ON COLD NUCLEAR FUSION

Ignatovich V.K.

JINR, FLNP, Joliot-Curie 6, Dubna, Moscow reg., 141980 Russia

Abstract

The cold nuclear fusion (CNF) can be explained by usually omitted singular part of the wave function in description of the hydrogen atom. This singular part can appear as an admixture to usual state when an atom is confined inside matter, or it can constitute a real bound state of a free atom with continuous spectrum. The probability of CNF because of the singular states is estimated.

1   Introduction

The last publications on real devices for CNF [1, 2], existence of the European patent on some analogous device [3], a lot of recent reviews on experiments and theoretical researches [4–7], repeating of the Rossi's generator in Russia [8] and report [9] about huge isotopic changes in fuel of Rossi's generator deeply impress the readers. The attitude toward CNF as a pathological science seems to be changed, though with some reservation, because all the activity in this area may stem from a greedy desire to get as much money as possible from venture capital or hedge funds.

The common opinion of enthusiasts in this area is that explanation of CNF reactions requires some new physics. However the old physics also can be useful here. It is only necessary to look carefully, what was missed in the old physics. May be we need to use some singular wave functions, which screen nuclear charge at small distances and help to overcome the huge Coulomb barrier in nuclear reactions.

The presented here idea was induced by the paper [10], where a spectrum of a confined atom was calculated. The authors considered an atom confined in a spherical well with unrealistic infinitely high potential walls. A more realistic confinement is presented by a potential step of a finite height U_w at $r = R_w$ as is shown in fig.1. In the interval $0 < r < R_w$ a solution of the Schrödinger equation in the Coulomb field is found. Earlier [11] it was attempted to find a localized singular radial eigen wave function Ψ_s, which is matched to the exponentially decaying wave function of the type $C \exp(-q(r - R_w))/r$ inside the wall at $r > R_w$. The matching at R_w gives an equation for determination of the bound level of the singular state. However calculations involved very complicated hypergeometric functions [12], difficult for interpretations. Here another approach is used. In the Coulomb range $r < R_w$ the common solution of lowest level hydrogen atom is accepted and the singular linear independent part is added to it. A careful investigation of this problem, however, shows that it is possible to find singular bound states even in a free atom without confinement, and the spectrum of such states is continuous. Here we use only nonrelativistic approach, and relativistic corrections considered in [11] are omitted.

In the next section an admixture of a singular function to the lowest state of a confined hydrogen atom is considered. In the third section the singular bound states
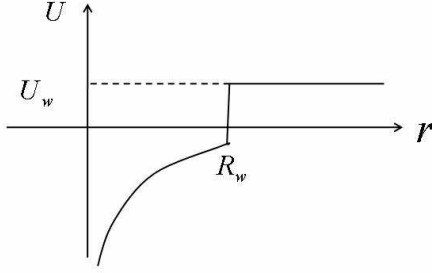
Figure 1: Total potential for the electron in presence of confinement. Radius of the confinement is R_w. The potential U_w can be of any sign.

in free atom and their spectrum are investigated. In fourth section the probability of screening of the Coulomb field at the nuclear radius because of singular wave function is estimated and the the possibility of heat release because of CNF is discussed. In fifth section the usual objection against singular states is refuted and in the last section the summary of the paper is presented. The main goal of this presentation is to persuade scientists to check whether isotopes composition really changes in some experiments.

2 Solution of the Schrödinger equation inside Coulomb potential

The Schrödinger equation for a confined atom looks

$$\left(\frac{\hbar^2}{2m}\Delta + \frac{Ze^2}{r}\Theta(0 < r < R_w) - U_w\Theta(r > R_w) - E \right)\Psi(\boldsymbol{r}) = 0, \tag{1}$$

where $E > 0$, i.e. $-E$ is the bound state energy, and Θ is the step function equal to unity, when inequality in its argument is satisfied, and to zero otherwise. The composite potential function looks as shown in fig. 1.

After substitution of $\Psi(\boldsymbol{r}) = \Psi_L(r)Y_L^M(\theta, \phi)$ Eq. (1) is transformed to

$$\left(\frac{d^2}{dr^2} + \frac{2}{r}\frac{d}{dr} - \frac{L(L+1)}{r^2} + \frac{2m}{\hbar^2}\left[\frac{Ze^2}{r}\Theta(0 < r < R_w) - U_w\Theta(r > R_w) - E \right] \right)\Psi(r) = 0. \tag{2}$$

and after substitution

$$\Psi(r) = u(r)/r \tag{3}$$

Eq. (2) for $L = 0$ and $Z = 1$ is transformed to

$$\left(\frac{d^2}{dr^2} + \frac{2m}{\hbar^2}\left[\frac{e^2}{r}\Theta(0 < r < R_w) - U_w\Theta(r > R_w) - E \right] \right)u(r) = 0. \tag{4}$$

The radial Schrödinger equation for $u(r)$ can be rewritten in dimensionless units: $r = [r]/a_B$, $E = [E]/E_H$, $U = [U]/E_H$, where square brackets denote usual dimensional values, $a_B = \hbar^2/me^2 = 0,525$ Å is the Bohr radius, and $E_H = e^2/a_B = 27.2$ eV is the Hartree energy. In dimensionless units the equation is:

$$\left(\frac{d^2}{dr^2} + \frac{2}{r}\Theta(0 < r < R_w) - 2U_w\Theta(r > R_w) - 2E \right)u(r) = 0. \tag{5}$$

A solution of this equation for $r < R_w$ can be found by Laplace method [13]:

$$u(r) = \oint_C dt\, e^{rt} Z(t), \tag{6}$$

where C is some contour in complex plane of t. Substitution of (6) into (5) gives

$$\oint_C dt [r(t^2 - 2E) + 2] e^{rt} Z(t) = (t^2 - 2E) e^{rt} Z(t)\big|_C + \oint_C e^{rt} dt \left[-\frac{d}{dt}(t^2 - 2E) + 2 \right] Z(t) = 0. \tag{7}$$

If contour C is closed, or the first term on the right hand side is equal to zero, then (7) is equivalent to equation

$$\frac{d}{dt}(t^2 - 2E) Z(t) = 2Z(t). \tag{8}$$

Solution of this equation is

$$\ln[(t^2 - 2E) Z(t)] = 2 \int \frac{dt}{(t^2 - 2E)} + c, \tag{9}$$

or

$$Z(t) = \frac{c}{(t^2 - 2E)} \left(\frac{t-k}{t+k} \right)^{1/k} = c \frac{(t-k)^{1/k-1}}{(t+k)^{1/k+1}}, \tag{10}$$

where $k = \sqrt{2E}$, and c is a constant, which is determined by a normalization. For now it will be put to $c = 1$. Substitution into (6) gives

$$u(r) = \oint_C dt\, e^{rt} \frac{(t-k)^{1/k-1}}{(t+k)^{1/k+1}} = \frac{1}{k} \oint_C dt\, e^{krt} \frac{(t-1)^{1/k-1}}{(t+1)^{1/k+1}}. \tag{11}$$

In Hydrogen atoms usually $2E = 1/n^2$, where n is integer, therefore $1/k = n$, and

$$u(r) = u_n(r) = \oint_C dt\, e^{rt} \frac{(t-k)^{n-1}}{(t+k)^{n+1}} = n \oint_C dt\, e^{rt/n} \frac{(t-1)^{n-1}}{(t+1)^{n+1}}, \tag{12}$$

i.e. the expression under the integral has a simple pole, though, in general, when n is not integer, there are two branch points, and two independent solutions correspond to two different contours around branch points. Here, for now, it is accepted that $n = 1$. Then C is a circle around the pole, and

$$u_n(r) = r e^{-r}. \tag{13}$$

The second independent solution $u_s(r)$ is defined with the help of Wronskian, which satisfies the equation

$$\frac{d}{dr}[u_s' u_n - u_n' u_s] = 0 \rightarrow u_s' u_n - u_n' u_s = c. \tag{14}$$

The constant c again is related to normalization and for convenience is put $c = -1$. Then the equation for u_s becomes

$$u_s' - \frac{u_n'}{u_n} u_s = -\frac{1}{u_n} \equiv -\frac{e^r}{r}. \tag{15}$$

Solution of this equation can be represented in the form

$$u_s(r) = A(r)u_n \rightarrow A' = -\frac{1}{u_n^2} \equiv -\frac{e^{2r}}{r^2}. \tag{16}$$

Integration of this equation by parts 3 times gives

$$A = \frac{1}{r}e^{2r} - \int^r dr' \frac{2}{r'}e^{2r'} = \left(\frac{1}{r} - 2\ln(r) + 4r(\ln r - 1)\right)e^{2r} - 8\int_0^r r'dr'\,(\ln(r') - 1)\,e^{2r'}. \tag{17}$$

The integral at the right hand side is numerically well integrable. With account of (16) the solution of (15) after substitution of (17) becomes

$$u_s(t) = \left(1 - 4r^2 - 2r\ln(r)(1 - 2r)\right)e^r - 8re^{-r}\int_0^r r'dr'\,(\ln(r') - 1)\,e^{2r'}. \tag{18}$$

Let's suppose that the total wave function is $u(r) = A(-\alpha u_s(r) + u_n(r))$, where A is a normalization constant. It can be shown that for some values of the constant α the logarithmic derivative of $u(r)$ in some range of r is negative. Therefore, choice of the confinement radius R_w in this range makes it possible to match u with the function $u_w(\rho) = u(R_w)\exp(-q(r - R_w))$ in the confining wall, where $q = \sqrt{2U_w + 2E}$. In the case
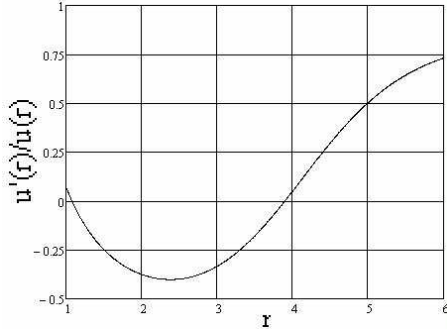

Figure 2: Logarithmic derivative of the function $u_n - \alpha u_s$ for $\alpha = 0.01$.

of $\alpha = 0.01$ the logarithmic derivative is -0.335, for instance, at $r = 3$. If $R_w = 3$ is accepted, then $U_w = -0.44$ of the Hartree energy, which is a little bit higher than the binding energy of hydrogen ground state $E = -0.5$.

3 Singular bound states in free atoms

Above a singular admixture to a usual hydrogen state in a confined atom was considered. However singular states can exist even in free atoms. More over they represent bound states with continuous spectrum. Indeed, let's accept that in (5) $R_w \rightarrow \infty$, then we get a free atom, and the general solution for it can be represented as in (11)

$$u(r) = \frac{1}{k}\oint_C dt e^{rt/\beta}\frac{(t-1)^{\beta-1}}{(t+1)^{\beta+1}}, \tag{19}$$

where $\beta = 1/k$. If $\beta = n$, an integer, it corresponds to usual spectrum of bound states $E = 1/2\beta^2 = 1/2n^2$. However, because of Ridberg corrections [14], β is not integer.
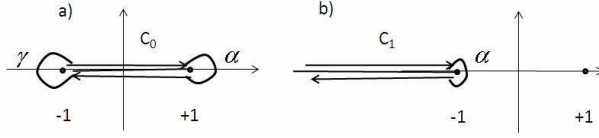
Figure 3: Two different paths for integration in complex plane t in the integral (19) to find two independent solutions of (5) at $R_w \to \infty$. a) For wave function regular at the origin. b) For wave function singular at the origin.

Therefore $Z(t)$ under the integral has two branch points at $t = \pm 1$, and the two different solutions can be found by choosing contours around them as shown in fig.3. Integration in (19) over the closed contour C_0 after integration by parts [15] leads to

$$u_0(r) = \left(1 - e^{2\pi i \beta}\right) \frac{r}{2\beta^2} \int_{-1}^{+1} dt\, e^{rt/\beta} \frac{(t-1)^\beta}{(t+1)^\beta}. \tag{20}$$

If $\beta > 1$, the (19) can be integrated by parts once more to be reduced to a well numerically integrable function. But in all the cases the integration along the path C_0 gives a function which is $\sim r$ at $r = 0$, therefore $u_0(r)/r$ is regular function at the origin. However this function exponentially grows at $r \to \infty$. Indeed, in the interval $0 < t < 1$ the integral at $r \to \infty$ looks like

$$u_0(r) \approx \Gamma(\beta + 1) r \left(\beta/r\right)^{\beta+1} e^{r/\beta}, \tag{21}$$

where irrelevant constant factor was omitted. Since the function u_0, which is $\sim r$ in the origin, diverges at $r \to \infty$, one can expect that the linearly independent one, $u_1(r)$, which is obtained by integration over contour C_1, should be constant at $r = 0$ and exponentially decaying at infinity, to make Wronskian (14) a constant for all r.

To get the integral over C_1 one can consider the closed contour shown in fig.4. Since the integrand is analytical inside it, the integral over it is zero. It contains integrals over two arcs, which are zero for infinite radius. Therefore Integral over the cut C_1 is equal to sum of the integrals over two lines B+A, where they are convergent and well numerically integrable. If, for instance, $\phi = \pi/4$ the sum of the integrals over B+A is

$$u_1(r) = c \int_0^\infty \text{Im}\left[e^{-i\pi/4} \exp\left(-\frac{r}{\beta} s e^{-i\pi/4}\right) \frac{\left(s e^{-i\pi/4} + 1\right)^{\beta-1}}{\left(s e^{-i\pi/4} - 1\right)^{\beta+1}} \right] ds. \tag{22}$$

Since the integrand at large s is $\sim 1/s^2$, the integral converges for any positive r, and at $r = 0$ is a constant. Therefore $u_1(r)/r$ is singular and at the same time it exponentially
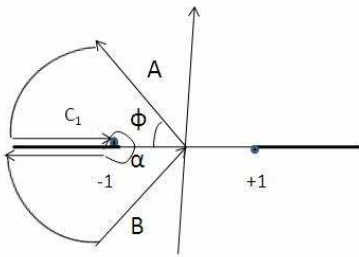

Figure 4: An integration of (19) over the closed paths A,B,C_1 and two arcs of infinite radius is zero, because there is no singularity of the integrand inside this contour. Integration over arcs are zero. Therefore the integral over C_1 is equal to the sum over A and B.

decreases at infinity. So it represents a singular bound state. It is clear that the spectrum of such bound states is continuous, because it exists for all β except $\beta = n$.

4   Contribution of singular states to CNF

With the wave function (3) it is possible to define the probability of screening of the nuclear charge. For the confined function $u_n - \alpha u_s$ with $\alpha = 0.01$ such a probability is

$$Q = \int\limits_0^{R_N} dz |u(r)|^2 / \int\limits_0^\infty dr |u(r)|^2 = 1.8 \times 10^{-8}, \tag{23}$$

if $R_n = 10^{-4}$. In the case of singular state of free atom one obtains $Q = 6.613 \times 10^{-5}$ for $\beta = 0.6$, $Q = 7.401 \times 10^{-6}$ for $\beta = 0.8$, and $Q = 2.248 \times 10^{-6}$ for $\beta = 1.2$. Without the singular part the parameter Q is of the order 10^{-12}, which is many orders of magnitude smaller.

CNF reactions are not observed in everyday practice at room temperature, because of small probability. Usually atoms in solids are hold apart at some distance of order 3 Angstrom, and the screened nucleus cannot approach another one. Thermal vibrations have an amplitude of the order of 0.1 Å. For closer distance between nuclei the vibration amplitude should be an order of magnitude larger, which is possible only at high temperatures above the Debye one. But at such temperatures solids transform to liquids. Only at some special conditions like, possibly, those reached in Rossi generator [8], the nuclear reaction can take place.

There are so many unknown parameters that one can make only rough estimation of possible heat production from CNF. Suppose there are 10^{23} of hydrogen or deuterium atoms per cm^3 that can penetrate to a neighboring site because of self diffusion, and enter neighboring nucleus. The distance l, which atom can pass during time t, is equal to $\sqrt{Dt}$, where D is diffusion coefficient. Let's suppose that $l = a$, which is the distance between atoms. Then the time needed for atom to enter the neighboring site, $l = a$, can be estimated as $t = a^2/D$. Probability of a nuclear reaction can be estimated as $N_0 \sigma a$, where $N_0 \sim 10^{23}$ is atomic density, and σ is cross section, which can be estimated as 10^{-24} cm^2. If probability of approaching atom to be in picohydrogen state is $Q = 10^{-6}$ and the released energy is of the order $w = 1$ MeV, then the total energy obtained from a single atom is $E_1 = N_0 \sigma a Q w \sim 10^{-28}$ J. The total power released in 1 cm^3 by all the hydrogen and deuterium atoms is therefore

$$P = E_1 N_0 / t = E_1 N_0 D / a^2 \sim 10^2 \text{ Watt/cm}^3, \tag{24}$$

if for D the value $D \sim 10^{-9}$ cm/s at temperature 1000 K is substituted.

5   Objections against singular wave function

The singular wave function (3) has asymptotic $1/r$. One of the reason to reject such a solution is that substitution of it in the Schrödinger equation creates a term [16]

$$\Delta \frac{1}{r} = -4\pi \delta(r), \tag{25}$$

which, seems, has no physical meaning.

However there are two objection against such a reason: 1) the right hand side term in (25) is purely mathematical and does not appear in real physical systems; 2) such a term has a physical meaning.

1) In real physical systems the nucleus is not a point like one. If one takes into account form factor of the nucleus, the asymptotics $1/r$ modifies and δ-function does not appear.

2) It is well known that there exist such a phenomenon as neutron-electron interaction [17], which can be described by the Fermi pseudo potential [18]

$$4\pi b_{ne}\delta(\boldsymbol{r}), \tag{26}$$

where b_{ne} is the point like neutron-electron scattering amplitude defined by quark or meson structure of the neutron. The similar amplitude should exist also in proton-electron scattering. Therefore such a term can be included in the Schrödinger equation for atoms. In fact, regular solution of the Schrödinger equation for Hydrogen has a unit amplitude of the electron field incident on the nucleus, therefore there should be the scattered field with asymptotics 1/r.

6   Conclusion

The most important result obtained here is a discovery of bound singular states in free atoms. They can be populated by scattering of free electrons on nuclei with loss of energy. These singular states should be quite a general phenomenon. They can explain CNF and, in particular, how stars start burning, when their density increases because of gravity, and why the hydrogen is burned out the first.

Acknowledgement

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