# ALPHA-CLUSTER FORMATION FACTOR IN (n,α) REACTION CROSS SECTIONS

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## 1. Introduction

Alpha particle clustering in nuclei is a one of important subjects for understanding of α-decay, α-particle emission reactions and nuclear structure [1]. The alpha clustering problem was considered by many authors who used different theoretical approaches. Bonnetti and Milazzo-Colli suggested [2] the preformed alpha particle model which was used in analysis of experimental data for alpha decay, (n,α) and (p,α) reactions. From this analysis the alpha particle preformation factor was found to be around $\phi_\alpha \approx 0.7 - 0.01$ for favoured α-transitions. Kadmensky and Furman developed the α-cluster model [3] and obtained the surface α-clustering probability $7\cdot10^{-4}$, $3\cdot10^{-5}$ and $8\cdot10^{-7}$ for favoured, semifavoured and unfavoured α-transitions, respectively.

It is seen that a discrepancy between these two approaches to α-clustering is estimated from ~10 to $10^3$ times for favoured alpha transitions. Tonozuka and Arima obtained a ratio of the calculated reduced α-width to the observed value and considered [4] surface α-clustering of α-decays for $^{212}$Po. It was shown that the surface α-clustering effect produces a tremendous enhancement of the α-decay widths of $^{212}$Po. Hogan calculated [5] the rate of nucleon-nucleon and nucleon-alpha interactions in nuclear matter. He found the alpha-particle preformation factors and number of alpha clusters in several complex nuclei. Zhang, Royer and Li using the semiclassical approach for frequency of the α-cluster motion inside mother nuclei obtained [6,7] the α-particle preformation probability for α-decay of some heavy nuclei to be around 0.0065 ÷ 0.244, which are close to the Bonetti and Milazzo-Colli results. It is seen from here that the authors used different assumptions and essential discrepancies between their results there are. So, from the unified view point more detailed study of the α-clustering problem is needed.

In this work we in the framework of the statistical model carried out a systematical analysis of fast neutron induced (n,α) reaction cross sections and found the α-clustering factor normalizing the theoretical cross sections to experimental data. Also, the α-clustering factors were obtained for slow neutron induced (n,α) reactions using the different approaches to this problem. Our results are compared with other values of α-clustering factor.

## 2. Alpha-clustering in the fast neutron induced (n,α) reactions

The statistical model based upon Bohr's postulate of the compound mechanism is used for systematical analysis of fast neutron induced (n,α) reaction cross sections. If we use Weisskopf-Ewing evaporation model [8], constant nuclear temperature approximation [9] and

Weizsäcker's formula [10] for binding energy can get [11] following expression for fast neutron induced (n,α) reaction cross section:

$$\sigma(n,\alpha) = C\pi\left(R + \lambda\!\!\!^{-}_{n}\right)^2 \exp\left(-K\frac{N-Z+0.5}{A}\right), \tag{1}$$

where

$$C = 2\exp\frac{1}{\theta}\left(-3\alpha + \beta\left[A^{2/3} - (A-3)^{2/3}\right] + \gamma\left(\frac{Z^2}{A^{1/3}} - \frac{(Z-2)^2}{(A-3)^{1/3}}\right) + \varepsilon_\alpha - V_\alpha\right) \tag{2}$$

and

$$K = \frac{2\xi}{\theta}. \tag{3}$$

Here $R = r_0A^{1/3}$ is the target nucleus radius ($r_0$=1.3·10$^{-13}$cm); $\lambda\!\!\!^{-}_{n}$ is the wavelength of the incident neutrons divided by 2π; *A, N* and *Z* are the nucleon, neutron and proton numbers in the target nucleus, respectively; $\alpha, \beta, \gamma$ are the Weizsäcker's constants; $\varepsilon_\alpha$ is the internal binding energy of α-particle: $\varepsilon_\alpha$=28.2 MeV; $V_\alpha$ is the daughter nucleus Coulomb potential for α-particle, which can be written in the following form [12]:

$$V_\alpha = 2.058\frac{Z-2}{(A-3)^{1/3} + 4^{1/3}} MeV; \tag{4}$$

$\theta = kT$ is the thermodynamic temperature, where *k* is the Boltzmann constant and *T* is the absolute temperature.

If we use Fermi gas model [13] for level density parameter the nuclear thermodynamical temperature is expressed [9] as follows:

$$\theta = \sqrt{\frac{U_\alpha^{\max}}{a}} = \sqrt{\frac{13.5(E_n + Q_{n\alpha})}{A-3}}, \tag{5}$$

where $U_\alpha^{\max}$ is the maximum excitation energy of daughter nucleus; *a* is the level density parameter; $E_n$ and $Q_{n,\alpha}$ are the neutron and (n,α) reaction energy, respectively.

The formulae (1–5) were used for systematical analysis of (n,α) cross sections at neutron energies of 2, 4, 4.5, 5, 5.5, 6, 6.5, 8, 10, 13, 14.5, 16, 18, and 20 MeV. A comparison of absolute values of the theoretical (n,α) cross sections calculated by formulae (1–5) with known experimental data [14] in depending on relative neutron excess parameter (N–Z+0.5)/A at neutron energies of 2, 10, 14.5, and 20 MeV is shown in Fig.1, as example.

It was seen that the theoretical (n,α) cross sections calculated by statistical model formulae (1–5) were higher than experimental data. At the same time the statistical model formulae for fast neutron induced (n,p) reaction cross sections give close to experimental data results [15]. So, we assume that these results for (n,α) cross sections are, perhaps, caused by α-clustering effect which was not considered in the theoretical formula (1).

If the α-clustering effect will be taken into account the (n,α) cross section formula (1) can be rewritten in the form:

$$\sigma(n,\alpha) = C\pi\left(R + \lambda\!\!\!^{-}_{n}\right)^2 \phi_\alpha \exp\left(-K\frac{N-Z+0.5}{A}\right), \tag{6}$$

where $\phi_\alpha$ is the α-clustering factor.

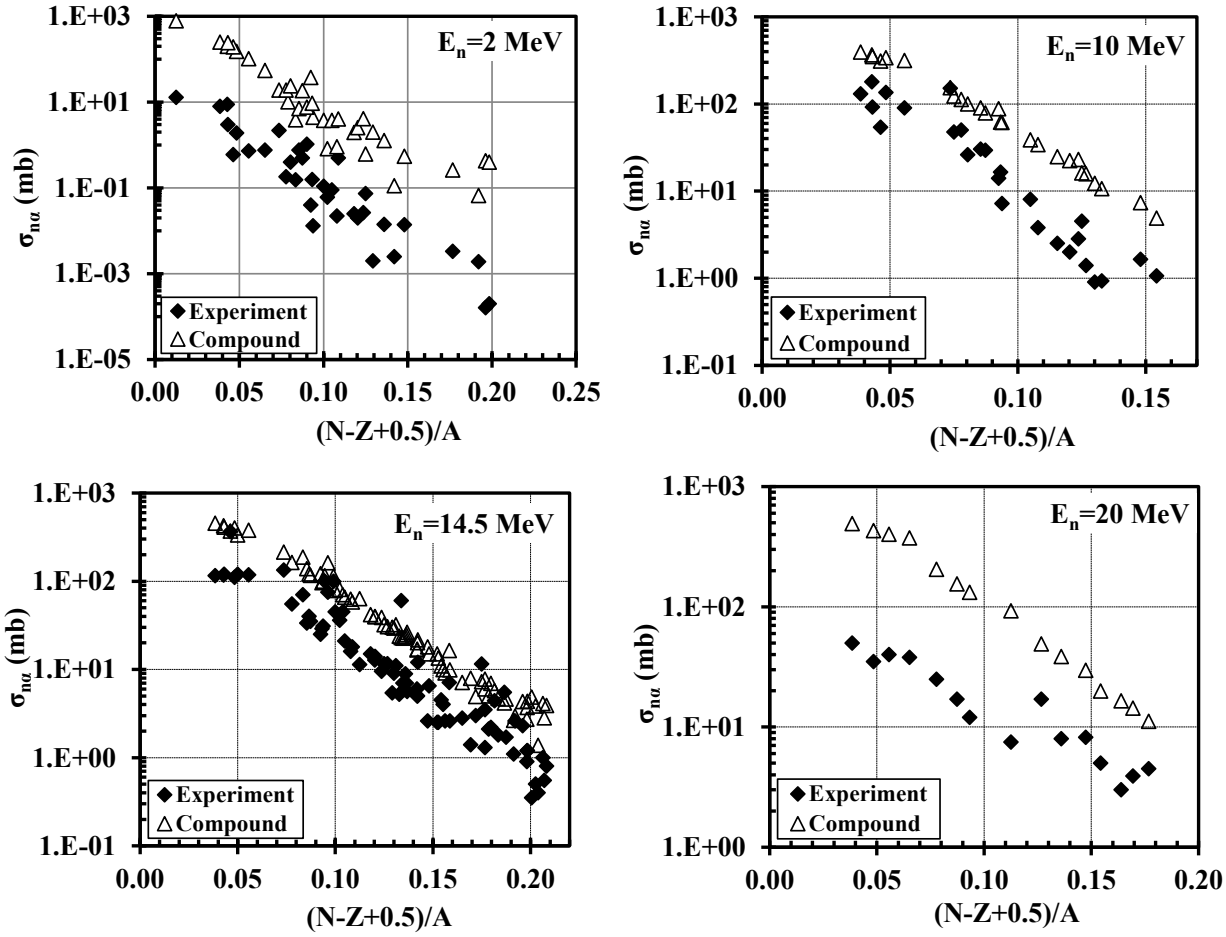


Fig.1. Comparison of theoretical (n,α) cross sections with experimental data at neutron energies of 2, 10, 14.5 and 20 MeV.

The α-clustering factor in comparison with proton (nucleon) emission probability can be determined from the ratio of calculated by formula (1) cross sections to experimental data (see Fig.2):

$$W_{p/\alpha} = \frac{\sigma^{th}_{(n,\alpha)}}{\sigma^{\exp}_{(n,\alpha)}} = \frac{1}{\phi_\alpha}. \qquad (7)$$

Some discrepancy in Fig.2 between the theoretical and experimental cross sections at $E_n$=20 MeV for asymmetry parameter (N–Z+0.5)/A>0.12 is, perhaps, caused by contributions from the pre-equilibrium and direct mechanisms to the (n,α) cross sections.

The values of the α-clustering factor $\phi_\alpha$ obtained by normalizing of the theoretical cross sections to experimental ones at their beginning points are given in Table.1 for neutron energy of 2 to 20 MeV.

It is interesting to note that our value of α-cluster formation factor $\phi_\alpha$ = 0.28 at $E_n$ = 6 MeV is lower 2 times than α-particle preformation probability 0.57 for $^{40}Ca(n,\alpha_0)^{37}Ar$ reaction obtained by Knellwolf and Rossel [16] in comparison with $(n,p_0)$ reaction.

Table.1. The values of α-clustering factor for different neutron energy

| $E_n$(MeV) | 2 | 4 | 4.5 | 5 | 5.5 | 6 | 6.5 | 8 | 10 | 13 | 14.5 | 16 | 18 | 20 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $W_{p/\alpha}$ | 50 | 4.5 | 4.5 | 4.0 | 3.5 | 3.5 | 3.5 | 3.0 | 3.0 | 3.0 | 3.5 | 4.5 | 5.5 | 10 |
| $\phi_\alpha$ | 0.02 | 0.22 | 0.22 | 0.25 | 0.28 | 0.28 | 0.28 | 0.33 | 0.33 | 0.33 | 0.28 | 0.22 | 0.18 | 0.1 |

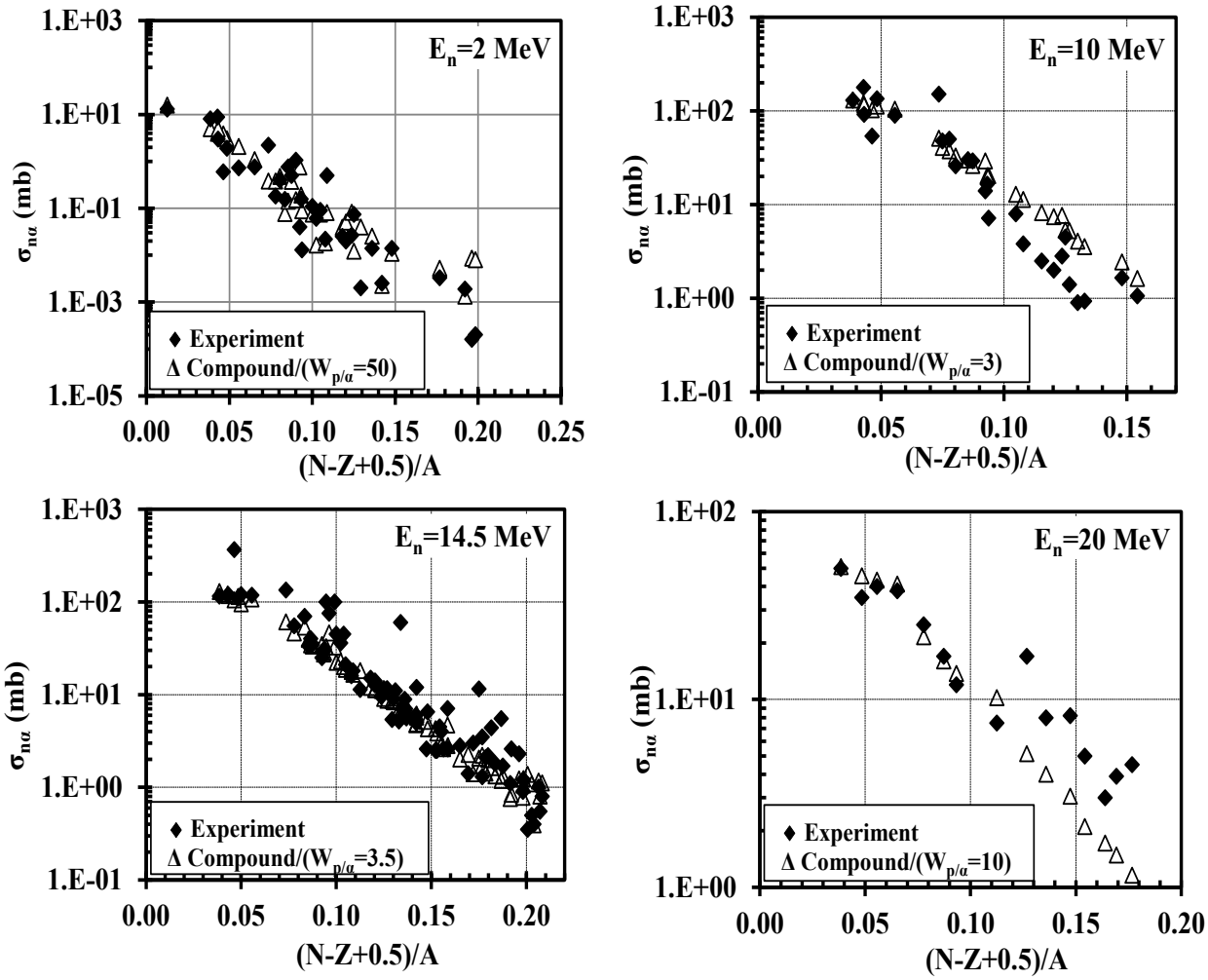


Fig.2. The theoretical (n,α) cross sections normalized to experimental data for $E_n$ = 2, 10, 14.5, and 20 MeV.

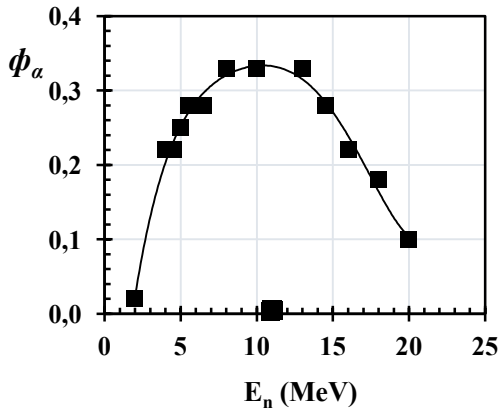


Fig.3. The energy dependence of the $\phi_\alpha$.

The dependence of the α-clustering factor $\phi_\alpha$ on the neutron energy $E_n$ is shown in Fig.3. It is seen that the α-clustering factor $\phi_\alpha$ is increased up to $E_n \approx 10$ MeV after that is decreased. Also, it can be seen that our values of the parameter $W_{p/\alpha} \approx 3 \div 10$ except at $E_n = 2$ MeV ($W_{p/\alpha}$=50) are close to results $W_{n/\alpha} \approx 2.8 \div 8.0$ of Yu.P. Popov *et al.* for slow neutrons [17–19] about which will be discussed below.

## 3. Bethe's hypothesis

In order to evaluate the α-particle formation probability Bethe suggested [20] to use the neutron emission probability for the same energy. It means that we can write following relation for reduced alpha-and neutron- widths:

$$\langle \gamma_n^2 \rangle \approx \langle \gamma_\alpha^2 \rangle . \tag{8}$$

This hypothesis is considered by Popov *et al.*, as mentioned above, by using the experimental data of (n,α) reactions for resonance neutrons [17–19] and found the following relation for the reduced average neutron- and alpha-widths:

$$W_{n/\alpha} = \frac{\langle \gamma_n^2 \rangle}{\langle \gamma_\alpha^2 \rangle} \approx 2.5 \div 8.0 . \tag{9}$$

From here we can get

$$\phi_\alpha = \frac{1}{W_{n/\alpha}} \approx 0.2 . \tag{9a}$$

These values are close to our results from the fast neutron induced (n,α) reactions and Bonetti and Milazzo-Colli α-particle preformation factor [2].

## 4. Alpha clustering in the slow neutron induced (n,α) reaction

### *4.1. Theoretical formulae*

In the frame work of the statistical model of nuclear reactions and taking into account the α-clustering factor the total α-width can be written in the following form:

$$\Gamma_\alpha = \hbar f_\alpha T_\alpha \phi_\alpha , \tag{10}$$

where $f_\alpha$ is the frequency of the α-particle motion in the daughter nucleus potential barrier; $T_\alpha$ is the transmission factor. The frequency $f_\alpha$ can be obtained by using the quantum and semiclassical approaches.

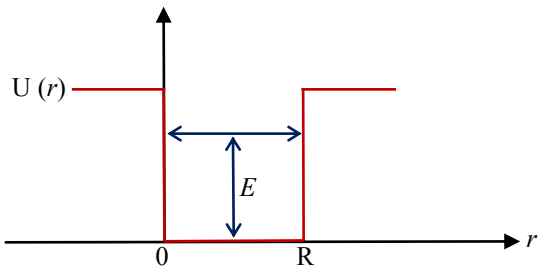


Fig.4. Alpha-particle motion in a square potential well. $E$ is alpha-particle energy; R is nuclear radius.

In the quantum approach, let us consider one dimensional motion of α-particle in a square potential well of nucleus, as a simple case, to obtain the frequency $f_\alpha$(Fig.4). In the region 0<r<R can be written Schrödinger's equation in the form:

$$\frac{d^2\Psi(r)}{dr^2} + \frac{2m_\alpha}{\hbar^2} E\Psi(r) = 0\,, \tag{11}$$

where $m_\alpha$ is the α-particle mass.

A solution of equation (11) inside the well is found as following [21]:

$$\Psi(r) = C\sin(kr + \delta)\,, \tag{12}$$

where

$$k = \sqrt{\frac{2m_\alpha E}{\hbar^2}} \tag{13}$$

is the wave number. The constant $C$ is determined to be $C = \sqrt{2/R}$ from normalization of the wave function. The condition $\Psi(r=0)=0$ gives the phase $\delta = 0$, and then the same condition for $\Psi(r=R)=0$ gives $\sin kR = 0$, whence $kR = n\pi$, $n$ being a positive integer. Then the alpha-particle energy is obtained as follows:

$$E_n = \frac{\hbar^2 k^2}{2m_\alpha} = \frac{\pi^2\hbar^2 n^2}{2m_\alpha R^2}\text{, and } n = 1, 2, 3\ldots, \tag{14}$$

where $E_n$ is the quantized energy of α-particle.

So, a distance between the neighbour states for α-particle in the well can be found by

$$\Delta E_n = E_{n+1} - E_n = \frac{\pi^2\hbar^2}{2m_\alpha R^2}\left[(n+1)^2 - n^2\right].$$

From here in the case of $n$ >>1 nuclear average level spacing $D$ for α-particle can be obtained:

$$D = \Delta E \approx \frac{\pi^2\hbar^2 n}{m_\alpha R^2}\,. \tag{15}$$

The α-particle motion frequency is determined using the classical kinetic energy formula as following

$$f_\alpha = \frac{1}{\tau_\alpha} = \frac{1}{\left(\frac{2R}{\upsilon_\alpha}\right)} = \frac{1}{R}\sqrt{\frac{E}{2m_\alpha}}\,, \tag{16}$$

where $\tau_\alpha$ is the period of the α-particle motion inside well; $\upsilon_\alpha$ is the α-particle velocity; $E$ is the α-particle energy.

Then from (14), (15) and (16) we can get following formula:

$$f_\alpha = \frac{D}{2\pi\hbar}. \tag{17}$$

Similar formula was given in [9]. From (10) and (17) the clustering factor is obtained as follows:

$$\phi_\alpha = 2\pi \frac{\Gamma_\alpha}{DT_\alpha}. \tag{18}$$

For the semiclassical case from (10) and (16) can be written the α-clustering factor in the following form:

$$\phi_\alpha = \frac{\Gamma_\alpha}{\hbar T_\alpha c} r_0 (A_D^{1/3}) \sqrt{\frac{2m_\alpha (MeV)}{E_\alpha (MeV)}}, \tag{19}$$

where $c$ is the light velocity; $A_D$ is the daughter nucleus mass number.

### *4.2. Alpha-clustering factor for slow neutron induced (n,α) reaction*

In the case of the quantum approach to calculate α-clustering factor $\phi_\alpha$ we will use the formula (18) in which values of the total alpha-width $\Gamma_\alpha$ were taken from (n,α) reaction data for resonance neutrons by Popov *et al.* [19] as average experimental α-widths $\langle \Gamma_\alpha^{\exp} \rangle$ because the total α-width has some fluctuation.

The transmission factor $T_\alpha$ was calculated by Rasmussen formula [22,23]. The average level spacing $D$ were chosen by different ways:

1. as nuclear surface vibration quantum energy [24]:
   $D_s \approx 1$ MeV;
2. average distance between the α-cluster levels in the daughter nucleus potential [3]:
   $D_\alpha \approx 20$ MeV;
3. from Fermi gas model and pre-equilibrium mechanism of nuclear reactions [2]:

$$D_F = \frac{4}{g_0} = \frac{4}{\left(\frac{6}{\pi^2} a\right)} = \frac{2\pi^2 13.5}{3A} MeV, \tag{20}$$

where $g_0$ is the single particle level density of the Fermi gas model [13].

Results of our calculations for the α-clustering factor of the slow neutron induced (n,α) reactions for some isotopes are given in Table.2. It is seen from Table.2 that the α-clustering factors varied from $\sim 10^{-3}$ to $\sim 10^{-7}$, which are satisfactorily in agreement with Kadmensky and Furman cluster model conclusions: $7 \cdot 10^{-4}$, $3 \cdot 10^{-5}$ and $8 \cdot 10^{-7}$ for favoured, semifavoured and unfavoured α-transitions, respectively. At the same time, our calculations show that cluster model level spacing $D_\alpha = 20$ MeV [3] gives lower values of $\phi_\alpha$ than other two cases (Fermi gas $D_F = 4/g_0$ and $D_s = 1$ MeV).

Table. 2. Alpha-clustering factors from slow neutron induced (n,α) reactions

| Compound nuclei | | Number of resonances | $\langle\Gamma_\alpha^{exp}\rangle$ (μeV) [19] | $E_\alpha$ (MeV) | $l_\alpha$ | $T_\alpha$ | $\phi_\alpha$, by formula (18) | | | $\phi_\alpha$, by formula (19) |
|---|---|---|---|---|---|---|---|---|---|---|
| Isotope | $J^\pi$ | | | | | | $D_s$=1 MeV | $D_\alpha$=20 MeV | $D_F = \frac{4}{g_0}$ | |
| $^{65}$Zn | 1/2 | 1 | 12 | 3.626 | 1 | 6.24E-08 | 1.2E-03 | 6.0E-05 | 8.7E-04 | 2.2E-04 |
| $^{68}$Zn | $3^-$ | 6 | 580 | 4.578 | 3 | 5.54E-06 | 6.6E-04 | 3.3E-05 | 5.0E-04 | 1.1E-04 |
| $^{96}$Mo | $2^+$ | 4 | 26 | 6.127 | 2 | 7.87E-07 | 2.1E-04 | 1.0E-05 | 2.2E-04 | 3.4E-05 |
| $^{124}$Te | $0^+$ | 7 | 7.3 | 7.331 | 0 | 2.32E-07 | 2.0E-04 | 9.9E-06 | 2.7E-04 | 3.2E-05 |
| $^{144}$Nd | $3^-$ | 15 | 21 | 9.453 | 3 | 1.49E-06 | 8.9E-05 | 4.4E-06 | 1.4E-04 | 1.3E-05 |
| $^{146}$Nd | $3^-$ | 5 | 0.32 | 8.507 | 3 | 4.84E-08 | 4.2E-05 | 2.1E-06 | 6.8E-05 | 6.7E-06 |
| $^{148}$Sm | $3^-$ | 20 | 2.3 | 9.856 | 3 | 1.75E-06 | 8.3E-06 | 4.1E-07 | 1.4E-05 | 1.2E-06 |
| $^{150}$Sm | $3^-$ | 13 | 0.21 | 9.183 | 3 | 1.85E-07 | 7.1E-06 | 3.6E-07 | 1.2E-05 | 1.1E-06 |

## Conclusions

1. In the framework of the statistical model we carried out a systematical analysis of known fast neutron induced (n,α) reaction cross sections and found the α-particle formation factor normalizing the theoretical cross sections to experimental data. This α-clustering factor is determined in comparison with proton (nucleon) emission probability.
2. The α-particle formation factors were obtained for slow neutron induced (n,α) reactions using the different approaches.
3. It was shown that our values of α-clustering factor for fast neutron induced (n,α) reactions are close to the Popov *et al.* [17–19] and Bonetti and Milazzo-Colli [2] results. At the same time these results were different from values of α-clustering factor for slow neutron induced (n,α) reactions.


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