Remarks on the modern status of the neutron charge radius problem

A.B. Popov, L.V. Mitsyna

Frank Laboratory of Neutron Physics (FLNP) of Joint Institute for Nuclear Research (JINR),
Joliot Currie 6, 141980 Dubna, Moscow region, Russia

The modern status of the neutron mean square charge radius $<r_e^2>_n$ problem is discussed. Experimental estimates of the (n,e)-scattering length accumulated over many years and the $<r_e^2>_n$ values extracted from them (under the assumption of non-subtraction of the Foldy term) have statistical dispersion in the interval $-(0.114\pm0.138)\cdot10^{-3}$ fm^2. Existing theoretical models of the nucleon quark structure have difficulties in simultaneous description of their magnetic moments and charge distributions, but provide estimates of $<r_e^2>_n$ that do not contradict to the experimental results. There is still an interest in precision measurements of the (n,e)-scattering length due to a large dispersion of existing experiment results. However, these potential new measurements can hardly clarify the problem under discussion.

1. Problem history

The neutron charge structure and its mean square charge radius (MSCR) are the objects of attention of physicists – both theorists and experimenters – for more than 70 years. In 1941 D. Ivanenko [1] pointed out that the model of nuclear forces must lead to interaction between neutron and electron. In 1947 E. Fermi and L. Marshall [2] fist tried to obtain the length of (n,e)- scattering imagining the neutron in a dissociative state as proton and π^--meson. During last decades the experimental evaluations of the (n,e)-scattering length b_{ne} were obtained by different groups used plenty of methods. The b_{ne} values extracted from experiments, as it is customary to assume up to now, forms two group near

$$b_{ne} = -(1.31\pm0.03)\cdot10^{-3}\ \mathrm{fm}\quad [3,4,5] \tag{1}$$

and

$$b_{ne} = -(1.59\pm0.04)\cdot10^{-3}\ \mathrm{fm}\quad [6]. \tag{2}$$

Comments to the results and their uncertainties can be found in [6,7,8,13].

In 50-es L. Foldy [9] supposed that (n,e)- scattering length has to be divided into two parts, one of which (b_I) is caused by neutron charge distribution and the other part is connected with an interaction between the anomalous magnetic moment of neutron and the Coulomb field. This second part is determined by the expression

$$b_F = \frac{\mu e^2}{2M_n c^2} = -1.468\cdot10^{-3}\,\mathrm{fm}.$$

It was named as the Foldy-term, and physicists started to accept that $b_{\mathrm{ne}} = b_I + b_F$. Correspondingly, the MSCR of neutron was divided into two parts as well

$$<r_e^2> = <r_{in}^2> + <r_\mu^2>.$$

And the mean charge radius was interpreted as

$$<r_{in}^2>=\frac{3\hbar^2}{M_n e^2}(b_{\mathrm{ne}}-b_F)\,.$$

There was an intrigue that under such an assumption the majority of experimental estimates of b_{ne} (1) led to positive values of "intrinsic"$<r_{in}^2>$, and this fact contradicted to the proton-pion model of neutron provoking Yu.A. Alexandrov's criticism [6,10]. In 1995 L. Koester and coauthors [7] abandoned Foldy's approach accepting the quark model of neutron structure proposed by A.W. Thomas [11]. In this model Foldy-term should not be separated, and as J. Byrne [12] revealed, the MSCR of neutron is connected only with the electrical form-factor $G_E^n(q^2)$ (and with b_{ne}) by the expression

$$<r_e^2>_n=-6\frac{dG_E^n(0)}{dq^2}=Cb_{ne}\,.$$

In this determination the $<r_e^2>_n$ value is always negative if $b_{ne}<0$. The authors of [7] prefer this approach and comment that their result

$$<r_e^2>_n=-(0.113\pm0.003)\ \mathrm{fm}^2$$

is in agreement with evaluations of the neutron MSCR from experiments on elastic scattering of electrons by deuterons.

It is not enconceivable that firstly uselessness of dividing b_{ne} into two parts was declared in the paper of G.G. Bunatian and coauthors [13], where description by Dirac equation of interaction of neutron with electromagnetic field was revised. As it was shown in [13], instead of Foldy definition

$$\varepsilon\sim e\frac{e}{6}<r_{in}^2>_n$$

it is necessary to use the expression

$$(\varepsilon+\frac{\hbar\mu}{2M_n c})=\frac{e}{6}<r_e^2>_n,$$

because it is impossible to recognize values in the multiplier at the $div\overline{E}$ term, which describes the interaction of neutron with electric field. So the MSCR is connected with a full coefficient at $div\overline{E}$ and

$$<r_e^2>_n=\frac{3\hbar^2}{M_n e^2}b_{ne}\,.$$

From middle of 90-es the $<r_e^2>_n$ value is defined in just such a way by all authors, except of Yu.A. Alexandrov.

In [13] the problem of (n,e)-scattering was considered within the framework of the cloudy bag model, where it was shown that the neutron MSCR is connected with "total" (n,e)-scattering length.

The review of theoretical investigations of the neutron charge structure was considered twice by T.Yu. Tretyakova [14,15]. Unfortunately, in different theoretical approaches up to now the simultaneous evaluations of $\mu_p, <r_e^2>_p, \mu_n, <r_e^2>_n$ nucleon's parameters, which agree with the experimental data, are not successful. And $<r_e^2>_n$ evaluations are scattered in $\sim-(0.11\div0.26)\ \mathrm{fm}^2$ interval.

In [15] there are results of averaging practically all known experimental b_{ne} values and $<r_e^2>_n$ appropriated to them, which are presented in Table 1. There are no new experimental data on b_{ne} in the recent years.

Table 1. b_{ne} values and the mean square charge radius of neutron

Experiment	Year	$b_{ne}\cdot 10^{-3}$ fm	$<r_n^2>$ fm^2
W. Havens, liquid Bi σ_t [17]	1951	–1.89±0.36	–0.163±0.031
Huhges, mirror Bi/O$_2$ [18]	1953	–1.39±0.13	–0.120±0.011
Melkonian, Bi cryst spectr σ_t [19] Re-estimation, Koester Re-estimation, Kopecky [5]	1959 1976 1997	–1.56±0.05 –1.49±0.05 –1.44±0.03±0.06	–0.135±.004 –0.127±.004 –0.124±0.003±0.008
Krohn, angle distribution on gases Re-estimation, Krohn [3]	1966 1973	–1.34±.03 –1.33±0.03	–0.116±0.003[**] –0.115±0.003[*]
Alexandrov, ^{186}W [20]	1975	–1.60±0.05	–0.138±0.004
Koester, filters - b_{coh} Pb [21] Re-estimation, Nikolenko [22]	1976 1990	–1.364±.025 –1.32±0.03	–0.118±0002[**] –0.114±0.003
Koester, filters - b_{coh} Bi [21] Re-estimation, Nikolenko [22]	1976 1990	–1.393±.025 –1.33±0.03	–0.120±0002[**] –0.115±0.003
Alexandrov, TOF σ_t - b_{coh} Bi [23] Re-estimation, Nikolenko [22]	1986 1990	–1.55±.11 –1.40±0.04	–0.134±.009[*] –0.121±0.004
Koester, filters - b_{coh} Pb, Bi [24]	1986	–1.32±0.04	–0.114±0.003[**]
Kopecky, liquid ^{208}Pb TOF σ_t [25]	1995	–1.31±0.03±0.4	–0.113±0.002±0.003[**]
Koester, $^{208}Pb, Bi$ TOF [7]	1995	–1.32±0.03	–0.114±0.003[*]
Kopecky, liquid ^{208}Pb TOF σ_t [5]	1997	–1.33±0.03±0.03	–0.115±0.003±0.003[*]
Kopecky, liquid Bi TOF σ_t [5]	1997	–1.44±0.03±0.04	–0.124±0.003±0.005[*]
Magli, diffraction on liquid Kr [26]	2006	–1.40±0.10	–0.121±0.009
Waschkowski+FLNP JINR [27]	2006	–1.56±0.18	–0.135±0.016
Estimation of Particle Data Group[16]	2006		–0.1161±0.0022

[*] Data used for estimation of average $<r_n^2>$ by Particle Data Group

[**] Data included in Table of Particle Data Group but not used for estimation of $<r_n^2>$

It is useful to give ones again the results of averaging b_{ne} ($<r_n^2>$) from [15] in Table 2.

Table 2. Averaged values of $<r_e^2>_n$

Number of points	χ^2	$<r_n^2>$ fm^2	Confidence interval
14 points	39.4	–0.1172±0.0012(0.0021[*])	67%
14 points	39.4	–0.1172±0.0023(0.0040[*])	95%
without ^{186}W	9.87	–0.1153±0.0012	67%
without ^{186}W	9.87	–0.1153±0.0024	95%
17 points including ^{208}Pb and Bi from [23]	46.5	–0.1178±0.0022(0.0037[*])	95%
17 points, errors are increased by $\sqrt{\chi^2/(n-1)}$	16.1	–0.1178±0.0037	95%

[*] Errors are corrected by factor $\times\sqrt{\chi^2/(n-1)}$

The values of $<r_e^2>_n$ presented in Table 1 are shown in Fig.1 together with recommend by Particle Data Group (PDG).

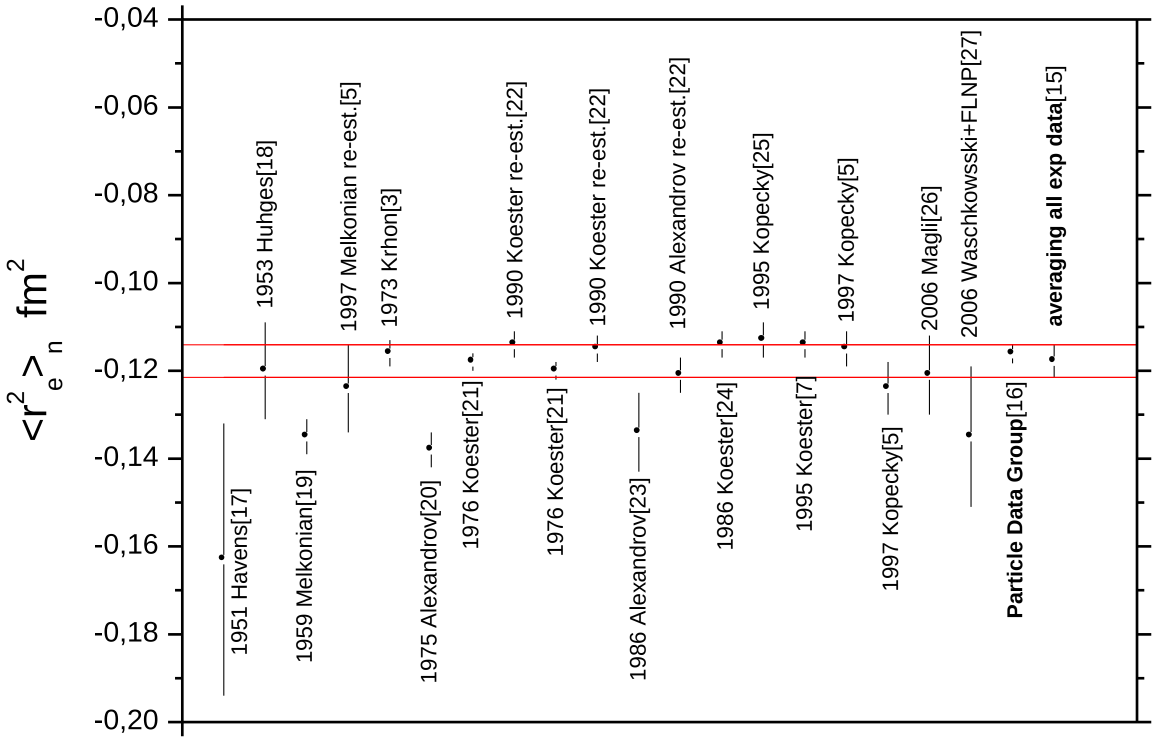
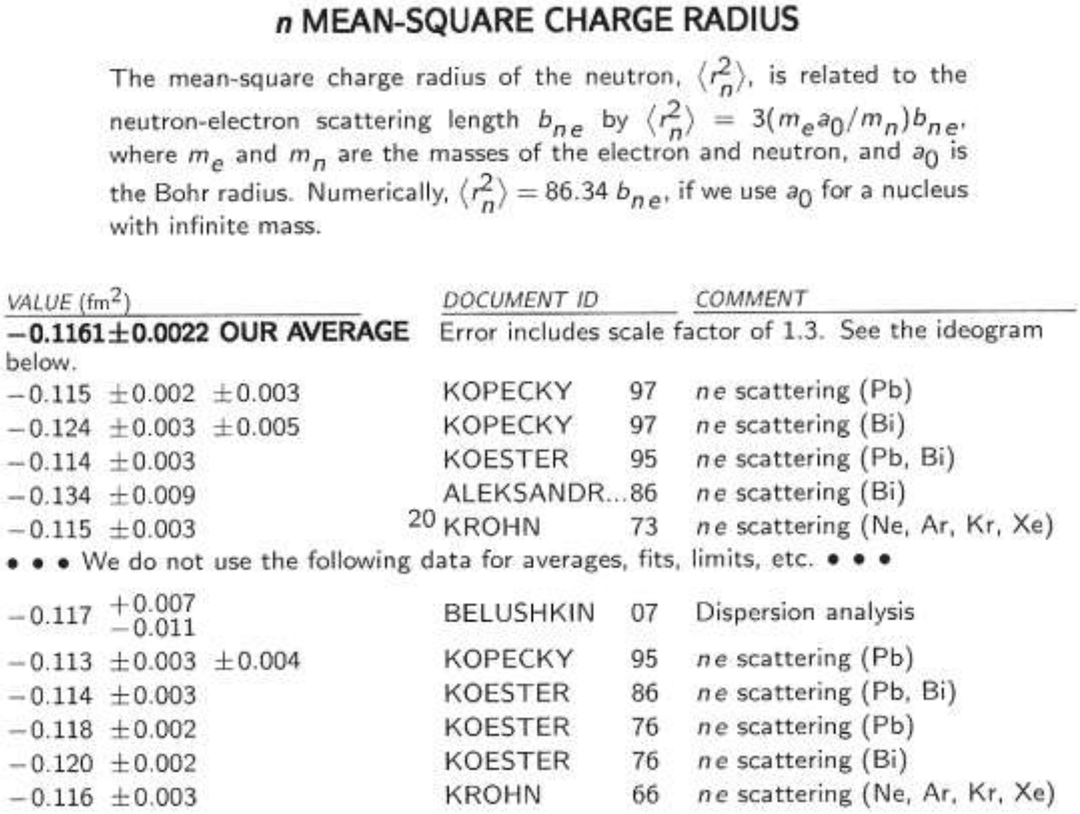


Fig.1. The neutron mean square charge radius $<r_e^2>_n$ from different experiments and its average. Lines show the 95 % confidence interval.

It is possible to draw a conclusion that all available experimental estimates of b_{ne} (or $<r_n^2>$) are not in contradiction with each other under an assumption that the authors underestimate the uncertainties of their experiments by a factor of ~2. This assumption is quite realistic taking into account the presence of essential corrections from different sources in each experiment. Accuracies of all experiments require a serious reassessment. All evaluations of the neutron MSCR agree in uncertainties limits with the value advised by PDG. This value remains valid up to 2014 [16]. In Fig.2 there is a set of experimental data used by PDG for estimation of the averaged $<r_e^2>_n$ value.

Fig.3 demonstrates the details of abovementioned averaging.



n MEAN-SQUARE CHARGE RADIUS

The mean-square charge radius of the neutron, $\langle r_n^2 \rangle$, is related to the neutron-electron scattering length b_{ne} by $\langle r_n^2 \rangle = 3(m_e a_0/m_n)b_{ne}$, where m_e and m_n are the masses of the electron and neutron, and a_0 is the Bohr radius. Numerically, $\langle r_n^2 \rangle = 86.34\ b_{ne}$, if we use a_0 for a nucleus with infinite mass.

VALUE (fm^2)	*DOCUMENT ID*		*COMMENT*
−0.1161±0.0022 OUR AVERAGE	Error includes scale factor of 1.3. See the ideogram below.		
$-0.115 \pm 0.002 \pm 0.003$	KOPECKY	97	*ne* scattering (Pb)
$-0.124 \pm 0.003 \pm 0.005$	KOPECKY	97	*ne* scattering (Bi)
-0.114 ± 0.003	KOESTER	95	*ne* scattering (Pb, Bi)
-0.134 ± 0.009	ALEKSANDR...	86	*ne* scattering (Bi)
-0.115 ± 0.003	[20] KROHN	73	*ne* scattering (Ne, Ar, Kr, Xe)
• • • We do not use the following data for averages, fits, limits, etc. • • •			
$-0.117 {}^{+0.007}_{-0.011}$	BELUSHKIN	07	Dispersion analysis
$-0.113 \pm 0.003 \pm 0.004$	KOPECKY	95	*ne* scattering (Pb)
-0.114 ± 0.003	KOESTER	86	*ne* scattering (Pb, Bi)
-0.118 ± 0.002	KOESTER	76	*ne* scattering (Pb)
-0.120 ± 0.002	KOESTER	76	*ne* scattering (Bi)
-0.116 ± 0.003	KROHN	66	*ne* scattering (Ne, Ar, Kr, Xe)

Fig.2. $<r_e^2>$ value estimated by Particle Data Group [16].

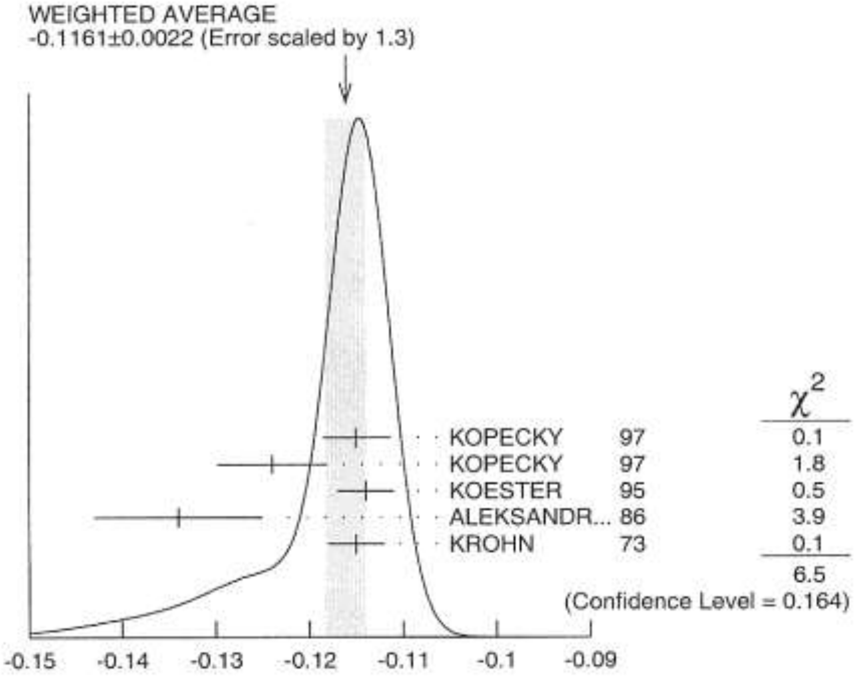


Fig.3. The result of Particle Data Group’s estimation [16].

2. The last year activity

From the above discussion we can conclude that the problem of experimental determination of the (n,e)-scattering length is quite solved: there are no dramatic contradictions in the available set of experimental $b_{\rm ne}$ values. This conclusion is confirmed by Fig.4, where there are all the data on $<r_e^2>_n$ from table 1 with uncertainties, which are multiplied by 1.7 in accordance with results of averaging (table 2). The Gaussian curve corresponds to distribution

$$C\exp[(r-<r>)^2/(2\sigma^2)],$$

where $\sigma = 0.0037\ fm^2$. As it is seen in Fig.4, it is unjustified to separate the set of experimental $<r_e^2>_n$ evaluations into two subgroups.

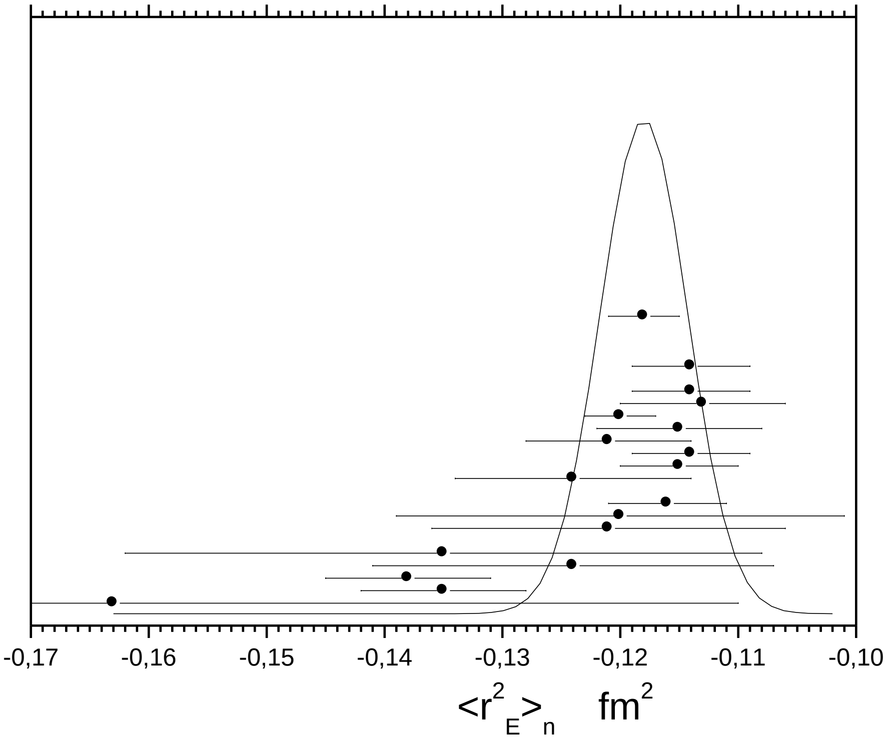


Fig.4. Values of $<r_e^2>_n$ from Table.1 for different experiments with uncertainties multiplied by a factor of 1.7. The curve is Gauss with σ=0.0037 fm^2 dispersion.

Knowledge about the relation between the $<r_e^2>_n$ and $b_{\rm ne}$ values was also defined more precisely. Apparently, it allowed PDG to fix the recomended $<r_e^2>_n$ value in the last decade.

Nevertheless, the fact, that there is a scatter of $b_{\rm ne}$ values from different experiments of more than 3–5 standard errors, stimulates new experiments for its extraction with a high accuracy. So, J.M. Sparenberg and H. Leeb proposed [28] precision measurements of neutron diffraction in a transmission geometry on the perfect monocrystal of silicon. They showed that in measuring a large number reflexes from a thermal reactor spectra it is possible to reach an accuracy of $\Delta b_{\rm ne} \approx 0.06\cdot 10^{-3}$ fm. F.E. Wietfeld and M. Huber with coauthors announced [29] realization of an experiment at neutron interferometer in National Institute of Standard and Technology (NIST). They hoped to improve the accuracy on b_{ne} by a factor of five. Up to now the experiments [28,29] were not completed.

A new method of extracting the (n,e)- scattering length from the neutron diffraction on noble gases was developed by L.V. Mitsyna and colleagues in [30] and verified in [31] by analysis of literature data on structure factors for gaseous Kr and ^{36}Ar and for liquid Kr. In some variants of analysis the real possibility for achieving the accuracy not worse than 2–3% was shown. In order to remove some systematic uncertainties and to reach such good accuracy the execution of comparative diffraction measurements was proposed with a pair gases having close atomic properties and different (n,e)- scattering contributions, such as $^{\mathrm{nat}}$Ar and ^{36}Ar or Kr and Xe. These proposed experiments also are not carried out up to now.

A new experiment on the angular anisotropy of scattering slow neutrons by noble gases using time-of-flight method [13] has been discussed for a long time. Such analogue of Krohn- Ringo experiment allows to extract more surely the (n,e)- scattering contribution from energy dependence of angular anisotropy. Now the experimental setup AURA [32] was prepared and already tested. But the new measurement of b_{ne} was not performed because project intensity of the IREN neutron source was not achieved.

With regard to new theoretical investigations of the charge structure of neutron we should note the paper of T.R. Gentile and C.B. Crawford [33], where an analysis of existing experimental data on the electrical form-factor of neutron $G_E^n(q^2)$ with simultaneous inclusion of the $<r_e^2>_n$ value was done. The authors proposed variants of $G_E^n(q^2)$ parametrization, which allows a good description of the experimental $G_E^n(q^2)$ and $<r_e^2>_n$.

3. Summary

We think that up to now obtained in different laboratories experimental b_{ne} evaluations and theoretical representations about connection between this value and the mean square charge radius of neutron allow to agree with recommend by PDG value of b_{ne}, but with a reservation that its uncertainty must be two times larger. The executed analysis of the full set of experimental data has pointed out an expediency of such uncertainty's increment.

The further interest to the problem of $<r_e^2>_n$ is linked on developing theoretical models about intrinsic structure of nucleons and model's ability to describe well the known nucleon's parameters.

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