ABOUT PARAMETER ESTIMATION FOR AN ARBITRARY NUCLEUS IN THE MODERN PRACTICAL MODEL OF CASCADE GAMMA-DECAY

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The intensities of the two-steps cascades in 43 nuclei of $28 \leq A \leq 200$ mass region were approximated with a high accuracy by the modified variant of the cascade gamma-decay practical model. In this variant a rate of decreasing the model density of vibrational levels is equal for every breaking Cooper pairs. The required values of the radiative strength functions both of $E1$- and $M1$-transitions are obtained using one or two peaks on a smooth model dependence on gamma-transition energy. The main result of analysis is a statement that the Cooper pairs breaking thresholds have higher values for spherical nuclei than for deformed ones. The process parameters are determined now with accuracy, which allows to notice their difference for nuclei with various parity of neutrons and protons.

1. Introduction

Analysis of the experimental data on the intensities of the two-step gamma-decay cascades of neutron resonances [1–4] unambiguously shows, that statistical theory of nucleus depictured, for example, by model of non-interactive Fermi-gas can describe the experimental data with a large uncertainty only. For all explored data the level density ρ and the partial radiative widths Γ [5] are obtained with uncertainties up to 1000% in the region of exciting energies about 0.5 B_n (B_n is the neutron binding energy). However, in practical calculations the uncertainties of spectra intensity and cross sections ΔS don't exceed, in the worst case, a few tens percents, as transfer coefficients of $\Delta\rho$ and $\Delta\Gamma$ uncertainties to ΔS are very small. Nevertheless, it is necessary to determine the ρ and Γ values from experiment more accurately in order to make clear physical picture of the processes in nucleus. For example, a step-like structure in the level density, which was first discovered at model-free approximation of the two-step cascade intensity by different sets of random functions ρ and Γ [1], cannot be described by smooth and monotonous function like that from Fermi-gas model. The other nuclear parameters also cannot be determined correctly and exactly without taking into account a sharp local change of nuclear structure at several excitation energies.

In the up-to-date views on nucleus the discovered effect of a step-like structure in the level density may be a consequence of breaking some Cooper pairs below B_n. Investigation of this effect at an arbitrary nucleus is possible in coincidences experiments only. These experiments are based on recording intensities in coincidences both of cascades of two or more gamma-transitions and of cascades with nucleon products of reaction [6] during decay of high excited nuclear levels with energy of ~5–10 MeV. The partial widths and the level density are extracted from data of these experiments with a systematically high reliability. It ensures to evaluate really, how interaction between normal and superfluid phases of nuclear matter influences on the parameters of nuclear reaction in the wide diapason of nuclear excitation energies.

2. Optimization of phenomenological assumptions in the practical model

The precise determination of ρ and Γ is need also for exact evaluation of nuclear data with help of modern models of level density and radiation widths.

In [7, 8] is shown that intensities of measured two-steps cascades can be described with pinpoint accuracy using simple practical model of the cascade gamma-decay of neutron resonance for wide diapason of masses of stable nuclei-targets. Within the framework of the likelihood method it means that ρ and Γ for any nucleus-product of any nuclear reaction can be extracted now with errors, which are as small as possible.

Exploring the two-step gamma-cascade's intensities for obtaining the most exact model parameters it is absolutely necessary to determine [9] a part of the primary transitions in any energy interval with a precision not worse than 10–20%. The form of measured spectra is determined by convolution of the $\rho(E_{ex})$ and $\Gamma(E_\gamma)$ functions ($E_{\rm ex}$ is excitation energy and E_γ is an energy of any gamma-quanta). Since, the total energy of reaction is not depends on the ρ and Γ parameters, the high accuracy of experimental data during recording coincidences is required.

Researching the superfluity properties of excited nucleus it is need to solve two principal problems:

1) to recognize the most important latent parameters, which determine both the level density and/or the partial widths of emission of reaction products at any energies of intermediate levels of cascades;
2) to determine the most probable values of all parameters.

The existence of the step-like structure in the level density demands that Γ should be dependent on the structure of initial and final levels in nuclear transition. That is the only possibility to explain a smoothness of evaporated nucleon spectrum [10].

The optimal number of parameters for any phenomenological model cannot be determined experimentally. Because of that, the most possible form of likelihood function may be find only by comparing different variants of calculations of high-excited (~5–10 MeV) level decays for all set of experimental data. Now 43 compound nuclei in $28 \le A \le 200$ mass region are available for this purpose.

In the new variant of the practical model it is necessary to take into account three concrete results of previous analysis [7, 8].

1) Firstly, for explored nucleus the same parameters E_μ and E_η are fitted for all iterations in the coefficient C_{coll} of enhancement of vibrational level density [7, 8]:

$$C_{coll} = A_l \exp(\sqrt{(E_{ex} - U_l)/E_\eta} - (E_{ex} - U_l)/E_\mu) + \beta \qquad (1)$$

Here E_μ is a changing rate of the nuclear entropy, and E_η is a changing rate of quasi-particles states energy [11]. This condition is not related to parameters A_l, which determine the absolute values of density of vibrational levels above the breaking point of each l-th Cooper pair. The breaking thresholds U_l are the model parameters for density of quasi-particles exciting [12, 13].

2) The peak's centers of radiative strength functions for E1- and M1-transitions usually correspond to the different excitation energies.
3) In the approximation process it is necessary to fix relation between M1- and E1-transitions in the energy interval, which is some hundred keV above the ground state. It can be done on a base of the known data on partial widths of primary transitions in cascades following the thermal neutrons capture.

3. Some aspects of the likelihood maximum fitting

For a fixing energy of primary transition E_1 the extraction of the ρ and Γ parameters from the intensities $I_{\gamma\gamma}(E_1)$ of two-step cascades between neutron resonance (or other compound-state) λ and some group f of low-lying levels of nucleus through any intermediate levels i is executed using expression:

$$I_{\gamma\gamma}(E_1) = \sum_{\lambda,f} \sum_{i} \frac{\Gamma_{\lambda i}}{\Gamma_{\lambda}} \frac{\Gamma_{if}}{\Gamma_i} = \sum_{\lambda,f} \frac{\Gamma_{\lambda i}}{<\Gamma_{\lambda i}> m_{\lambda i}} n_{\lambda i} \frac{\Gamma_{if}}{<\Gamma_{if}> m_{if}}. \qquad (2)$$

Here $m_{\lambda i}$ is a number of initial excited primary transitions in energy intervals from the energy of primary level λ to the intermediate level i, m_{if} is a number of initial excited secondary γ-transitions in energy intervals from the energy of the intermediate level i to the lower level f, $n_{\lambda i}$ is a number of intermediate levels of cascades in small intervals of the energies of initial transitions.

Practical extraction of ρ and Γ is complicated by necessity to solve a system of nonlinear and strongly correlated equations, which connect ρ and Γ with $I_{\gamma\gamma}$. (At the excitation energy $E_{ex}< E_d$ (E_d is a border of the level "discrete region") the system (2) contains only experimental data on energies and quantum numbers of known to date levels and their decay modes. The middle value of E_d varies from 0.5 to 2 MeV in odd-odd, even-odd and even-even nuclei, correspondingly.) The nonlinearity (2) inevitably leads to false likelihood maxima. And any deviation of the gamma-transition widths from the mean value for a certain excitation energy (because of different structure of wave functions of levels E_λ, E_i and E_f connected by a cascade) is compensated by ρ and Γ deviations in any intervals of $I_{\gamma\gamma}$ even for equal χ^2.

As in previously analysis [7, 8], for solving of the system (2) the Monte Carlo method is used to preset components of random vector of sought corrections for model parameters in each iteration. In the executed calculations the modules of maximal values for all components of this vector usually not exceed 1% of current value for any parameter.

The middle values of components of vector of sought corrections may decrease in each iteration. So, for each of 43 nuclei the mean value of ratio $R^- = \rho(\pi_-)/(\rho(\pi_+)+\rho(\pi_-))$, which is a part of the level density of negative parities, was varied for different variants of iteration processes from 0.1 to 1% of the current value. Initial value R^- for level λ in this analysis equal 50% was taken and for E_d the most probably value was determined. Correspondingly, R^- parameter in this variant of model is linear dependent on excitation energy.

A typical picture of the iteration process is presented in Fig.1 for ^{193}Os, where ratios χ^2/f are shown (f is a number of intervals of $I_{\gamma\gamma}$ averaging). Such approximation [7, 8] is realized in the best way if the cascade intensities have a smooth form and a difference between the experimental and fitting final data is minimal. The most complex case for the considered model is a nucleus ^{28}Al with minimal χ^2/f=7 and the largest fluctuations of the cascade intensities in comparison with the rest investigated nuclei. The lowest level of negative parity in ^{28}Al is observed at E_{ex}=3.4 MeV. By this reason, E1-strength function in this nucleus is determined only for a part of energies of cascade transitions. As it is seen in Fig. 2 the presented variant of the practical model allows obtaining an excellent approximation of gamma-spectra even for light nuclei.

As parameters of models [7, 8] it is necessary to determine the thresholds of all Cooper pairs breaking and the coefficients, which specify form and values of density of vibrational levels (1). Radiative strength functions are determined by the model type of [15] with fitted thermodynamic temperatures and local peaks for possible dipole transitions. At that, the wave function's structure of the excited intermediate levels has a strong influence on the radiative strength functions through the level density. Such influence was firstly investigated in [10].

The optimal width of averaging the intensity $I_{\gamma\gamma}(E_1)$ over an excitation energy of intermediate nucleus is expected for ΔE=100–200 keV, when the some hundreds of thousands events of capturing the total energy of two-steps cascade to final level with excitation energy less than ~0.5–1 MeV are recorded [18]. In the case of $\Delta E \approx 100$ keV a ratio of model parameter number to a number of degrees of freedom f in approximation will be essentially less than number of intervals of averaging $I_{\gamma\gamma}$ spectrum. The system of nonlinear equations (2) sometimes may be degenerated even at maximal f. But even infinite number of solutions of (2) will be located [1] in a bounded interval of parameters of model explored here.

4. Results of using modified practical model

Unlike the previous variant of model [7, 8] the energy interval of initial transitions for ^{184}W and ^{191}Os was enlarged on 0.5 MeV due to including very intensive cascades to the levels of "discrete region". For all 43 nuclei an approximation quality (Figs. 3-5) is not worse than in [7, 8]. For the majority of nuclei in the best approximations of this model variant the breaking threshold of the 4-th Cooper pair (for spherical and near-magic nuclei – of the 3-rd pair) is pointed near B_n.

A main conclusion from the $I_{\gamma\gamma}(E_1)$ data analysis is that the pairing energy Δ determines dynamics of the decay process of any excited state up to B_n (and possibly some higher energy).

The level densities determined in this work (Figs. 6–8) and the level densities obtained from spectra of different multipolarities in the resonances of ^{111,113}Cd measured in Los Alamos [19] suggest that for the interpretation of obtained radiative strength functions (Figs. 9–11) it is necessary to take into account the large variations in their energy dependence.

The existing a structure in cascade spectra and/or very intense peaks of gamma-transition in near-magic nuclei demand using a complex form for description of the strength functions both of E1- and M1-transitions. Such function form is performed as a superposition of distribution as in [15] and relatively narrow peaks. For the total cascade energies more than some MeV the pure higher multipolarities practically are not observed in experiment.

For peak shaping two exponents with varied index are used instead of Laurence-curve. It was done for taking into account a fragmentation of different states of nuclear potential (n quasi-particles and/or m phonons) with a minimal number of fitting parameters. Because of fragmentation these states must have "tails" to the region of high excitation energies of nucleus [20]. Naturally, a number of peaks must be minimal, and their initial positions must differ for electrical and magnet transitions.

Variation of initial fitting parameters (Fig. 1) and selection of approximation variants with minimal χ^2 allow us to manage with 4 peaks of strength functions for nuclei ^{28}Al, ^{64}Cu, ^{71}Ge, ^{177}Lu, and ^{193}Os and with 3 peaks for ^{52}V, ^{60}Co, ^{74}Ge, ^{156}Gd, ^{160}Tb, ^{166}Ho, ^{176}Lu, ^{184}W, ^{191}Os, and ^{200}Hg. For the best fitting the data on remaining 28 nuclei one peak of the strength functions both of electrical and of magnet transitions is enough. The fitting results are presented in Figs. 6–11. According to [7, 8] the full radiative strength function for both multipolarities is determined from the data of Figs. 9–11 by multiplication of its best value in each point of excitation energy by a coefficient ρ_{FG}/ρ_{ex} (ρ_{FG} is the model density, ρ_{ex} is the best fit [14]).

From these data (Figs. 9–11) we can draw a conclusion that a sensitivity of the modern practical model of gamma-decay allows to consider systematically and evaluate the parameters of local changing the strength functions of $E1$- and $M1$-transitions at any excitation energy including transitions between high-lying levels. For $M1$-transitions it was done for the first time.

It is necessary to note that for many nuclei the exact approximation of $I_{\gamma\gamma}$ by smooth functions for $E_1 < 1\div2$ MeV is impossible without taking into account a strong increasing the strength functions of low-energy primary gamma-transitions. For these energies the errors of the experimental $I_{\gamma\gamma}$ spectra, which is divided by parts of primary and secondary transitions [9], are minimal (especially at good statistics and low background [18]). So it is impossible to explain large fluctuations of the strength functions by experimental errors only.

5. Superfluidity of an excited nucleus and its gamma-decay process

As it follows from the results of the model [7, 8], the main nuclear parameters are number of breaking Cooper pairs and thresholds of their breaking U_l. The same conclusion was done using different techniques [1–4, 10] realized earlier.

In all variants of analysis the best approximation of gamma-cascade spectra is reached taking into account breaking of 3–4 pairs, when the breaking threshold U_l of the last pair is placed near B_n. Correspondingly, a neutron capture by nucleus leads to exciting some number of phonons and up to 6–7 quasi-particles at $E_{\rm ex} \le B_n$.

In Fig. 12 the relations U_l/Δ_0 (l=2,3) and B_n/Δ_0 are presented, where $\Delta_0 = 12.8\sqrt{A}$ is a pair energy of the last nucleon in nucleus with mass A. The result of approximation of cascade's intensities shows that model parameters are noticeable different for even-even, even-odd and odd-odd nuclei. Therefore, a comparison of U_l and B_n was done for them separately (odd-even ^{177}Lu was include to the set of even-odd nuclei).

The result of the comparison confirms that in the majority of spherical nuclei (and also transitional as ^{191,193}Os) near B_n there is the breaking threshold for the 3-rd pair, and in deformed nuclei it take place for 4-th Cooper pair only. The main result of analysis performed is a statement that the breaking thresholds of Cooper pairs for spherical nucleus lie higher in comparison with thresholds for deformed nucleus. Of course, this conclusion is valid only within the limits of an accuracy of practical model parameters and phenomenological representations about entropy of exciting nucleus. There are two possible reasons for appearance of result errors (Fig. 12) in this model:

1) a reduction of a precision of the threshold's determination (due to changing the energy dependence of the level density at the next breaking pair) for the 3-rd and 4-th Cooper breaking pairs in the model [12] or
2) an impossibility to determine a ratio of probabilities of breaking neutron or proton Cooper pairs at a short distance of nuclear excitation energies.

6. Interpolation of the model parameters an arbitrary nucleus

The parameters founded in the model are scattered for different nuclei. Mainly, it is happened because of fluctuations of nucleon pairing energy [21] and because of ρ and Γ dependency on details of the wave functions structure of cascade levels. Inevitable existence in likelihood function of false χ^2-minima, which weakly differ from a principal one, influences on this scatter also.

The parameters of the modified practical model for an arbitrary nucleus may be obtained from the data of any neighboring nuclei (Figs. 3–11) with the same nucleon parity by linear interpolation for

- breaking thresholds;
- ratios $R^{-}=\rho(\pi_{-})/(\rho(\pi_{+})+\rho(\pi_{-}))$;
- coefficients A_l of enhancement of the density of vibrational levels higher than breaking thresholds for 2-nd and 3-rd Cooper pairs;
- absolute values of sums of radiative strength functions for the energy of the cascade primary transitions with E_1~1 MeV or slightly less;
- excitation energies, for which an experimental strength function is equal to one from model [15].

The required parameter of spin cut-off factor for describing the level density is taken from model [14].

In Fig. 13 the best values of the E_μ and E_η parameters and of the breaking Cooper pairs for all nuclei are presented and compared with the mean pairing energy Δ_0. Maximal deviations of E_μ from expected $\Delta_0 = 12.8\sqrt{A}$ are observed for ^{60}Co, ^{150}Sm, ^{165}Dy, ^{168}Er, ^{174}Yb, ^{193}Os and ^{198}Au. These deviations may be a result of breaking threshold's proximity for neutron and proton pairs, which influences to an amplitude of vibrational levels through the C_{coll} coefficient (1).

In Fig. 14 there is the best ratio of density of negative parity levels R^{-} to the total level density in a changeover point to concrete nuclear levels determined by spectroscopic methods. The average value of $\langle R^{-}\rangle$ = 0.43(33) shows that there is no dominance of level density of any parity. But almost a half of investigated nuclei have $R^{-}<0.15$ or $R^{-}>0.85$.

The noticeably worse situation is for parameters A_l, which determine a density of vibrational levels. Models of the vibrational level density like [12] for quasi-particles are absent. Furthermore, A_l parameters and the thresholds U_l anticorrelate. So, searching the factors, which define A_l values, it is need to take into account this connection.

In Fig. 15 a parametric dependence

$$V_l = U_l - 2\Delta_0(l-l_0) - \ln(A_l) \qquad (3)$$

is shown. For l=2 (the second breaking pair) l_0=2 for even-even and even-odd nuclei and l_0=1 for odd-odd nuclei. For l=3 (the next pair) l_0=4 for even-even nuclei, l_0=3 for even-odd and l_0=2 for odd-odd nuclei. The value of V_l is small enough, that allows to expect a fairly simple relation between densities of vibrational and quasi-particle types.

Errors of phenomenological presentations of the strength functions of dipole transitions and errors of mixed model-phenomenological descriptions of excited level's density [7, 8] are partly reciprocally compensated. The absence of data with noticeable deviations from the mean spacing D_0 between *s*- and *p*-resonances [13] allows for all explored nuclei using a hypothesis that densities of levels of different parities are the same near B_n.

For exact practical calculations it is advisable to approximate the sum $k(E1)+k(M1)$ by superposition of two different functions: higher and lower than a value of energy of primary gamma-transitions $E_1=L_k$ (L_k is a cross point of model functional dependences and approximated ones). For $E_1<L_k$ logarithm of sum $k(E1)+k(M1)$ descends linearly with regard to extrapolation [15]. As it is seen in Fig. 16, at E_1=1 MeV the strength functions are 2–6 times smaller relative to the data of existing models [15–17].

The data of Fig. 16 shows also that distortion of the strength functions extracted in experiment and ones calculated by models is maximal for even-even nuclei, and it is minimal for even-odd nuclei.

7. Possible experiments for a study of superfluidity

Experiments on recording the cascades of two gamma-transitions of radiation capture of thermal neutrons were carried out in Dubna (Russia), Riga (Latvia), Rez (Czech Republic) and Dalate (Vietnam). Unfortunately, gamma-quanta cascades at thermal neutron capture allow to determine ρ and Γ parameters only in a fixed area of nuclear excitations, for a fixed spin interval and for one parity of decayed resonance (or two spins for nuclei with a small spacing D_λ between resonances).

Up to now, in analysis a nucleus is usually imagined as a statistical system. Real uncertainty of this nuclear model is unknown, so new experiments (as [8]) are needed. An experiment can be fulfilled not only at sources of thermal and resonance neutrons, but at any accelerators of charged particles, if a scatter of energies of excited levels in a target and an energy resolution of HPGe-detectors are comparable.

The best possibilities for a study of the cascades of gamma-transitions of decaying levels excited by gamma-quanta can be realized at any source of gamma-radiation (type of ELBI [22] or S-Dalinac [23]) with fixed energy. At fixed energy $E_{\max}$ of the gamma beam it is possible to apply the model [8] in interval of excitation energies of level λ from $E_{\max}$ to $E_{\max}$ – 511 keV. This allows to exclude an out-of-date representation of cascade decay by a statistical process.

A background conditions during cascade recording for a beam of gamma-quanta are essentially better than for a neutron beam. For experiments of type [22] or [23] a singular requirement is that detectors must be placed in a back hemisphere relatively to a target and close to it. At that, radiation transfer between HPGe-detectors must be significantly reduced. It is also possible in this experiment to determine separately radiation strength functions for gamma-transitions both to the ground state of a nucleus-target and to its excited levels. Information content of such experiment will exceed, at the least, ten times the results of (n,2γ) reaction investigation.

Unlike the cascades of gamma-transitions, the cascades with nucleons emission provide a significant statistics increment due to high efficiency of recording charged products of reaction. Mathematically a spectrum of primary gamma-transitions of decaying levels below the emission threshold for nucleon products of the reaction and a spectrum of evaporated nucleons (light nuclei) above the binding energy are identical. So analysis of cascade “evaporated nucleon & gamma-quantum” is similar to analysis of cascade of gamma-transitions. Intensity of “nucleon product & gamma-quantum” cascade to low-lying level can be strongly dependent on orbital moment of evaporated nucleon. At that, components of the wave functions of levels excited during nucleon emission are determined by fragmentation of different *n*-quasi-particle or *m*-phonon states [20].

Recording the two-step cascades of gamma-transitions in accelerator beam at a small target thickness (not more than 10 – 20 keV) gives a possibility for unambiguous determination of emission order of quanta in the two-step cascade. A peak width of recorded primary transition is a convolution of Ge-detector resolution and of target thickness, and a peak width of secondary gamma-transition is determined only by intrinsic resolution of spectrometer.

8. Conclusion

To describe the measured parameters of cascade gamma-decay of neutron resonances with a high accuracy the theoretical models for the level density and for the radiative strength functions are needed. In these models a dynamics of exciting quasi-particles and phonons interactions must be taken into account starting with their minimal number. For a practical application of such models the parameters of breaking some Cooper pairs of nucleons (including maybe neutron-proton pairs) can be used as a basis.

There is no base for a doubt that such representation is also suited for calculations of spectra of any nuclear reactions with nucleon products emission. The data taken such a way can give fundamental information about superfluidity of nuclear matter, at least, below the energy of giant dipole resonance.

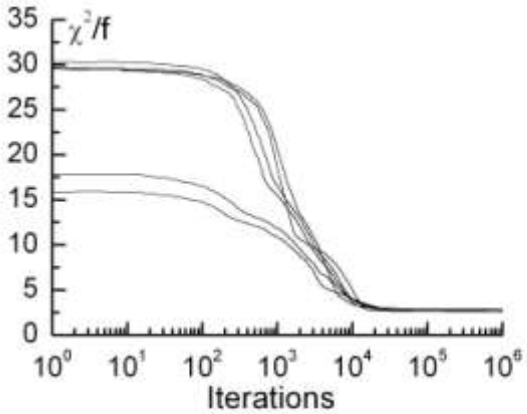

Fig.1. χ^2/f dependence on iteration number for 6 variants for different initial parameters (f is number of averaging intervals of cascade intensity).

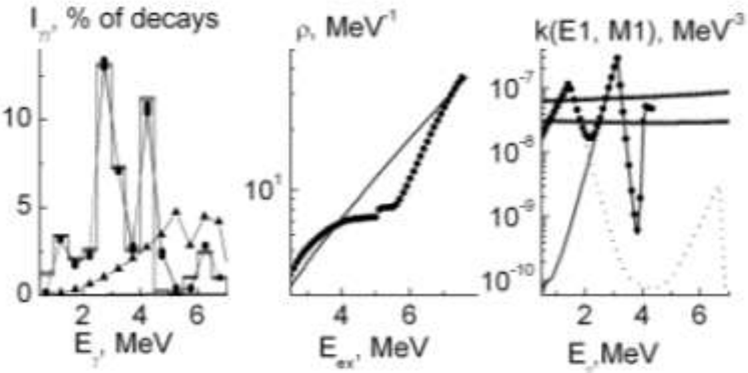

Fig.2. In the left picture there are the experimental intensities (histograms with experimental errors) and the best approximation results (points). Triangles are calculation on the base of statistical model. In the middle picture there are the most probable model level density (points with a random scatter of values from different fittings) and its expected value according to model [14] (solid line). In the right picture there are the most probable sums of the model radiative strength functions of $E1$- and $M1$-transitions (points with errors), strength functions $k(E1)$ (solid line), $k(M1)$ (dotted line) and calculations by models [15–17] (triangles).

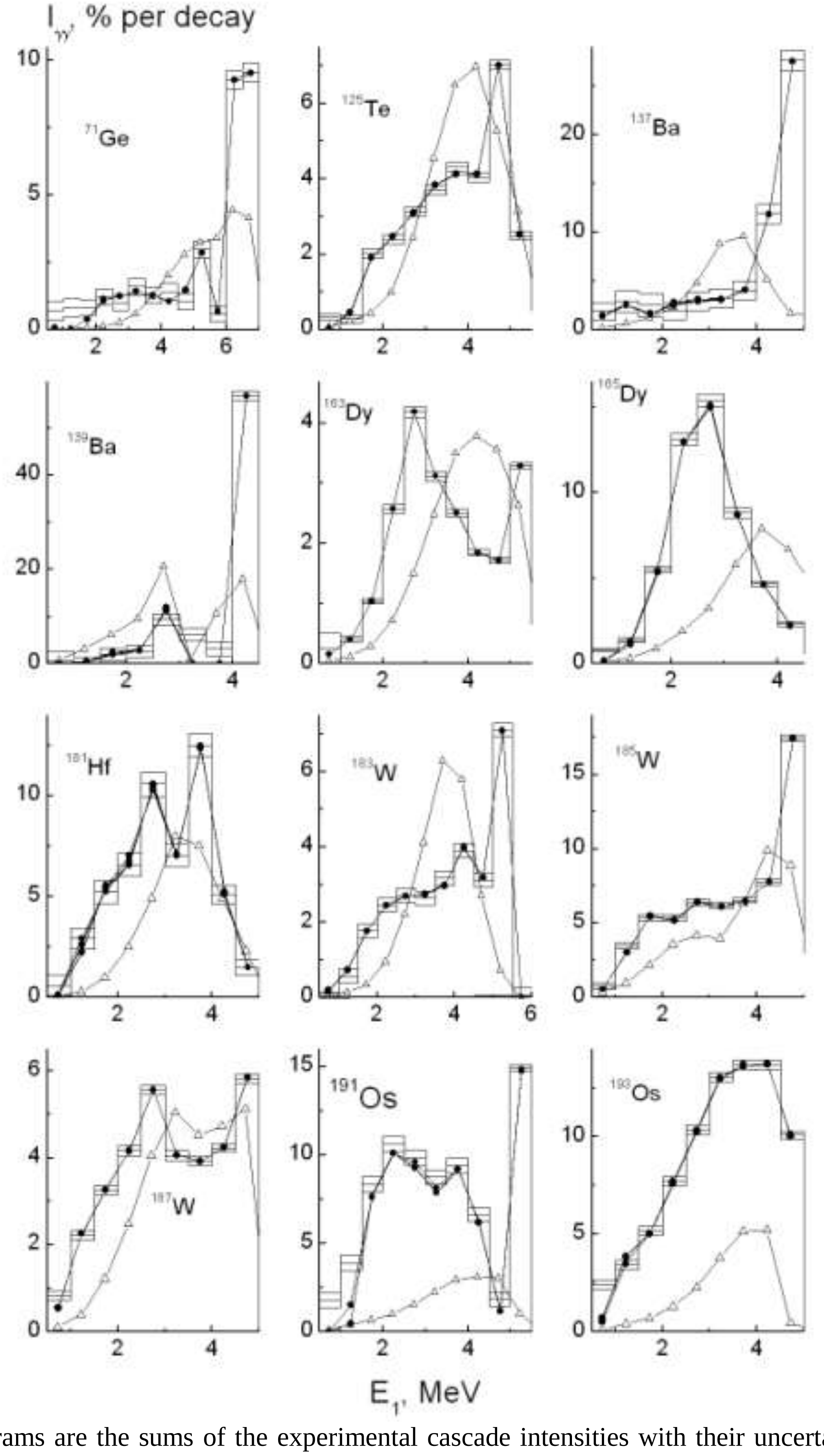

Fig.3. Histograms are the sums of the experimental cascade intensities with their uncertainties in 0.5 MeV bins for even-odd nuclei. Full points are the best fit for 6 different variants of approximation, triangles are the calculated spectra for models [14, 15] with $k(M1)$=const.

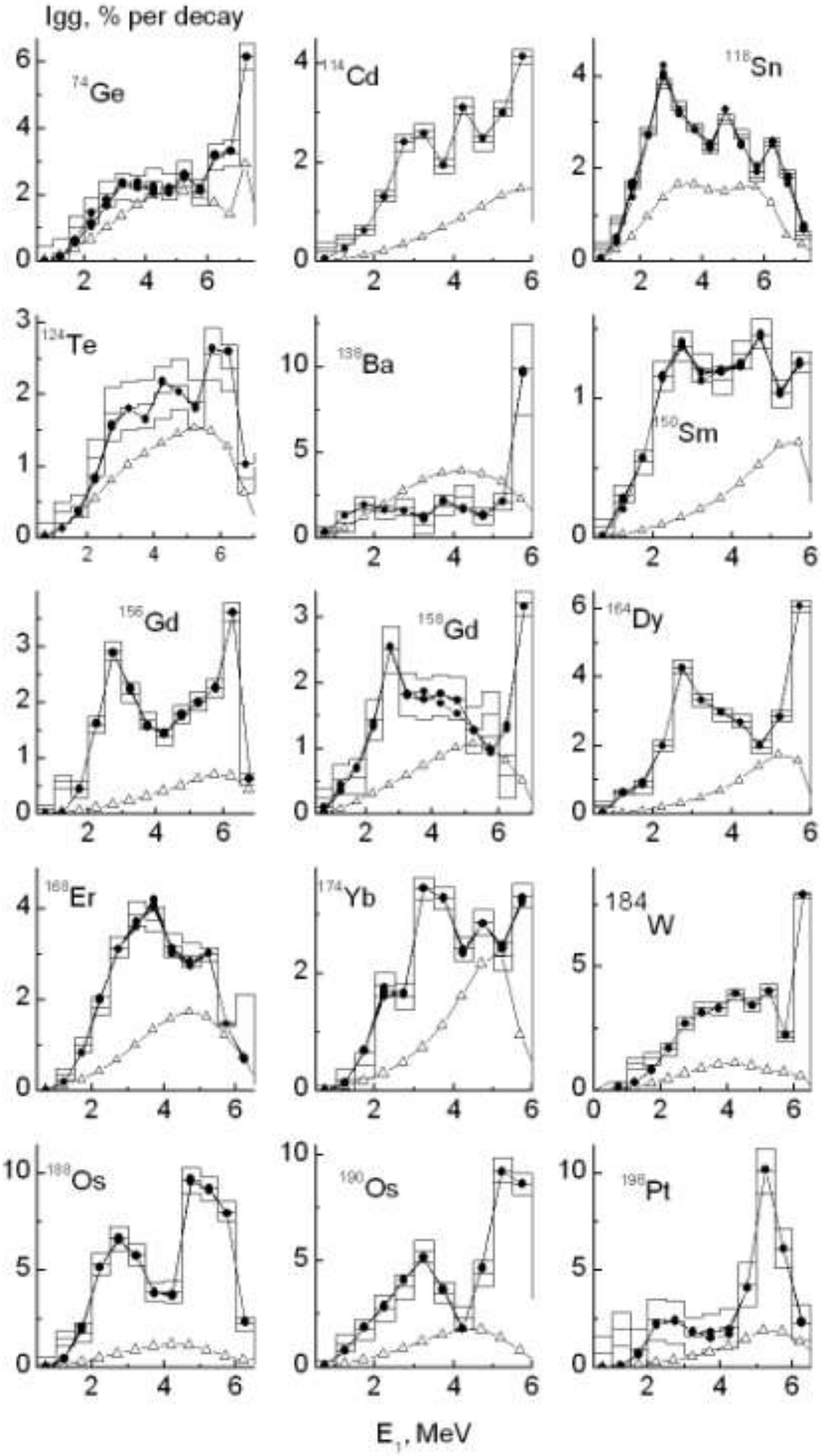

Fig.4. Histograms are the sums of the experimental cascade intensities with their uncertainties in 0.5 MeV bins for even-even nuclei. Full points are the best fit for 6 different variants of approximation, triangles are the calculated spectra for models [14, 15] with $k(M1)$=const.

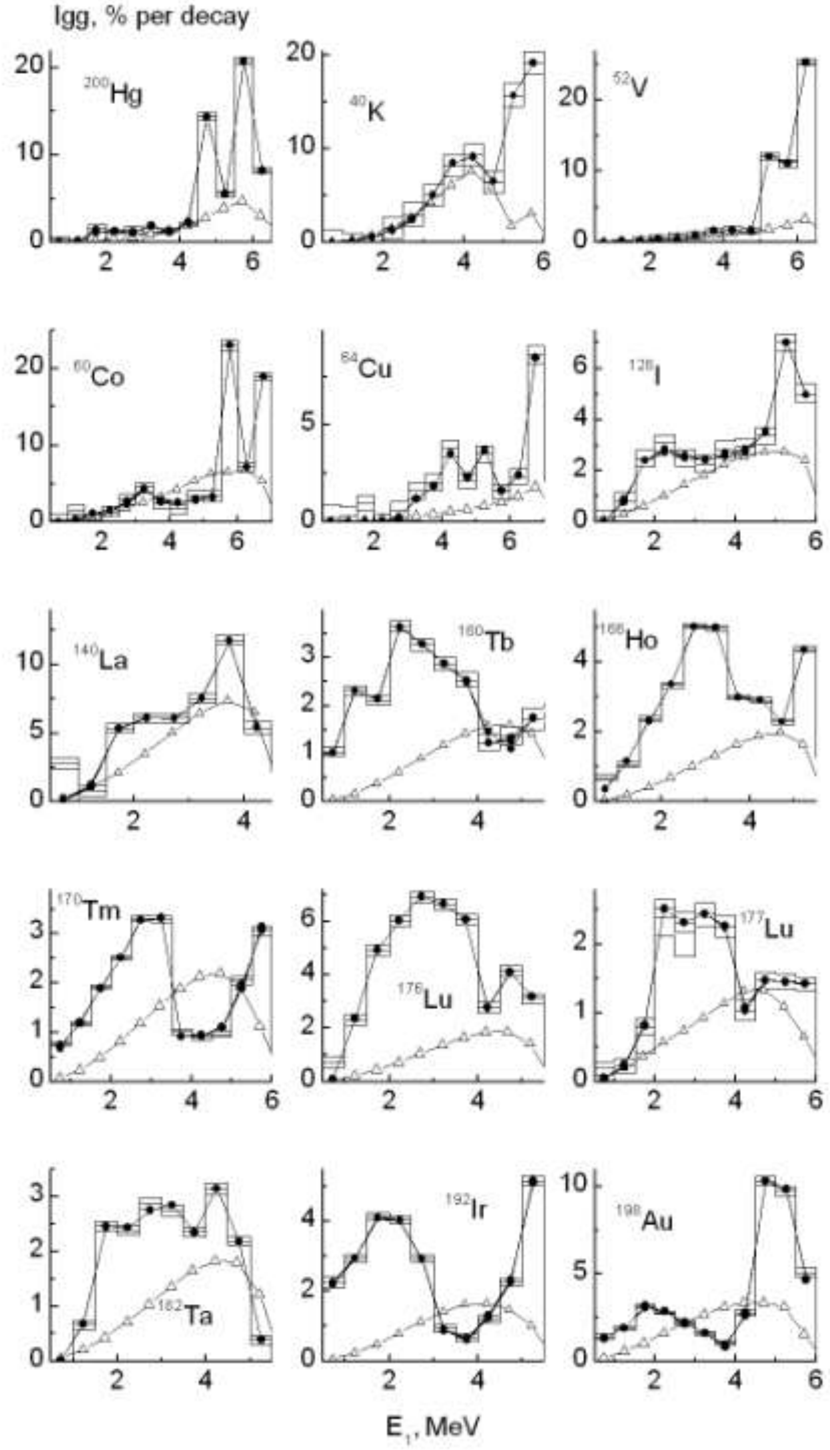

Fig.5. Histograms are the sums of the experimental cascade intensities with their uncertainties in 0.5 MeV bins for odd-odd nuclei (and for odd-even ^{177}Lu). Full points are the best fit for 6 different variants of approximation, triangles are the calculated spectra for models [14, 15] with $k(M1)$=const.

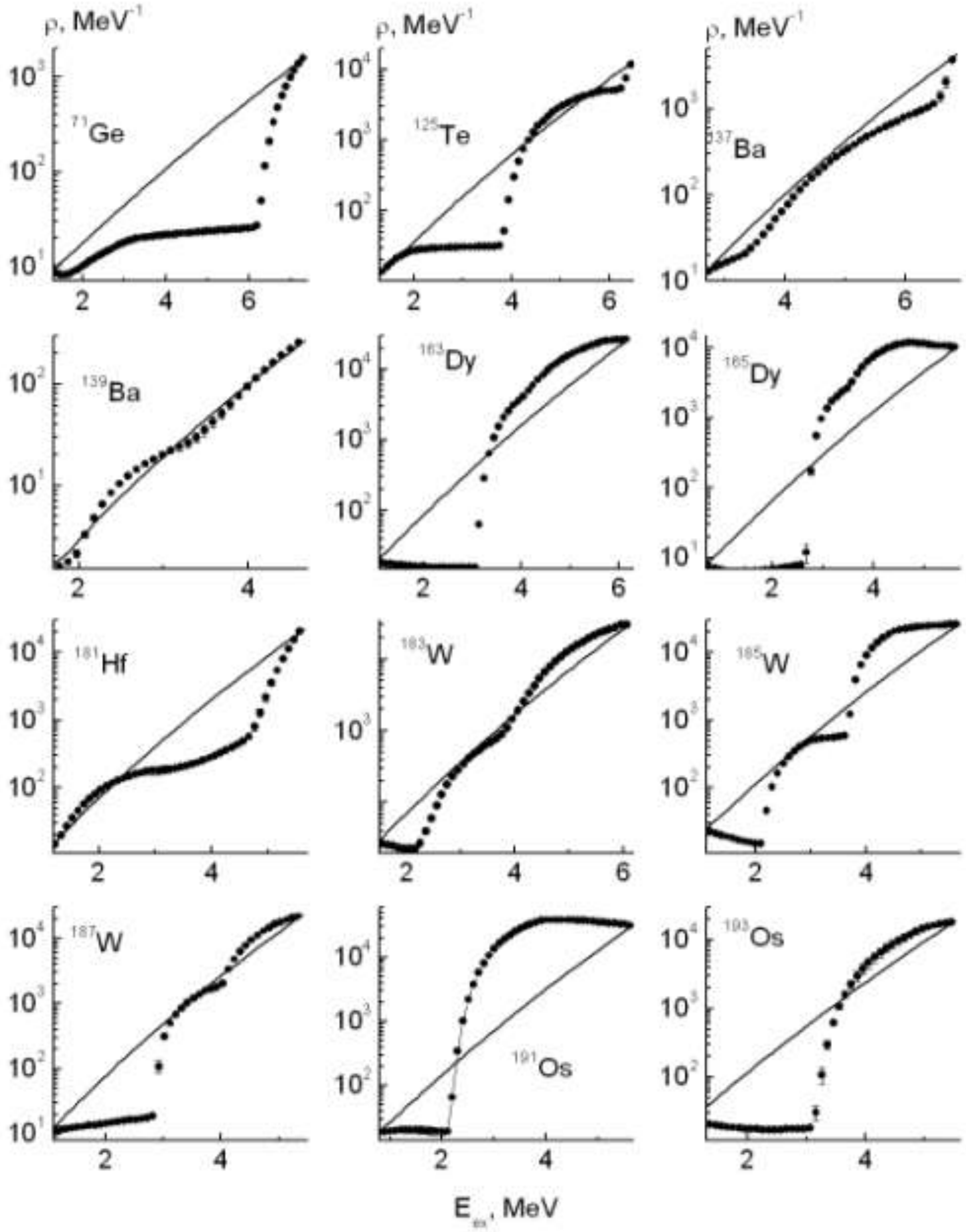

Fig.6. The most probable average density of intermediate levels of the two-step cascades in even-odd nuclei (full points with errors) and their fluctuations in some approximation variants with lowest χ^2. Solid line is the model [14] calculation.

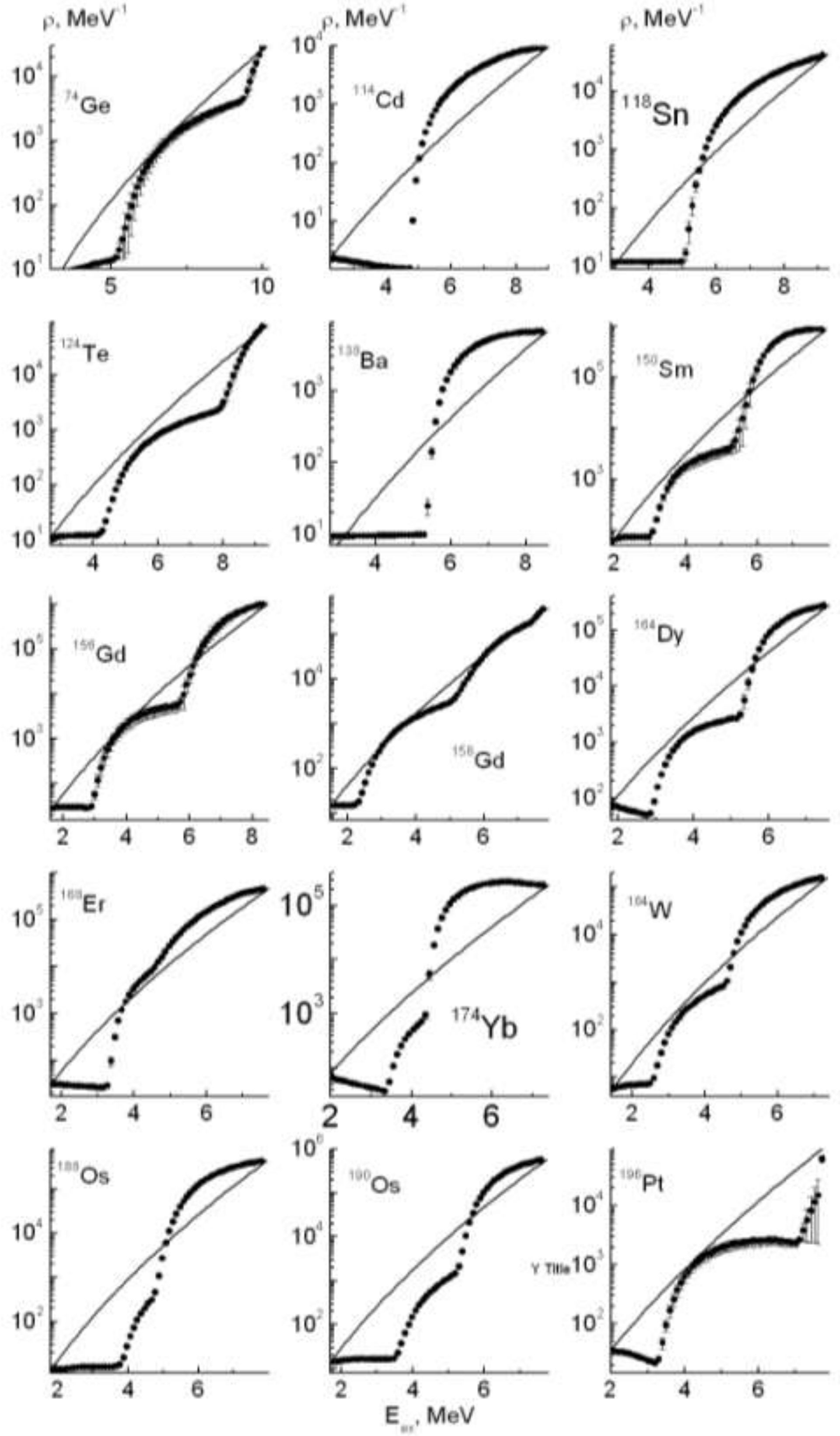

Fig.7. The most probable average density of intermediate levels of the two-step cascades in even-even nuclei (full points with errors) and their fluctuations in some approximation variants with lowest χ^2. Solid line is the model [14] calculation.

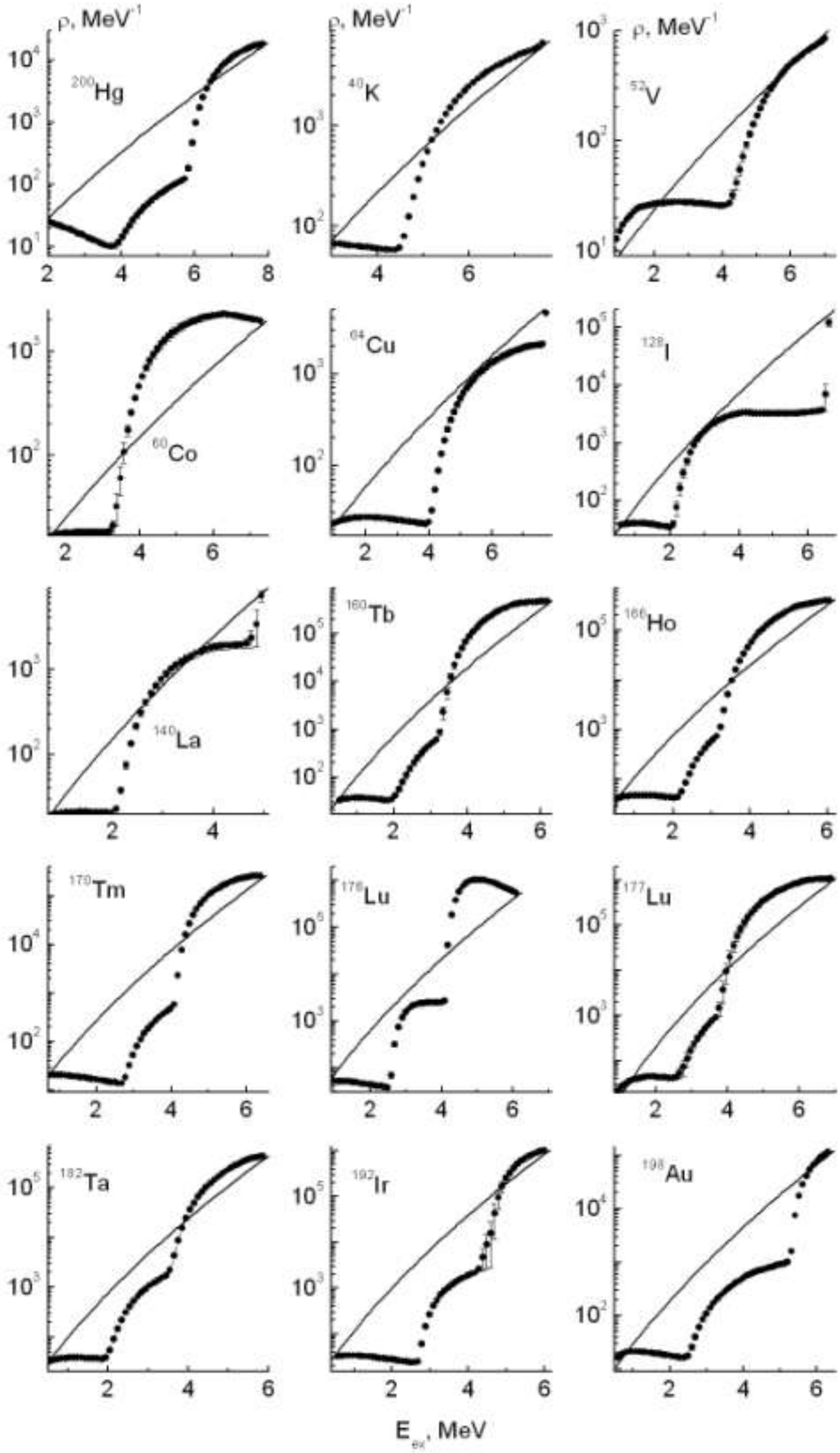

Fig.8. The most probable average density of intermediate levels of the two-step cascades in ^{177}Lu, ^{200}Hg and odd-odd nuclei and(full points with errors) and their fluctuations in some approximation variants with lowest χ^2. Solid line is the model [14] calculation.

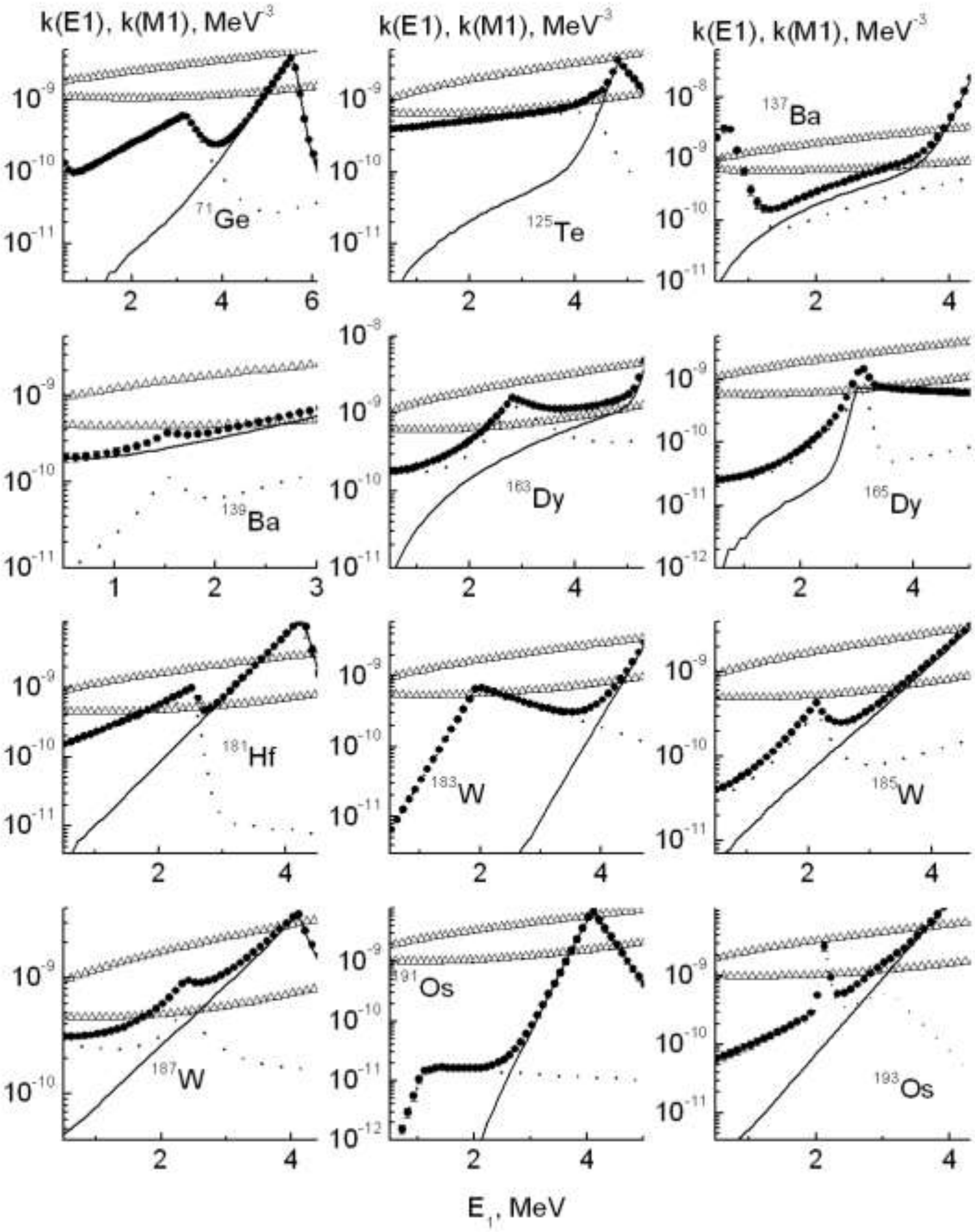

Fig.9. Strength functions of $E1$-transitions (solid lines) and $M1$-transitions (dotted lines) for even-odd nuclei. Points with errors are their sums. Upper triangles are the data of models [16, 17], down triangles are model [15] calculations with $k(M1)$=const.

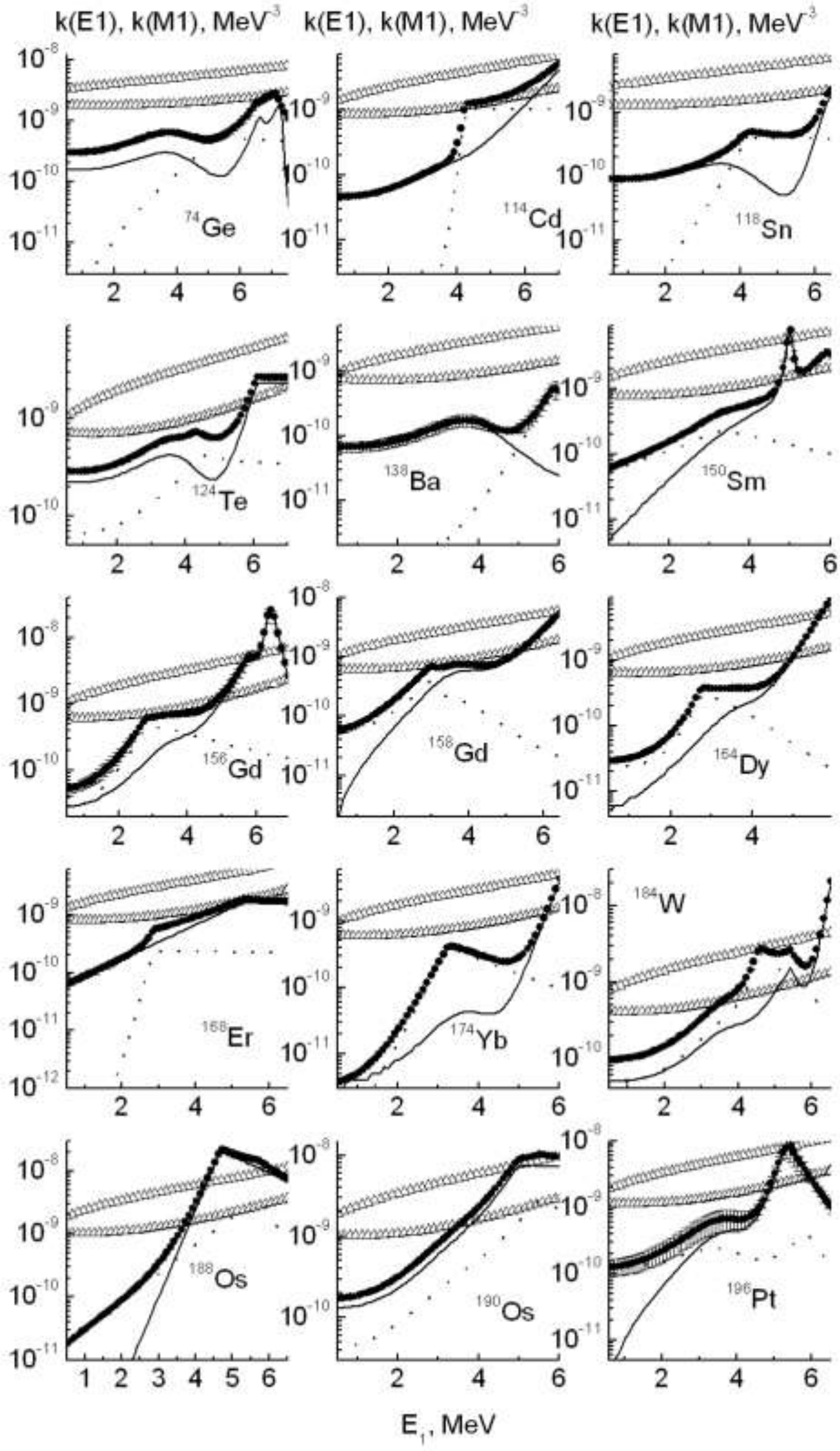

Fig.10. Strength functions of $E1$-transitions (solid lines) and $M1$-transitions (dotted lines) for even-even nuclei. Points with errors are their sums. Upper triangles are the data of models [16, 17], down triangles are model [15] calculations with $k(M1)$=const.

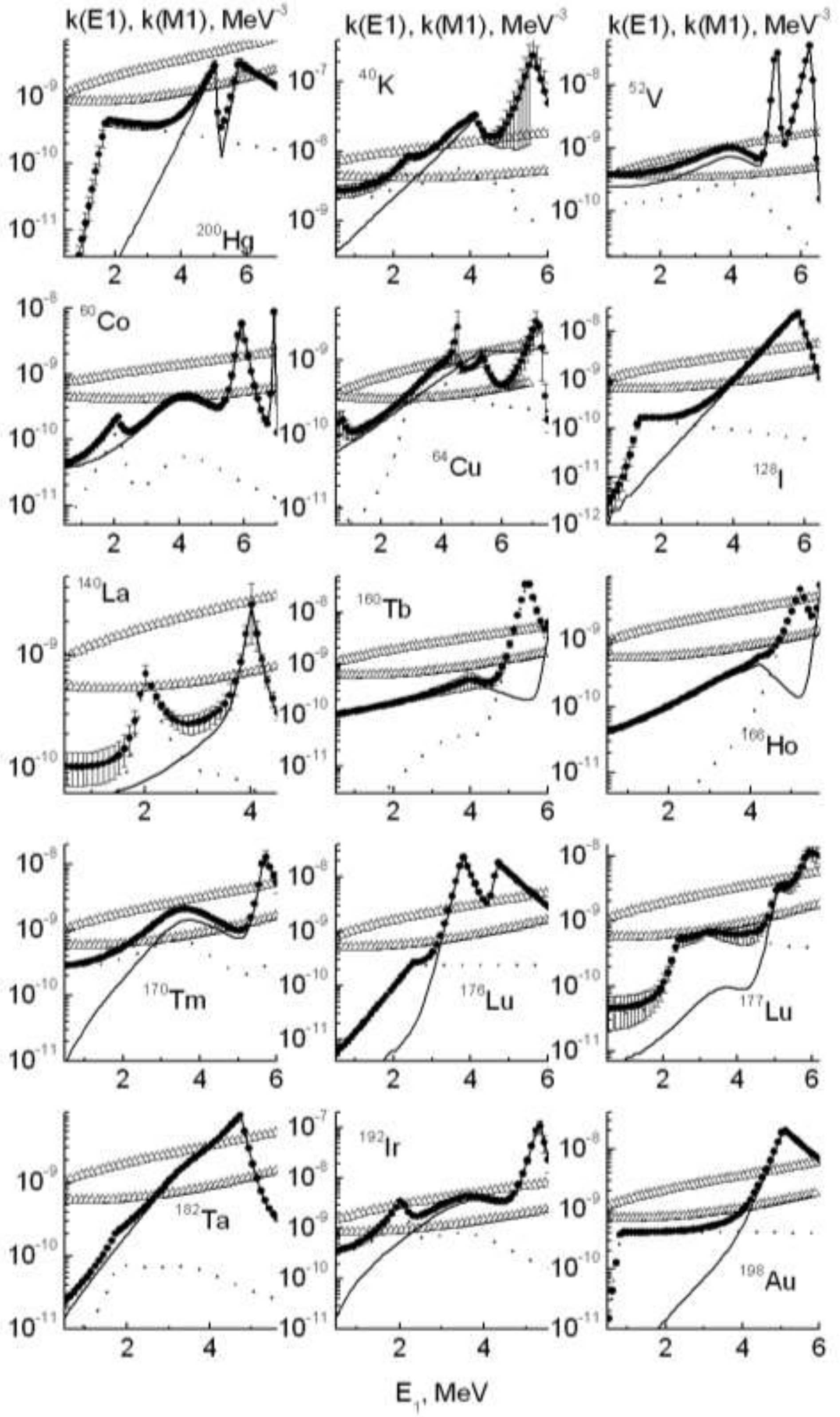

Fig.11. Strength functions of *E*1-transitions (solid lines) and *M*1-transitions (dotted lines) for ^{177}Lu and odd-odd nuclei. Points with errors are their sums. Upper triangles are the data of models [16, 17], down triangles are model [15] calculations with *k*(*M*1)=const.

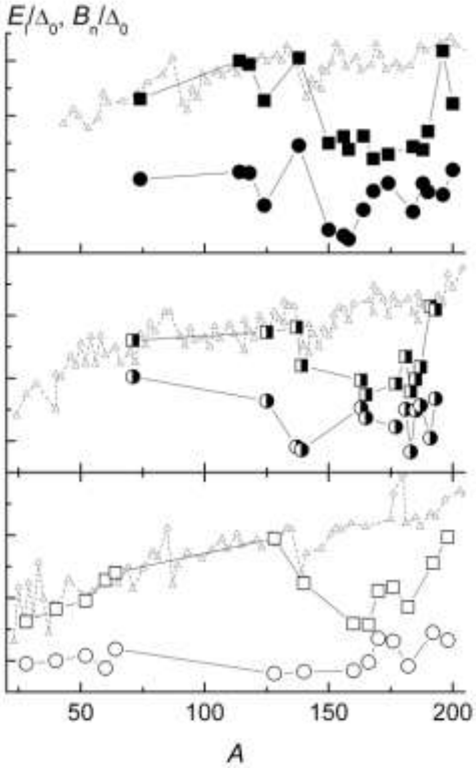

Fig.12. Dependence of breaking thresholds of the second (points) and the third (squares) Cooper pairs on the nuclear mass *A*. Full points are even-even, half-open points are even-odd and open points are odd-odd compound nuclei. Triangles are mass dependence of B_n/Δ_0 (of binding neutron energy divided by middle value of pairing energy of the last nucleon).

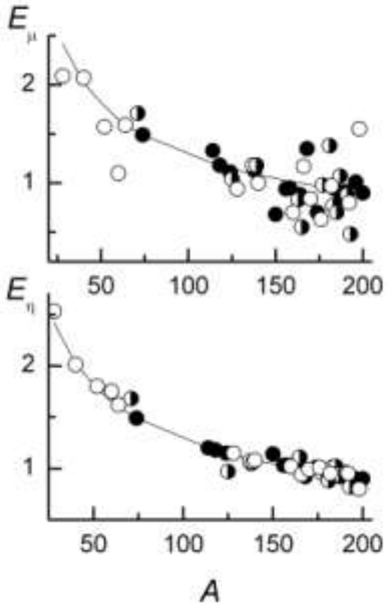

Fig.13. Mass dependence of parameters E_μ (speed of the nuclear entropy changing) and E_η (speed of changing the energy of quasi-particle's states). Full points are even-even, half-open are even-odd and open points are odd-odd nuclei. Line is the middle value of pairing energy of the last nucleon.

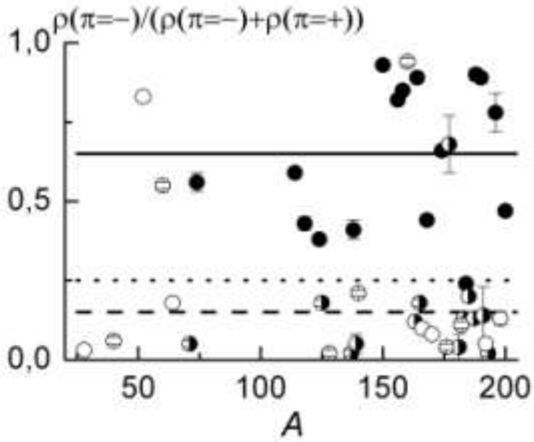

Fig.14. Mass dependence of the ratio of the level density with negative level parity to the common level density in the point E_d (upper border of the level "discrete region") and its middle value for even-even nuclei (solid lines), even-odd (dashed lines) and odd-odd nuclei(dotted lines).

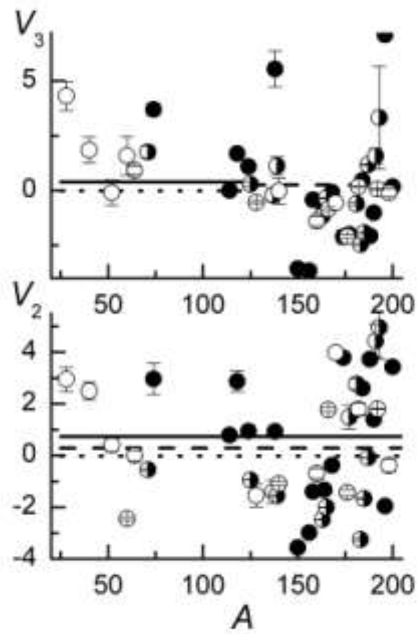

Fig.15. Mass dependence of parameter V_l of conjectured connection of values from the parametrical relation (3) $U_{l,}$, Δ_0 and ln(A_l) for the second (V_2) and the third (V_3) Cooper pairs.

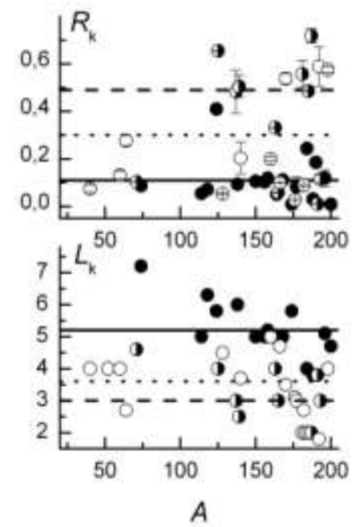

Fig.16. Mass dependence of the ratio R_k of the best approximation of the strength function's sum to its calculation value by model [15] at E_1=1 MeV and of energy L_k of the primary transition, at which the model and approximated strength functions are equal. Full points are even-even, half-open are even-odd and open points are odd-odd compound-nuclei.

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