RESONANCE STATES OF THE ALPHA-ALPHA SYSTEM

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Abstract. In this work we investigate the 0^+, 2^+, 4^+, 6^+, 8^+ and 10^+ resonant states of the α+α system and each advantage of Gaussian and Harmonic Oscillator basis wave functions in the complex scaled orthogonal condition model (CSOCM).

Keywords: Complex scaling method, orthogonal condition model, alpha-alpha system.
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1. INTRODUCTION

The complex scaling method (CSM) [1-4] and the orthogonal condition model (OCM) [5] have been successfully utilized in the description of resonance states in light nuclei. In this work, we apply the CSOCM [4,6] to ^{8}Be and investigate two-body resonances for α+α system. We calculate resonance energies in the complex energy plane applying the CSM to the relative motion between two α-clusters. From the viewpoint of a microscopic description of the relative motion between the α+α clusters it is important to take into account the Pauli exclusion principle in the inter-cluster motion of nucleons. The Gaussian and Harmonic Oscillator wave functions are applied. The calculation procedures of using the Pauli principle are different in these basis functions, however, the same results for two-body system are expected. Our calculated results of the resonance energy and decay width are satisfactorily in agreement with experimental data for the $J^\pi=0^+$, 2^+ and 4^+ states [7]. The purpose of this work is to calculate experimentally unknown 6^+, 8^+ and 10^+ higher excited states of ^{8}Be system.

2. THEORY

2.1. Complex scaling method

In the CSM the relative coordinate is rotated as like $r \longrightarrow re^{i\theta}$ in the complex coordinate plane. Therefore, the Schrödinger equation

$$\hat{H}|\Psi\rangle = E|\Psi\rangle \qquad (1)$$

is rewritten as

$$\hat{H}(\theta)|\Psi^\theta\rangle = E^\theta|\Psi^\theta\rangle, \qquad (2)$$

where $\hat{H}(\theta)$andΨ^θ are the complex scaled Hamiltonian and wave function, respectively. The θ is scaling angle being a real number, $U(\theta)$ operate on a function Ψ^θ, that is

$$\Psi^\theta = U(\theta)\Psi(r) = e^{\frac{3}{2}i\theta}\Psi(re^{i\theta}). \qquad (3)$$

The eigenvalues and eigenstates are obtained by solving the complex scaled Schrödinger equation Eq.(2). The eigenvalues of resonance states are found as $E^{\theta} = E_r - i\Gamma_r/2$, where E_r is resonance energy and Γ_r is the width of resonance. More detailed explanation of the CSM is given in Refs.[1-2].

2.2. Two-body interaction

For the alpha-alpha system the Hamiltonian is expressed as

$$\hat{H} = \sum_{i=1}^{2} \hat{T}_i - \hat{T}_{c.m.} + V_{\alpha\alpha}^{Nucl}(r) + V_{\alpha\alpha}^{Coul}(r). \quad (4)$$

As mentioned at the beginning, in this work we use two different basis sets as follows: (i) A Gaussian basis for the radial part is given as

$$\phi_l^i(r) = N_l^i r^l exp\left(-\frac{1}{2\,b_i^2} r^2\right) Y_{lm}(r). \quad (5)$$

Here $i = 0,1,2,\ldots,$ and N_l^i are thenormalization constants expressed as $N_l^i = \frac{1}{b_i^{l+3/2}} \left\{\frac{2^{l+2}}{(2l+1)!!\sqrt{\pi}}\right\}^{1/2}$ and b_i is the size parameter of Gaussian function described as $b_i = b_0 \gamma^{i-1}$. Where b_0 and γ are the first term and a common ratio in the geometric progression, respectively. (ii) Harmonic oscillator wave function for radial part is

$$\phi_{nl}(r) = N_l^n \left(\frac{r}{b_F}\right)^l L_n^{l+\frac{1}{2}}\left(\left(\frac{r}{b_F}\right)^2\right) exp\left(-\frac{1}{2\,b_F^2} r^2\right) Y_{lm}(r), \quad (6)$$

where $L_n^{l+1/2}$ are Laguerre polynomials for the angular momentum l and N_l^n denotes the normalization constants as given by $N_l^n = \left\{\frac{2\Gamma(n+1)}{b_F^3 \Gamma\left(l+n+\frac{3}{2}\right)}\right\}^{1/2}$. The size parameter of relative motion of two alpha-cluster b_F is taken as 0.967 fm which corresponds to a single particle size parameter $b_0 = 1.3975$ fm employed to fit the observed r.m.s. radius of ^{4}He. In the case (i), we introduce the Pauli-potential $V_{\alpha\alpha}^{P}(r) = \lambda|\chi_F\rangle\langle\chi_F|$, where the strength λ is chosen as 10^7 MeV, which is enough to push up the Pauli-forbidden states into the unphysical energy region.

3. CALCULATION AND RESULTS

3.1. Calculation procedure

In the numerical calculation, we have used two different basis set: (i) Gaussian basis function, and (ii) harmonic oscillator wave function. In Eq.5, the Buck [8] and folding [9] potentials are applied for the Gaussian basis function, but also the folding potential is employed in the harmonic oscillator wave function. According to the Buck-potential, the Pauli-forbidden states need not involve on the alpha-alpha system because of the Pauli principle effect is estimated by an appropriate choice of alpha-alpha potential. However, it is important to take into account of the forbidden states when we use the folding potential of the effective nuclear interaction. From Eq.(2) the eigenvalues are obtained distributions, which on the complex energy plane are shown Figs. 1-2. Also, Figs. 1-2 show the complex energy

eigenvalues of 2^+ and 4^+ states, which are obtained by Buck and folding potentials for Gaussian basis at different θ on the complex energy plane. The resonance energy solution must be stationary for changing the values of θ as explained in Ref. [6].

According to this explanation, we can see that for different θ segregated energy points are observed, but also position of these are almost unchanged by various θ on the complex energy plane (see Figs. 1 and 2). Figs. 1-2 show that there is significant energy point segregation around the location resonance state at the complex scaled plane.

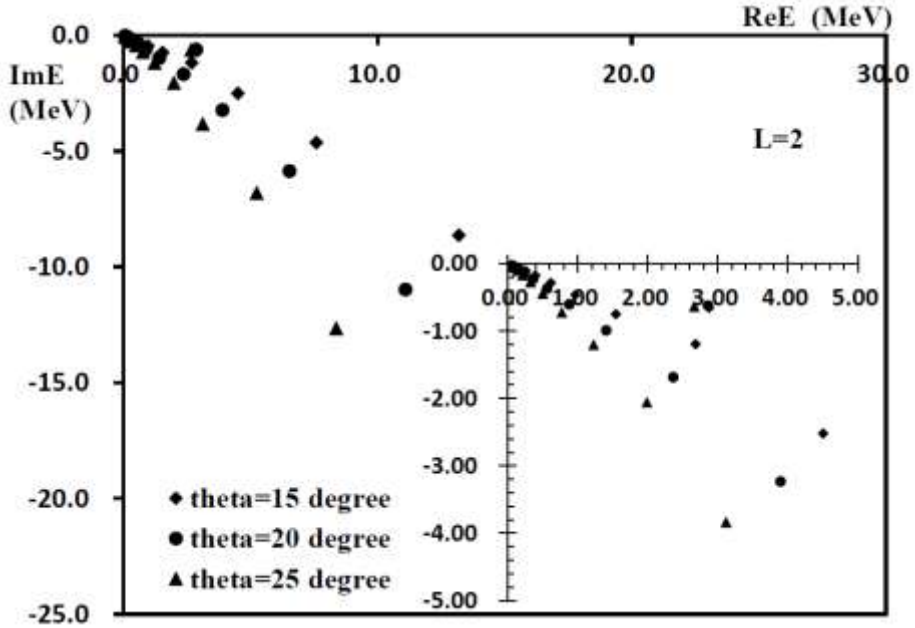

FIGURE 1. The resonance eigenvalues at $J^{\pi}=2^+$ for the different θ. Here Buck-potential is used for Gaussian basis.

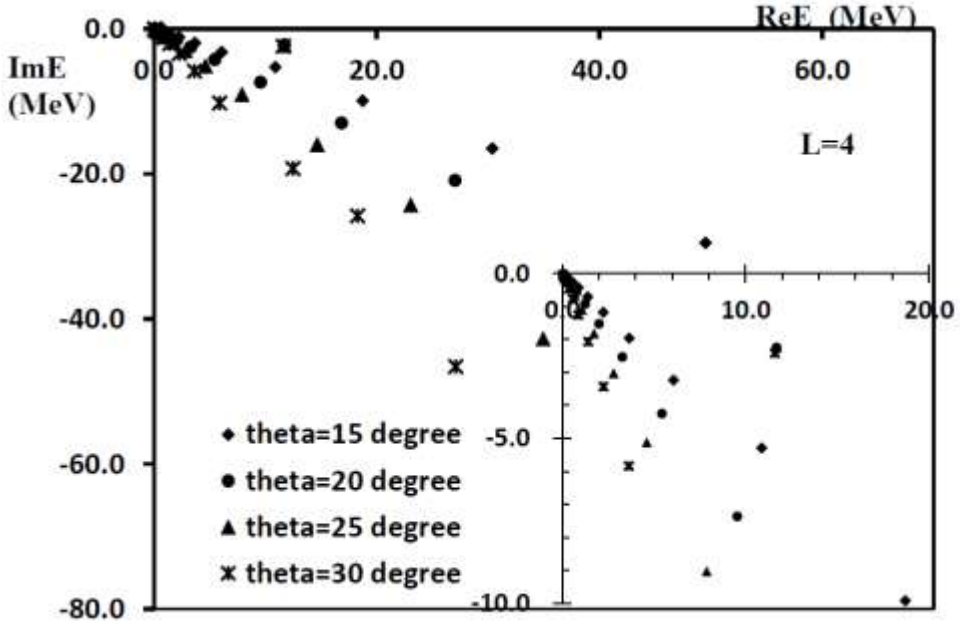

FIGURE 2. The resonance eigenvalues at $J^{\pi}=4^+$ for the different θ. The folding potential is used for Gaussian basis.

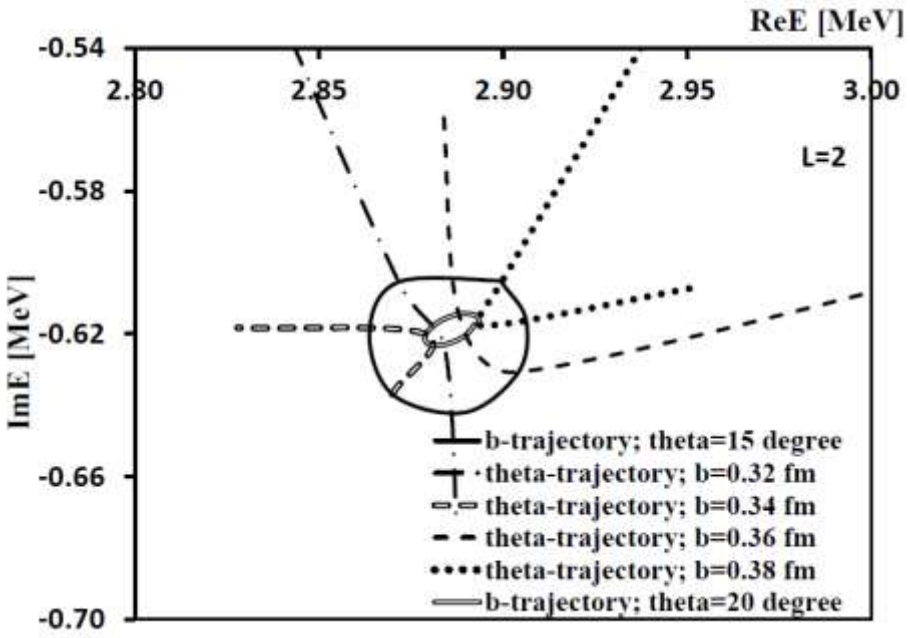

FIGURE 3. The θ and b-trajectory at $J^{\pi}=2^{+}$.The Buck-potential and Gaussian basis are used.

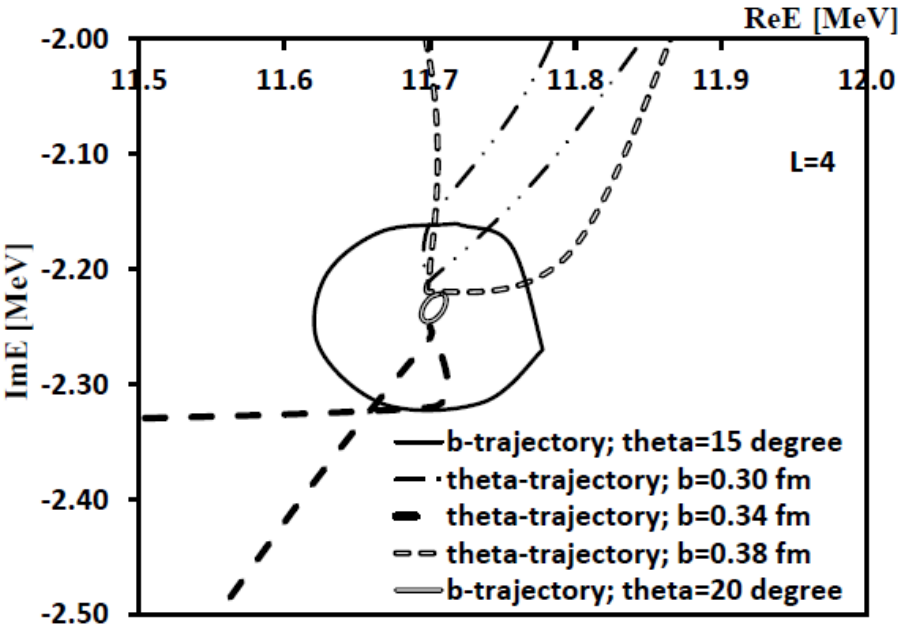

FIGURE 4. The θ and b-trajectory at $J^{\pi}=4^{+}$. The folding potential and Gaussian basis are used.

In addition, Figs. 1 and 2 present the outcome of the Gaussian basis function. Here Buck and folding potential parameters are applied at $\theta = 15^0, 20^0, 25^0$ and at $\theta = 15^0, 20^0, 25^0, 30^0$ on the complex energy plane, respectively. Furthermore, in order to describe accurately the energies and widths for resonance state should be to consider the θ and b-trajectories. In Figs. 3-4 are displayed eigenvalues, which were calculated by θ and b-trajectories.

Therefore, b-trajectories shapes are displayed as circles and θ-trajectories shapes like curves. The b-trajectories are performed two times for each different potential parameters as shown

Fig. 3-4 and the θ-trajectories are calculated 4 and 3 times for Buck and folding potentials, respectively. Moreover, we chose the steps of the b-trajectories by $b = b_0 + 0.1\kappa$. Here $\kappa = 1, 2 \ \ldots, 10$, which were calculated by two methods: changing parameter b and θ is fixed for each b-trajectories, and by the same b for every θ -trajectories, here θ is changing parameter and taken by $\theta = \theta_0 + \kappa,$ where $\kappa = 0, 2, 4 \ \ldots, 20$.

It can be seen from Figs. 3-4, the resonance states are accurately described when the behavior of the θ- and b-trajectories are created well. The calculated result of harmonic oscillator wave function for folding potential is displayed in Fig. 5.

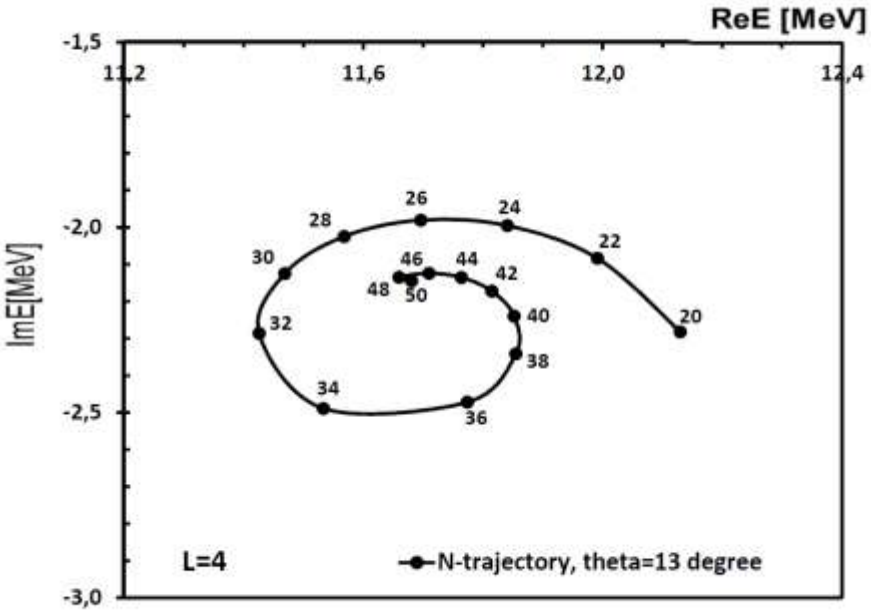

FIGURE 5. The N -trajectory at $J^{\pi}=4^{+}$. The folding potential and harmonic oscillator wave function are used.

Fig. 5 shows the N-trajectories at $\theta = 13$ and $N = N_0 + k,$ where $k = 0, 2, 4, \ldots, 30$. The accurate values of resonances are taken into account by θ- and N-trajectories for harmonic oscillator wave function. The spiral curve represents the N-trajectory followed by the basis states when its size increases. The approach of the energy point is round $N = 46 - 50$.

3.2. Results

The results of the calculated energies with decay widths for 0^{+}, 2^{+}, 4^{+}, 6^{+}, 8^{+} and 10^{+} states of ^{8}Be, experimental data and two different potential parameters are included in Table 1. The experimental data are taken from Ref. [7]. Calculated results are obtained by using the various bases within the Gaussian and harmonic oscillator wave functions. The low-lying calculated states are comparable with observed data [7]. However, there is a slight difference of energy at the 0^{+} state between the calculated result for folding potential [9] and measured data. Furthermore, there is a difference between the calculated decay width of the Gaussian basis and the harmonic oscillator wave functions at the 0^{+} state. In order to clarify a reason of this difference, it may necessary to check the convergence of solutions and increase the number of employed basis functions. In addition, the obtained results by Gaussian basis function and Buck potential are 33.4 MeV (37.2 MeV) and 51.5 MeV (92.4 MeV) energies (widths) at the 6^{+} and 8^{+} states, but as we know no experimental evidence to support this calculated results, however, these results are indicated a good agreement with other resent computed results [10],

which are 34.38 MeV (37.19MeV) and 53.65 MeV (93.74MeV) at the 6^+ and 8^+ states, respectively. Furthermore, the higher excited state 10^+ is computed by two bases functions applying both potential parameters. The calculated results are in good agreement with each other.

TABLE 1. The experimental and calculated resonance energies and widths of ^{8}Be.

	EXP [7]		**Gaussian Basis function**				**Harmonic oscillator wave function**	
	E_r (MeV)	Γ_r (MeV)	E_r (MeV)	Γ_r (MeV)	E_r (MeV)	Γ_r (MeV)	E_r (MeV)	Γ_r (MeV)
0+	9.1×10^{-2}	6.8×10^{-6}	9.13×10^{-2}	$\sim10^{-6}$	6.41×10^{-1}	3.8×10^{-5}	6.06×10^{-1}	3.0×10^{-3}
2+	3.132	1.5	2.75	1.24	3.01	1.2	2.90	1.4
4+	11.49	3.5	11.78	3.56	11.75	4.4	11.7	4.4
6+	-	-	33.4	37.2	30.5	35.7	30.5	36.8
8+	-	-	51.5	92.4	51.6	120	51.6	120
10+	-	-	70.7	160	70.0	180	70.0	180
V_0			122.6225		106.09			
β (fm^{-2})			0.22 [8]		0.2009 [9]			
α (fm^{-1})			0.75		0.5972			

SUMMARY

In this work we have presented different methods to calculate resonance state in the two-body system by CSOCM. Moreover, it can be seen that the different potential parameters able on both Gaussian and harmonic oscillator wave functions. The θ-, b- and N-trajectories are performed in order to determine the resonance states for different method.

The methods well explain all obtained resonance states of ^{8}Be except the width for the 0^+ state calculated by harmonic oscillator wave function. Furthermore, the results of the harmonic oscillator basis functions show that we need to employ a very large size basis functions in order to obtain converged resonance energies and widths.

It is remarkable that 6^+, 8^+ and 10^+ higher excited energies are calculated and broad decay widths are predicted. In connection with the broad decay widths these higher excited states may not able to observe as well defined resonances in experiment.

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