

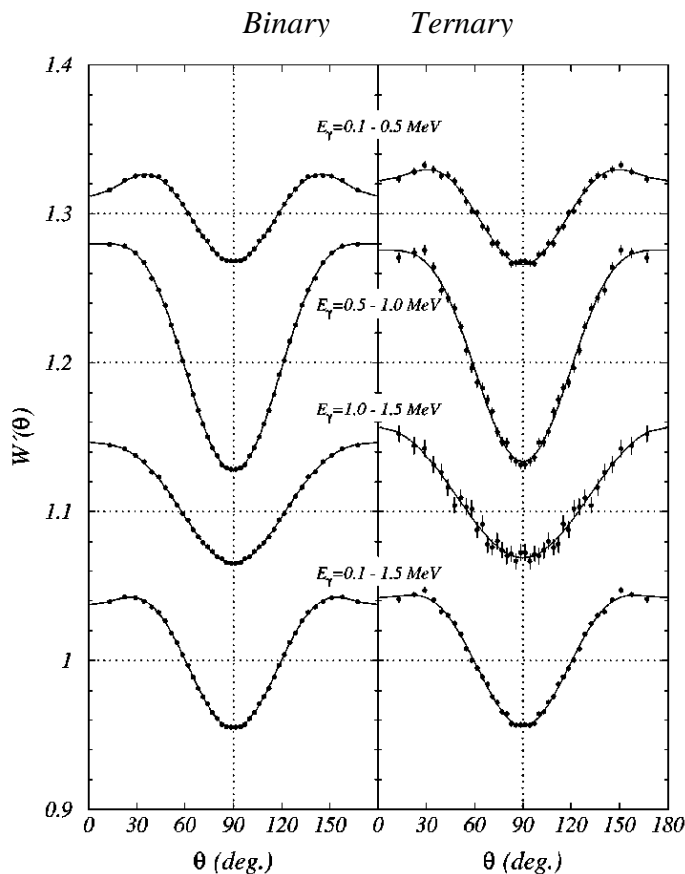
ANISOTROPY OF PROMPT NEUTRON'S EMISSION  
FROM THE FISSION FRAGMENT

I.S. Guseva

*NRC "Kurchatov Institute", B.P. Konstantinov Petersburg Nuclear Physics Institute,  
Gatchina, Leningrad District, 188300, Russia*

The large angular momentum of initial fission fragment can induce an appreciable anisotropy of neutron emission in its center-of-mass. But the kinematic focusing of evaporated neutrons due to the fragment motion does not allow so easy to see this effect by direct measurement. The aim of this study was to evaluate anisotropy for evaporated neutrons in the reference frame of fission fragment and then to investigate how such anisotropy is manifested in various experiments.

It is experimentally established that gamma rays accompanying spontaneous fission or fission induced by slow neutrons have quite pronounced anisotropy with respect to the fission axis. So the Fig.1 demonstrates the angular distributions of  $\gamma$ -rays for binary and  $\alpha$ -accompanied fission in  $^{252}\text{Cf}$  [1]. These data correspond to different  $\gamma$ -ray energy intervals.

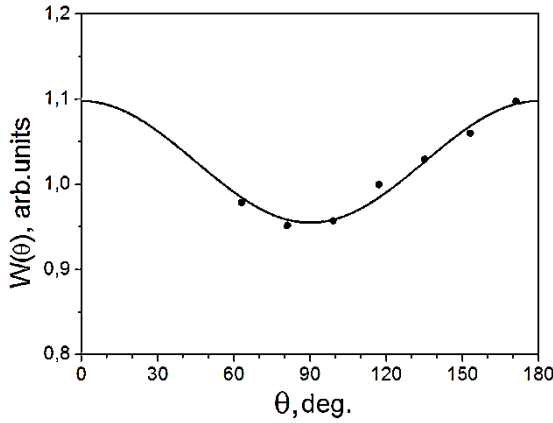


**Fig.1** Angular distributions  $W'(\theta)$  of  $\gamma$ -rays with respect to the fragment motion for binary and  $\alpha$ -accompanied fission of  $^{252}\text{Cf}$ .

The scale is correct for the lowest plots; others are shifted consecutively by 0.1 units. On the Fig.2 the angular distribution for the target of  $^{235}\text{U}$  is shown [2]. Note that the angular distributions for californium are given in the rest frame of fission fragment while the data for uranium correspond to laboratory system. Since the fragment velocity is significantly less than the speed of the light, therefore the anisotropy of gamma-emission from a stationary fission fragment is close enough to that observed in laboratory system.

According to Strutinskii [3] a reason for such anisotropy can be the presence of a large angular momentum of the primary fission fragment. This fragment spin can appear at the scission point and correlated with the direction of fragment motion.

The situation is quite different with the neutron evaporation. The average energy of emitted neutrons in



**Fig.2** Angular distribution  $W(\theta)$  of  $\gamma$ -rays with respect to the fragment motion direction for the reaction  $^{235}\text{U}(n_{th},f)$ .

the fragment's center-of-mass is comparable to kinetic energy per nucleon for a moving fragment. This leads to a large kinematic effect, which does not allow so easy to see the influence of similar anisotropy on neutron-fragment correlation. But, since neutron evaporation precedes the emission of gamma rays, we can expect that initial spin orientation of the fission fragment should also affect the angular distribution of neutrons. Thus, it is a natural desire to obtain in the beginning a mathematical evaluation of neutron anisotropy in the rest frame of fragment, and then considering the strong influence of the fragment

motion one can evaluate its manifestation in a particular form of correlation experiment.

T. Ericson and V. Strutinski have performed the quasi-classical approach for this task. In the classical limit it can be expected that the anisotropy  $A$  for angular distribution of neutrons

$$W \sim 1 + A \cdot \cos^2(\theta)$$

will depend on the ratio of the characteristic energies, namely: the centrifugal energy of a particle due to nucleus rotation and the average kinetic energy of an evaporated particle at the nuclear surface. In this case the angular distribution of emitted neutrons can be written as a function of the mass  $\mu$  of the emitted particle, of the angular velocity  $\omega$ , the nuclear radius  $R$  and the nuclear temperature  $T$ :

$$A = \frac{\mu \cdot \omega^2 \cdot R}{2 \cdot T}.$$

Then Gavron [3] has offered a statistical model calculation for consideration. In his paper the final angular distribution of evaporated neutrons in the fragment's centre-of-mass system was defined by the sum over spherical harmonics:  $W(\theta) = \sum_{lm} P_{lm} |Y_{lm}(\theta, \varphi)|^2$ .

The evaporation cascade is followed by a Monte-Carlo procedure in which the probability  $P_{lm}$  of emitting a neutron with given orbital angular momentum  $l$  and its projection  $m$  is proportional to the sum over all values of final fragment's spin  $J_f$  that can couple to initial spin  $J_i$  for the given  $l$  value:

$$P_{lm} \propto \sum_{J_f} \int_0^{E^* - B_n} \rho_{J_f}(E^* - B_n - \varepsilon) \cdot T_l(\varepsilon) \cdot \left| C_{J_f M_f l m}^{J_i M_i} \right|^2 d\varepsilon.$$

The neutron spin was neglected since it has no significant effect on the angular distribution but its inclusion considerably increases the amount of computer time.

The result of such approach depends on the fragment's level density  $\rho_{J_f}(E^* - B_n - \varepsilon)$ , neutron transmission coefficient  $T_l(\varepsilon)$  and vector coupling coefficient  $C_{J_f M_f l m}^{J_i M_i}$ .

Here:  $E^*$  – the excitation energy of the fragment;

$B_n$  – the neutron binding energy;

$e$  – the energy of evaporated neutron.

In his calculation the angular momentum ( $J$ ) and its projection on the fission axis ( $M$ ) were determined at each stage of cascade.

The initial spin projection was assumed to be equal zero ( $M=0$ ) in consequence of a suggestion that the angular momentum of primary fragments is aligned in a plain perpendicular to the direction of fission [5].

It is necessary to mention that Gavron could not obtain very accurate result at that time for the reason that computers were not fast enough for Monte-Carlo calculations. Similar calculations of anisotropy have been performed some years ago with better accuracy and these results were presented in proceedings of ISINN-13 [6]. In this paper the initial spin distribution of the fragment was taken in “standard” form:

$$\rho_J = (2J + 1) \cdot \exp\left(- (J + 0.5)^2 / B^2\right).$$

The parameter "B" for such distribution was determined using the average spin value for primary fragments.

Our calculations were performed not only with the “standard” form of angular distribution, but also with the form proposed by S. Kadmensky:

$$\begin{aligned} \rho_J &= (2J + 1) & J \leq J_{\max} \\ \rho_J &= 0 & J > J_{\max} . \end{aligned}$$

It was shown that the shape of spin distribution has no influence on the result of calculation. Everything depends on the mean value of the initial fragment spin.

The level density depends on the excitation energy as well as the angular momentum of the fragment. To determine kinetic energy of evaporated neutron we took into account that the level density is proportional to the square root of the excitation energy:

$$\rho \propto \exp\left(2\sqrt{aE^*}\right).$$

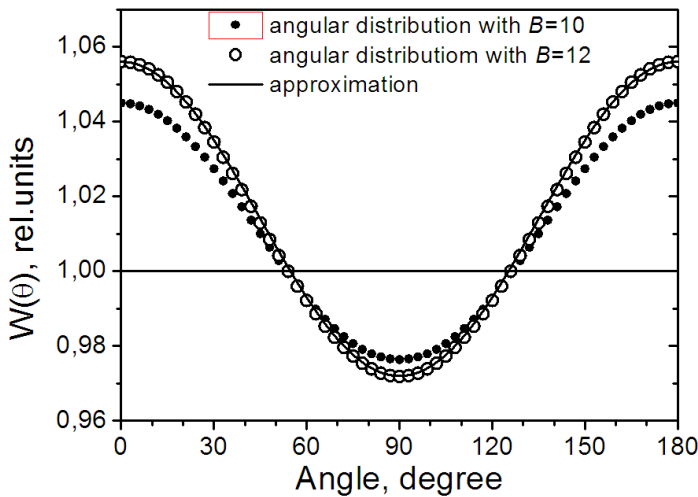
The spin dependency was used to find the angular momentum of a residual nucleus, which appeared after the neutron evaporation.

The excitation energy of fission fragments directly after scission notates by the subscript zero ( $E_0^*$ ). This subscript increases after neutron evaporation by a number one ( $i$  – serial number of evaporated neutron). The excitation energy decreases after emission on neutron binding energy and kinetic energy of evaporated particle ( $E_i^* = E_{i-1}^* - B_n - e_i$ ). Evaporation cascade is finished if the excitation energy is less than the binding energy of a neutron. To determine the primal excitation energy of fission fragments the averaged experimental values of neutron multiplicity for both light and heavy fragments were used.

The neutron transmission coefficients were calculated using well known Blatt-and-Weisskopf formulas [7]. For these calculations the nuclear radius was used, as well as the energy of emitted neutrons. The pictures of calculated neutron transmission coefficients for light and heavy groups of fission fragments were presented in the paper [6]. Taking into account neutron energy spectrum, we can ignore the values of neutron orbital momentum above 5 for both light and heavy regions of fragment mass.

The intermediate result of the Monte-Carlo calculations can be presented in the tabular form (see [6]), where the values in cells correspond to the amount of neutrons with given orbital momentum and its projection. Non-zero quantities in cells with a fixed orbital momentum  $l$  were accumulated consequentially by randomly choosing  $l$  value for a neutron with given kinetic energy through the proportion of transmission coefficient values with different orbital momenta. Vector coupling coefficient determines their distribution in each line. We can say that these values in cells are the non-normalized probability  $P_{lm}$  coefficients. These coefficients allow us to obtain the angular distribution of emitted neutrons in the frame of fission fragment.

The calculated angular distributions relative to the fission axis for two different values of the averaged initial spin for fission fragment are presented on the Fig.3. These curves are symmetric relative to 90 degrees and they can be sufficiently well approximated by the



**Fig.3** The calculated angular distributions of neutrons relative to the fission axis for different  $B$  values.

expression  $W(\theta) \sim 1 + b \cdot P_2(\cos\theta)$  with Legendre polynomial of the second degree. Such mathematical expression allows us to have normalized distribution of evaporated neutrons for different values of anisotropy parameter. This is why the Legendre polynomial in case of analytical description usually used. But for the Monte Carlo calculations we need only to know the shape of the angular distribution, so in this case we can use the angular dependence with the square of the cosine:  $W(\theta) \sim 1 + A_{nf} \cdot \cos^2 \theta$ . Here

$A_{nf}$  determined by the expression  $A_{nf} = \frac{W(0^\circ) - W(90^\circ)}{W(90^\circ)}$ . Taken into account that normalized

angular distribution is  $W(\theta) \sim 1 + b \cdot (3 \cos^2 \theta - 1) / 2$  the relation between two different

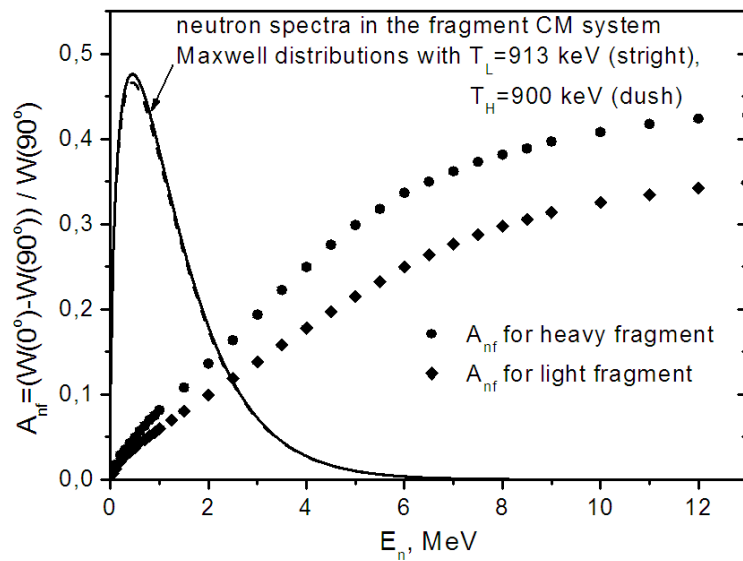
coefficients can be written this way:  $A_{nf} = \frac{3b}{2 - b}$  or  $b = \frac{2A_{nf}}{3 + A_{nf}}$ .

Previously [6] it has been shown the behavior of anisotropy  $A_{nf}$  for fixed orbital momenta of emitted neutrons and fixed neutron energies. It was understood later that for different experimental data descriptions the detailed energy dependence of the neutron anisotropy is very useful. This allows somebody to follow the conditions of the experiment in more detail including the experimental energy threshold. The Fig.4 demonstrates the example of calculated energy dependence of the anisotropy coefficient  $A_{nf}$  for light and heavy fragments  $^{252}\text{Cf}$ . Previously similar dependence has already been used in the description of the n-f correlation experiment for the neutron induced fission of  $^{235}\text{U}$  [8]. As we can see on the

Fig.4, that part of the curve, which corresponds to the large values of the neutron yields, can be approximated by a straight line. This approximation ( $b = A_2 \cdot E_n$ , where  $E_n$  is neutron energy in the fragment system) was used by A. Vorobiev during the description of neutron-fragment correlation. The best agreement of calculated curves with experimental data for neutron-fragment correlations obtained by Vorobiev [9] gives the value  $A_2 = 0.04$ , if neutron energy described in MeV. It means that for mean neutron energy 1.36 MeV in the fragment system the anisotropy  $A_{nf}$  is equal:

$$A_{nf}(E_{CM}) = \frac{3 \cdot b}{2 - b} = \frac{3 \cdot A_2 \cdot E_n}{2 - A_2 \cdot E_n} = 0.084,$$

what is very close to the result obtained on the base of a large initial fragment spin.



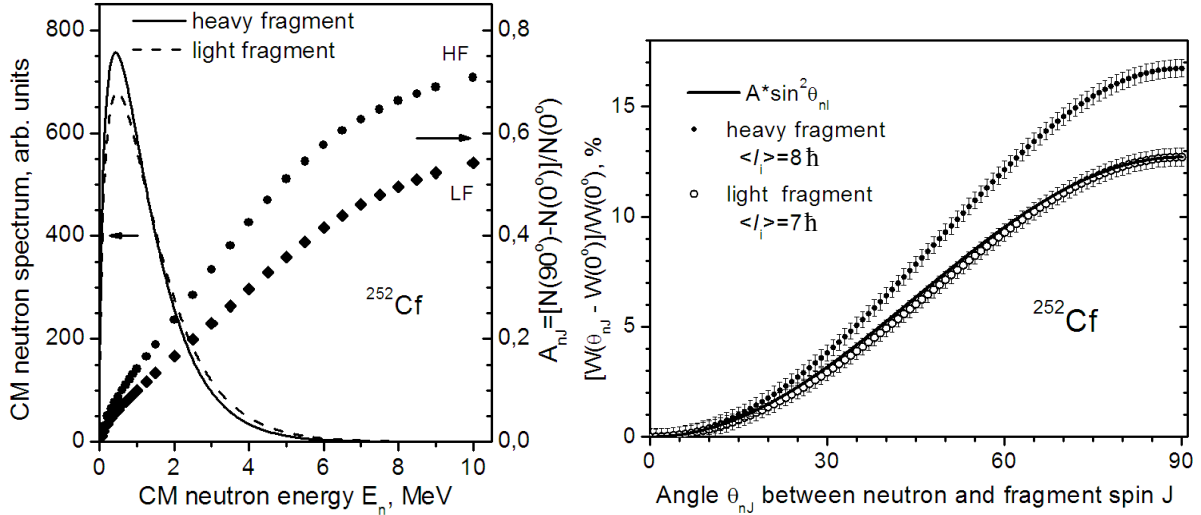
**Fig. 4** The energy dependence of neutron emission anisotropy  $A_{nf}$  for light and heavy fission fragments (CM system) relative to the fission axis.

The angular anisotropy in the rest frame of fragment relative to the fission axis was taken into account for better description of the neutron-fragment correlation data. Such coordinate system does not seem convenient for those experiments where we have interest in coincidences of two neutrons and any direction of fission axis is possible. For individual fission event it seems better to use the axis of reference lying in the plane perpendicular to the fission axis. And its orientation on this plane can be defined by the maximum value of the initial fragment spin projection  $J$ . The further calculations of the angular distribution of neutrons remain the same.

Choosing a reference axis this way we can find that the angular distribution of neutrons differs from the spherical on the term which can be well approximated by  $\sin^2(\theta)$  instead of  $\cos^2(\theta)$ . It is possible to calculate the energy dependence of coefficient  $A_{nJ}$  for such kind of anisotropy too (see Fig.5). If we compare two versions of anisotropy we can see that the values of the last anisotropy are two times larger than the values of anisotropy which was calculated against the fission axis. This is due to the fact that during the preparation of the

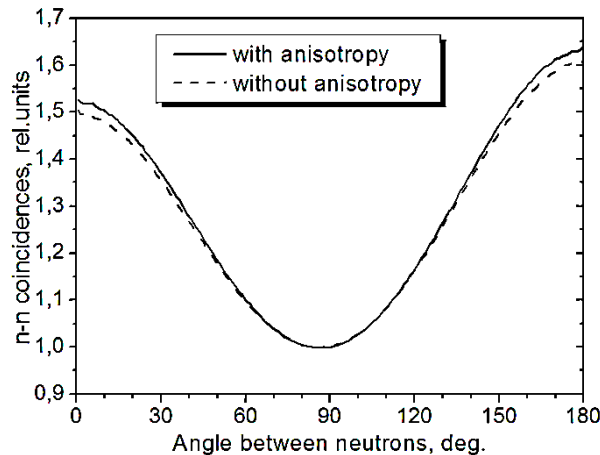
angular distribution of neutrons relative to the direction of the fragment motion the averaging over all orientations of the fragment spin takes place.

Due to Monte-Carlo simulation it is possible to realize how anisotropy of neutron emission can influence on different measurements in correlation experiments.



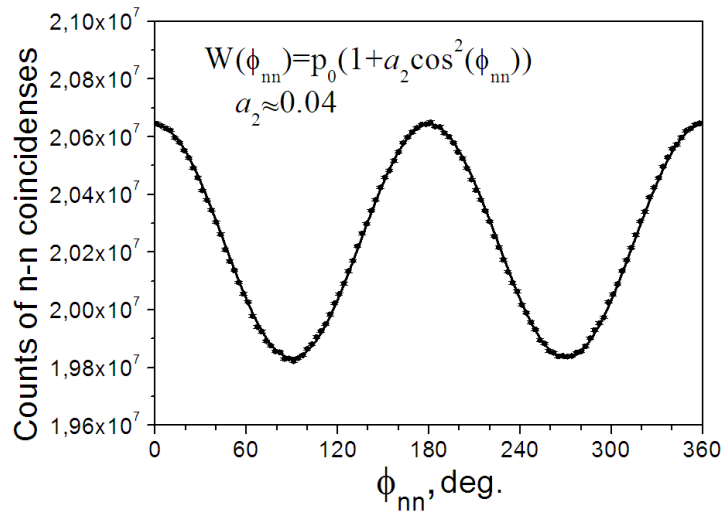
**Fig.5.** The energy dependence of neutron emission anisotropy  $A_{nJ}$  (a) and angular distribution of emitted neutrons (b) for light and heavy fission fragments (in their CM system) relative to fragment spin orientation.

The results of two versions of calculations for neutron-neutron coincidences are presented on the Fig.6. One of these curves was obtained with taking into account anisotropy of neutron evaporation in the fragment center of mass and second – without anisotropy. All other parameters used for calculations were the same. As we can see, the inclusion into analysis of the neutron-neutron correlation of such kind anisotropy slightly changes the shape of the curve. This curve becomes steeper, forcing to use additionally for the experimental data description (1÷2)% isotropic in laboratory system component associated with scission neutrons.

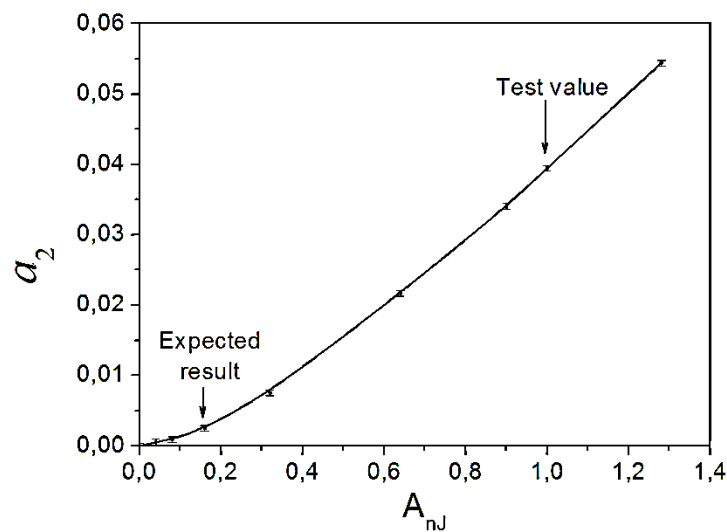


**Fig.6** The result of Monte-Carlo calculations for angular dependence of n-n correlations, which were obtained with and without anisotropy in fragment center of mass.

Ability to calculate the anisotropy of neutron emission from the fission fragments is especially important for the evaluation of angular dependence, which can be observed in the experiment with triple (n-n-f) coincidences. This CORA experiment (CORrelation Angles) was proposed by F. Goennenwein and undertaken to demonstrate the existence of the CMs anisotropy. Here was applied new approach, which is more sensitive namely to this effect [11]. During this experiment the counts of n-n coincidences against the angle  $\phi_{nn}$  between neutrons were measured. However, unlike the n-n correlations, which were discussed above, the angle  $\phi_{nn}$  is not measured between the directions of neutron motion, but between their projections on a plane perpendicular to the fission axis. This allows us to remove the influence on angular dependence of the kinematic component, which associated with a significant velocity of fragments. The form of new correlation can be predicted by the Monte-Carlo calculation:  $W(\phi_{nn}) = p_0(1 + a_2 \cos^2(\phi_{nn}))$ .



**Fig.7** Angular dependence of n-n coincidences in the plane perpendicular to the fission axis.



**Fig.8** Dependence of the CORA parameter  $a_2$  versus the average value of neutron emission anisotropy  $A_{nJ}$  in CM system of fission fragment.

Here parameter  $p_0$  for such distribution depends on the number of fission events while  $a_2$  determines by the value of neutron emission anisotropy in fragment CM system. A test version of the Monte-Carlo simulation (Fig.7) was performed with  $A_{n,f}=1$ . Obtained in this case CORA parameter  $a_2 \approx 0.04$ . But in reality the estimation gives the average value of the anisotropy  $A_{n,f} \approx 2 \cdot A_{nf} \approx 0.16$ . This leads to a smaller  $a_2$  and makes the measurement more complicated [12 - 14].

It is necessary to mention that the observation of another correlation between the motion directions of fission fragments and evaporated neutrons (so-called ROT-effect) would be impossible if there is no the anisotropy of neutron emission [15].

Although the inclusion during the calculation of the anisotropic evaporation of neutrons from the fragment changes the result of the correlation experiments insignificantly, but for adequate description of such experiments this should be taken into account.

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