra from the neutron-nucleus radiative capture and/or fission reactions, based on the statistical model of de-excitation of the formed compound nucleus.

1. Introduction

The method is applied to 4π -multidetector “Romashka” type systems. These multidetector systems are a new version of the compact arranged “Romashka” type detectors used by Muradyan et al. [1-3], with relatively free constructive arrangement of the detector modules. The present work deals with the response of the 24 modules *NaI(Tl)*-crystal system, which appears from arbitrary number of gamma-quanta striking in the detector at several geometries of the crystal disposition, in the light of the multiplicity spectrometry of the radiation capture of thermal and resonance neutrons [4,5].

The interest for an investigation of the gamma-multiplicity following neutron radiative capture is driven by the yet incomplete theoretical description and physical understanding of this process. The multiplicity spectrometry applied in coincidence with the time-of-flight technique supplies broad information about decaying of the compound-nucleus resonance levels.

2. Basic relations

Gamma quanta scattering in the detector leads to a difference between the physical multiplicity of the analyzed process noted as *PM* and the measured apparatus multiplicity noted as *AM*. *AM* and *PM* are related in the following way

$$f_k = \frac{1}{y_\Sigma} \sum_{\nu=1}^{\nu_g} \int_0^\infty dE_1 \int_0^\infty dE_2 .... \int_0^\infty dE_\nu G_k \left(E_1, E_2, .., E_\nu\right) Y_\nu \delta\left(U - \sum\nolimits_{p=1}^{\nu} E_p\right), \tag{1}$$

where y_Σ is explained in *Eq. (6)*, f_k is the probability of events, for which the k detector modules have operated simultaneously (*AM*), $G_k\left(E_1, ..., E_\nu\right)$ is the concrete response function, in which ν gamma-quanta with energies $E_1, ..., E_\nu$, irradiated uniformly in the space from a source fixed in the detector cavity, striking in the detector, consequently k detector modules have operated simultaneously. It is normalized to one

$$\sum_{k=0}^{n_S} G_k\left(E_1, ..., E_\nu\right) = 1 \tag{2}$$

The excitation energy U is approximately the neutron binding energy for the compound nucleus. $Y_1 = S_1\left(J_c^{\pi_c} \to J_g^\pi, E_1\right)$, S_1 is the spectral density of energy of the first gamma-quanta, J_c^π – spin and parity of the compound nucleus, J_g^π – spin and parity of the ground state. For $\nu = 2$ Y_ν is given as follows

$$Y_2 = \sum_{J_1^\pi} S_1\left(J_c^\pi \to J_1^\pi, E_1\right) S_2\left(J_1^\pi \to J_g^\pi, U - E_1\right), \tag{3}$$

where $0 < E_1 < U$, J_1^π is determined by spin, parity and multipolarity selecting rules. For $\nu > 2$, Y_ν is

$$Y_\nu = \sum_{J_1^\pi \to ... \to J_{\nu-1}^\pi} S_1\left(J_c^\pi \to J_1^\pi, E_1\right) \left\{ \prod_{p=2}^{\nu-1} S_p\left(J_{p-1}^\pi \to J_p^\pi, E_p\right) \right\} S_\nu\left(J_{\nu-1}^\pi \to J_g^\pi, E_\nu\right), \tag{4}$$

here the sum is on all the spin-parity decaying ways, permitted by the gamma-transition selecting rules. k and ν are counted in the following way: $k = 0, 1, 2, ..., n_S$, $\nu = 1, 2, ..., \nu_g$, n_S is the maximum number of detector modules, operating simultaneously, which can be observed during the experiment, ν_g is the maximum number of γ-quanta which can be obtained in a given process. The probability with which ν gamma-quanta are obtained *(PM)*, noted as g_ν, is defined as follows

$$g_\nu = \langle Y_\nu \rangle / y_\Sigma, \tag{5}$$

where $y_\Sigma = \sum\nolimits_{\nu=1}^{\nu_g} \langle Y_\nu \rangle$ for $\langle Y_\nu \rangle$:

$$\langle Y_\nu \rangle = \int_0^\infty dE_1 \int_0^\infty dE_2 .... \int_0^\infty dE_\nu Y_\nu \delta\left(U - \sum\nolimits_{p=1}^{\nu} E_p\right). \tag{6}$$

Detector efficiency is:

$$\varepsilon_D = \sum_{k=1}^{n_S} f_k = 1 - f_0 . \tag{7}$$

Using *Eq. (5)*, one rewrites *Eq. (1)* in the form,

$$f_k = \sum_{\nu=1}^{\nu_g} \langle G_{k\nu} \rangle g_\nu + \langle \Delta f_k \rangle , \tag{8}$$

where the averaged response function $\langle G_{k\nu} \rangle$ and the fluctuation term $\langle \Delta f_k \rangle$ are given as follows

$$\langle G_{k\nu} \rangle = \int_0^\infty dE_1 \int_0^\infty dE_2 .... \int_0^\infty dE_\nu G_k \left(E_1, E_2, .., E_\nu \right) \delta \left(U - \sum\nolimits_{p=1}^{\nu} E_p \right), \tag{9}$$

$$\langle \Delta f_k \rangle = \sum_{\nu=1}^{\nu_g} g_\nu \langle \Delta G_{k\nu} \rangle , \tag{10}$$

where

$$\langle \Delta G_{k\nu} \rangle = \int_0^\infty dE_1 \int_0^\infty dE_2 .... \int_0^\infty dE_\nu G_k \left(E_1, E_2, .., E_\nu \right) \frac{\langle Y_\nu \rangle - Y_\nu}{\langle Y_\nu \rangle} \delta \left(U - \sum\nolimits_{p=1}^{\nu} E_p \right).$$

Equation (8) gives a linear relation between *AM* and *PM*. It is possible if the following inequality

$$\left| \langle \Delta G_{k\nu} \rangle \right| << \langle G_{k\nu} \rangle , \tag{11}$$

is satisfied.

The solution of the problem defined in *Eqs. (1-10)* will be investigated by statistical modeling method based on Monte-Carlo simulations of the processes in the multidetector system.

3. Statistical modeling method

The main structural unit of this method is the act of modeling. It can be conditionally divided into two consecutive stages. The first stage: modeling the cascade of a given number of gamma-quanta irradiated from the source in the detector cavity, based on the previous information for gamma-transitions in the compound nucleus. The second stage: modeling the response of these gamma-quanta on the base of Monte-Carlo modeling their trajectories in the detector, from the source point to the absorption or leave of the detector. If one takes N_Σ modeling acts, *PM* and *AM* are derived from them consequently

$$g_\nu = F_\nu / N_\Sigma \tag{12a),}$$

$$f_k = N_k / N_\Sigma \tag{12b),}$$

$$\varepsilon_D = 1 - N_0 / N_\Sigma \tag{12c),}$$

where N_k is the number of events, for which the k detector modules have operated simultaneously and F_ν is the number of events, in which ν gamma-quanta are obtained.

3.1. Statistical modeling the concrete response function of the 24-module "Romashka"

The concrete response function $G_k \left(E_1, ..., E_\nu \right)$ is the detector response to a cascade of a ν number of gamma-quanta, with energies $E_1, E_2, ..., E_\nu$ at $\sum\nolimits_{p=1}^{\nu} E_p = U$, it is calculated as follows

$$G_k \left(E_1, ..., E_\nu \right) = f_k , \; g_\nu = 1, \; \varepsilon_D = 1 - G_0 \left(E_1, ..., E_\nu \right). \tag{13}$$

Table 1 and *Table 2,* show the concrete response function to a cascade of different number of gamma-quanta. The response is achieved by passing this cascade through the *ROMSD* code

[6], see *Fig. (1)*, 10^5 times each. The *ROMSD* code has functioned with the geometrical dimensions and characteristics of a 24-modules “Romashka” detector in multiplicity spectrometry mode with a “test geometry” *G1* and “compact geometry” *G2*, see. *Fig. 2*.

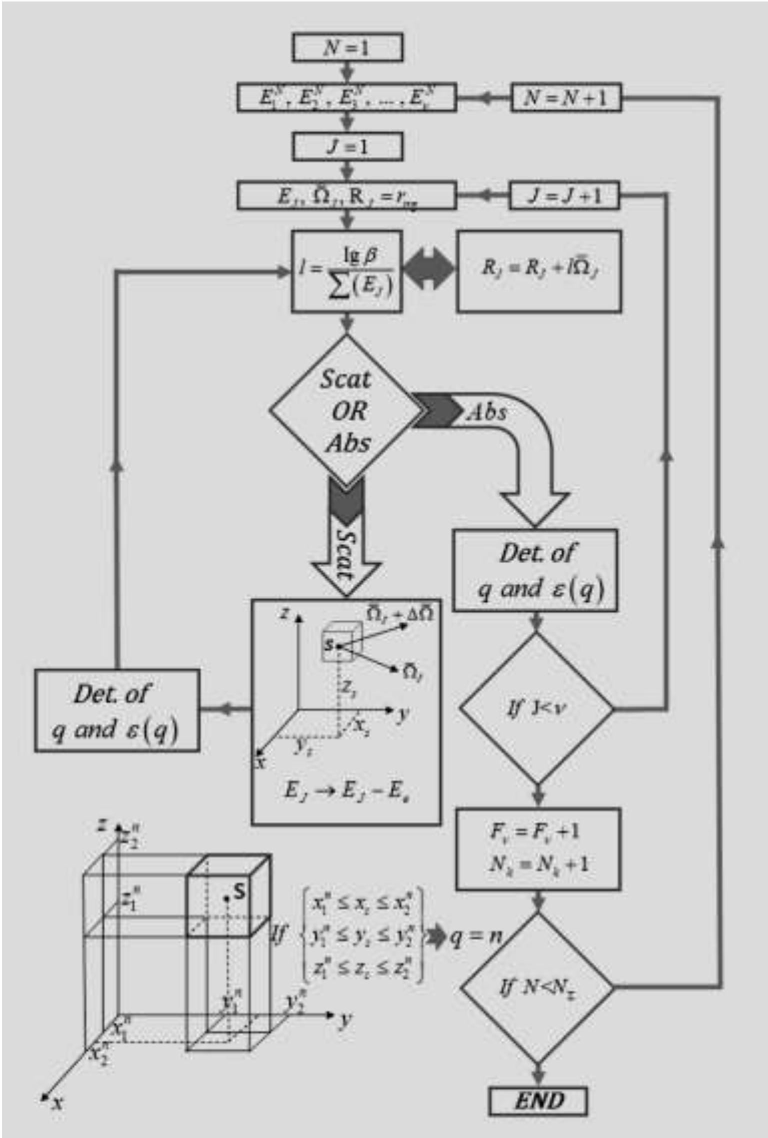


Figure 1. The block diagram of ROMSD code [5].

In this mode the registration of the counter and levels of discrimination are realized according to the conditions [6],

$$If \left\{ \begin{array}{c} \sum_{q=1}^{k} e(q) > E_{\Sigma} \\ e(q) \ge E_{in} \quad q=1,2,3,...,k \end{array} \right\} \rightarrow N_k = N_k + 1, \qquad (14)$$

where $e(q)$ is the energy release in the q detector module after passing all the gamma-quanta of the cascade with the initial energies $E_1, E_2, ..., E_\nu$, E_Σ is the upper summator threshold, E_{in} is the lower threshold of each detector module. The response is modeled and investigated at the following thresholds: $E_{\Sigma} = 1MeV$, $E_{in} = 0.1MeV$ and full energy $\sum_{p=1}^{\nu} E_p = 7.5MeV$, $\nu = 1,2,3,4$ for different internal energy structure of the cascades. The cases are noted as follows

C1 $E_1 = 7.5$, $\nu = 1$,

C2 $E_1 = E_2 = 3.75$, $\nu = 2$, *C3* $E_1 = 6.5$, $E_2 = 1$, $\nu = 2$,

C4 $E_1 = E_2 = E_3 = 2.5$, $\nu = 3$, *C5* $E_1 = 6$, $E_2 = 0.75$, $E_3 = 0.75$, $\nu = 3$,

C6 $E_1 = E_2 = E_3 = E_4 = 1.875$, $\nu = 4$,

C7 $E_1 = 5$, $E_2 = 1.5$, $E_3 = E_4 = 0.5$, $\nu = 4$.

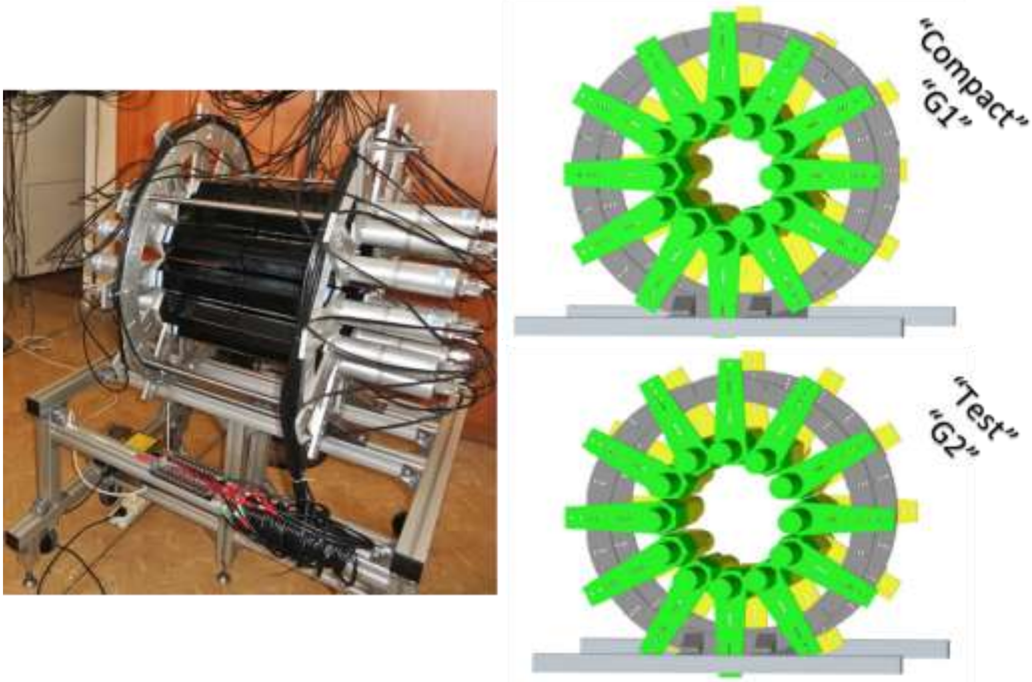


Figure 2. The detector "ROMASHKA" and the investigated geometries.

The derived values for *C1* to *C7*, for geometries *G1* and *G2* are listed in *Table 1* and *Table 2*. *Figure 3* contains the comparison of the response functions for geometries *G1* and *G2*.

Table 1. The concrete response function $G_{k\nu}$ *for the "test geometry" G1.*

k	$C1$	$C2$	$C3$	$C4$	$C5$	$C6$	$C7$
0	0.660	0.436	0.617	0.316	0.542	0.218	0.421
1	0.315	0.394	0.216	0.333	0.100	0.252	0.101
2	0.024	0.143	0.137	0.244	0.231	0.297	0.225
3	0.001	0.025	0.028	0.087	0.106	0.162	0.175
4	0.000	0.002	0.002	0.018	0.020	0.055	0.063
5	0.000	0.000	0.000	0.002	0.001	0.012	0.013
6	0.000	0.000	0.000	0.000	0.000	0.004	0.002

Table 2. The concrete response function $G_{k\nu}$ for the "compact geometry" G2.

k	$C1$	$C2$	$C3$	$C4$	$C5$	$C6$	$C7$
0	0.556	0.313	0.511	0.192	0.380	0.114	0.287
1	0.406	0.424	0.218	0.298	0.076	0.178	0.056
2	0.036	0.213	0.213	0.313	0.297	0.311	0.211
3	0.002	0.045	0.054	0.148	0.192	0.244	0.264
4	0.000	0.005	0.004	0.043	0.049	0.113	0.137
5	0.000	0.000	0.000	0.006	0.006	0.033	0.038
6	0.000	0.000	0.000	0.000	0.000	0.007	0.007

The comparison between the values given in *Table 1* and *Table 2* shows a better efficiency of the "compact geometry".
Deviations of the response have been observed in all types of "Romashka" detectors [6].

3.2. Modeling and investigation of the sensitivity of the 24-module "Romashka"

In the case of the "compact geometry" (*G1*), one models and investigates the response function at the thresholds of $E_{in} = 0.1 MeV$, $E_{\Sigma} = 1 MeV$ for the cascades *C1, C2, C3, C4, C5, C6, C7, C8, C9, C10, C11*. The new cascades *C8, C9, C10, C11*, corresponding to $\nu = 5$ and $\nu = 6$, are taken as follows:

$$C8 \quad E_1 = E_2 = E_3 = E_4 = E_5 = 1.5, \ \ \nu = 5$$
$$C9 \quad E_1 = 5, E_2 = 1.0, E_3 = E_4 = E_5 = 0.5, \ \ \nu = 5$$
$$C10 \quad E_1 = E_2 = E_3 = E_4 = E_5 = E_6 = 1.25, \ \ \nu = 6$$
$$C11 \quad E_1 = 6, \ E_2 = 0.5, \ E_3 = 0.5, \ E_4 = 0.3, \ E_5 = 0.1, \ E_6 = 0.1, \ \ \nu = 6$$

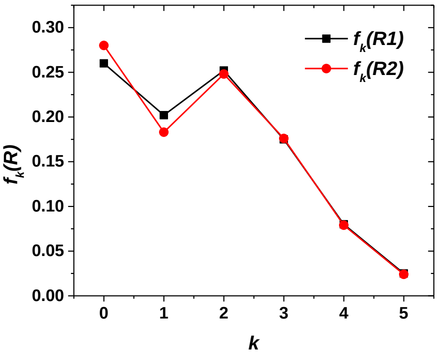
The physical multiplicity is given as follows: g_1 = 0.05, g_2= 0.15, g_3 = 0.30, g_4 = 0.35, g_5 = 0.10, g_6 = 0.05, i.e. $F_\nu = g_\nu N_\Sigma$ with $N_\Sigma = 500000$. The internal ratio of the cascades of ν-type, is given in the form of the sum $g_\nu = g_\nu(1) + g_\nu(2)$, as follows

$$R1 \Rightarrow \begin{cases} C1 \rightarrow g_1 = 0.05 \\ C2 + C3 \ \ \rightarrow g_2 = 0.10 + 0.05 \\ C4 + C5 \ \ \rightarrow g_3 = 0.23 + 0.07 \\ C6 + C7 \ \ \rightarrow g_4 = 0.30 + 0.05 \\ C8 + C9 \ \ \rightarrow g_5 = 0.02 + 0.08 \\ C10 + C11 \rightarrow g_6 = 0.02 + 0.03 \end{cases} \qquad R2 \Rightarrow \begin{cases} C1 \rightarrow g_1 = 0.05 \\ C2 + C3 \ \ \rightarrow g_2 = 0.05 + 0.10 \\ C4 + C5 \ \ \rightarrow g_3 = 0.20 + 0.10 \\ C6 + C7 \ \ \rightarrow g_4 = 0.25 + 0.10 \\ C8 + C9 \ \ \rightarrow g_5 = 0.04 + 0.06 \\ C10 + C11 \rightarrow g_6 = 0.01 + 0.04 \end{cases}$$

i.e. $F_\nu = F_\nu(1) + F_\nu(2)$, where $F_\nu(1) = g_\nu(1) N_\Sigma$, $F_\nu(2) = g_\nu(2) N_\Sigma$. The comparison between *R1* and *R2* responses for the "compact geometry" *G1* is given in *Table 3*.

Table 3

k	0	1	2	3	4	5
$f_k(R1)$	0.260	0.202	0.252	0.175	0.080	0.025
$f_k(R2)$	0.280	0.183	0.248	0.176	0.079	0.024



The response *R1* differs from the response *R2*

$$\left|f_k(R1)-f_k(R2)\right|>\delta_k(R1)+\delta_k(R2). \tag{15}$$

The differences are significant and the inequality is satisfied for $k=0,1,2$, at the binomial estimated statistical error $\delta_k=\sqrt{f_k(1-f_k)/N_\Sigma}$.

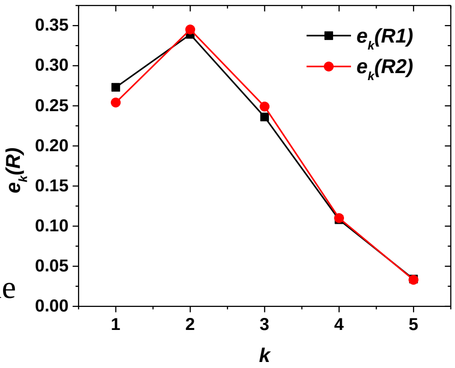
In the real experiment the normalization

$$e_k=\frac{N_k}{S_\gamma}, \qquad S_\gamma=\sum_{q=1}^{n_S}N_q \tag{16}$$

is used, where N_k is the experimentally measured number of events, for which the k detector modules have operated simultaneously. Usually in the real experiment the quantity N_0 is difficult to be estimated precisely. The comparison between *R1* and *R2* responses for the "compact geometry" *G1* for e_k is given in *Table 4*, where the same significant differences are observed.

Table 4

k	1	2	3	4	5
$e_k(R1)$	0.273	0.339	0.236	0.108	0.034
$e_k(R2)$	0.254	0.345	0.249	0.110	0.033



Obviously, the distribution of the energy in the cascade has very important role for the response of the detector.

3.3. Statistical modeling the efficiency of the 24-module "Romashka" and its dependence on the upper summator threshold

By modeling at different (several) upper summator thresholds, the efficiency of both *R1* and *R2* for the "compact geometry" *G1*, are obtained by *Eq. (12c)*:

$E_\Sigma[MeV]$	0.00	0.50	1.00	1.50	2.00	2.50
$\varepsilon_D(R1)$	0.86	0.79	0.74	0.71	0.67	0.60
$\varepsilon_D(R2)$	0.83	0.77	0.72	0.70	0.67	0.60

The efficiency is sensitively weak to the details of the process, if the upper summator threshold is relatively high.

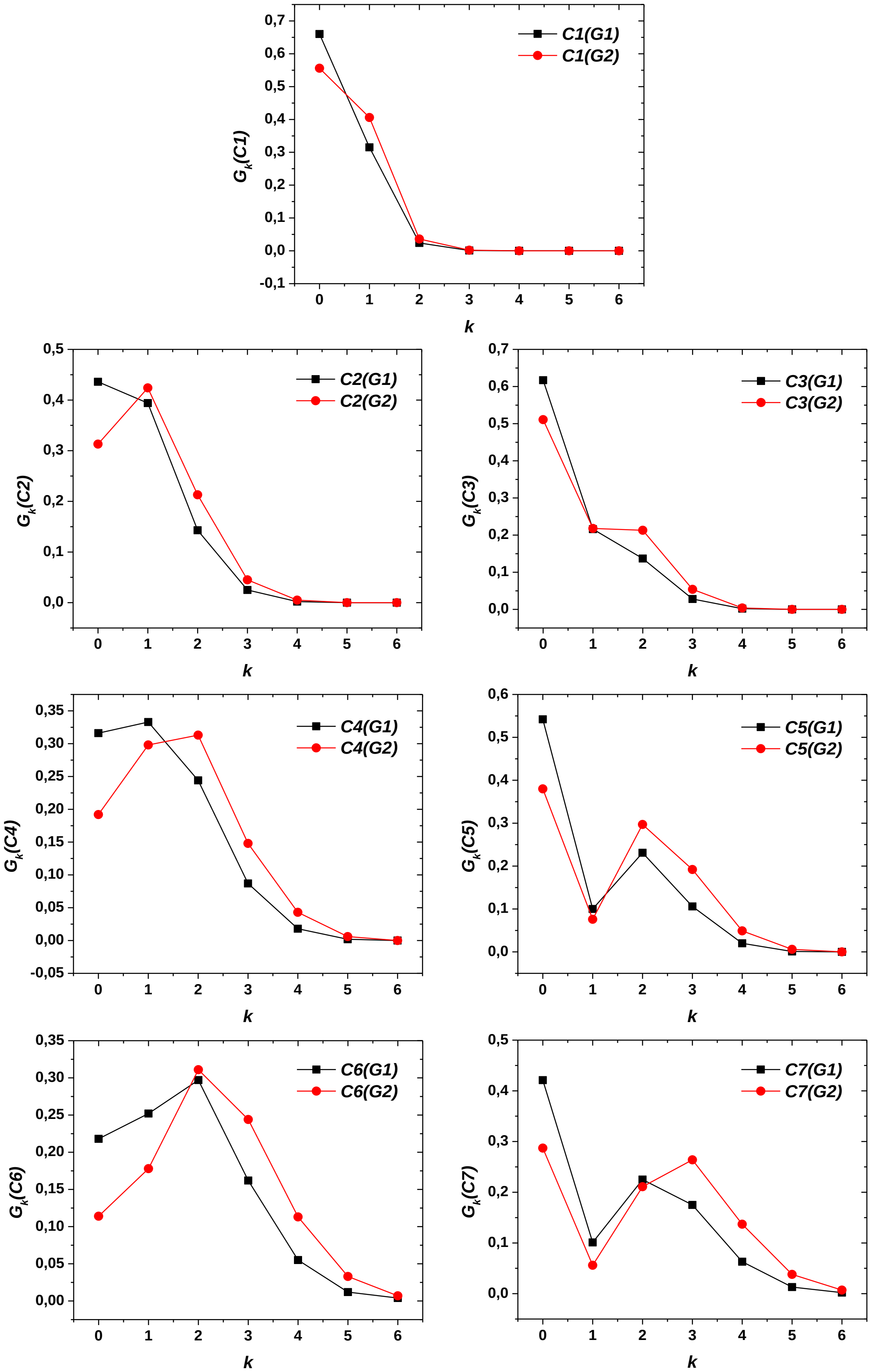


Figure 3. Comparison of the response functions for the geometries G1 and G2.

4. Concluding remarks

The obtained results have important methodological character. They are outcome of a statistical modeling the detector's response function and a simple model of mixing the cascades of γ-rays impinging the detector system, in Monte-Carlo treatment.

The detector's sensitivity data can be used for a further phenomenological analysis of the experimentally obtained γ-multiplicities and γ-spectra in the neutron inelastic scattering, radiative capture or nuclear fission, for verification of the compound nucleus decay model.

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