Verification of the Practical Model of Cascade Gamma-Decay

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Abstract

To determine simultaneously both the level density ρ and the partial widths Γ of nuclear reaction products is possible only by fitting the intensities of the cascades between fixed initial, any intermediate and some final levels. Experimental total gamma-spectra with their calculations by the practical model of the gamma-decay were compared. Verification of ρ and Γ values obtained earlier and evaluation of the achieved accuracy of the practical model were done. Determined using the practical model ρ and Γ systematic uncertainties led to a conclusion that the calculated accuracy of spectra of products from any nuclear reaction will be several percents.

Introduction

An adequate and correct mathematical model of nucleus is need for understanding and subsequent prediction of the nuclear-physical parameters for any nuclei. The basis of this model is experimental determination of both the excited level density ρ and the widths Γ of partial processes of excitation and decay of any given level. In the case if the space D_λ between excited levels is less than detector resolution FWHM only an average of ρ and Γ parameters may be determined.

Excitation energy E_{ex} determines unambiguously and simultaneously both the level density and the strength functions $k= \Gamma/(A^{2/3}\cdot E_\gamma^3\cdot D_\lambda)$ for any nucleus with A mass number and energy E_γ for emitted gamma-rays below neutron binding energy B_n. It means that level density and partial radiative widths (or corresponding strength functions k) must be determined simultaneously from the system of equations, which connects experimental data with the sought parameters of $\rho=f(q_1, q_2,...)$ and $k=\varphi(p_1, p_2,...)$ functional dependencies.

With regard to ordinary spectra of emission of reaction products and reaction cross sections at any energy E_{ex} they are determined by the ρ and Γ product normalized on a constant. As a strong correlation of ρ and Γ is inevitable in this case, the simultaneous determination of these values is impossible without using any subjective assumptions or untested hypothesis. Correspondingly, calculation of cross sections of nucleon reactions gives an addition to an uncertainty connected with errors of emission widths for nucleon products.

All experimental information about nuclear structure is determined only by shapes of ρ and Γ dependencies on energy. And low count of ρ and Γ absolute values measured in several energy points is not enough to understand all nucleus properties. Measuring the intensities of any two-step cascades between compound-state and some group of the low-lying nuclear levels allows to determine shapes both of level density dependence on the excitation energy

and of average dependence of decay widths of intermediate levels on energy of emitted products at the compound-state decay. Now it was done in Dubna for 43 nuclei in the mass region $28 \leq A \leq 200$ for a measured part of intensities of primary gamma-transitions of two-step cascades [1, 2].

Experimental data on Fermi- and Bose-system interaction

Experimental research of the dynamics of interaction of Fermi- and Bose-states of nuclear matter allows to obtain a new fundamentally information about this nuclear process, so properties of nucleus and of macro systems (as objects for this process investigation) differ in principle. For example, to date in Dubna the unique information is obtained [3, 4] about possible dependency of breaking thresholds of some Cooper pairs of nucleons on the nuclear shape, what is presented in Fig. 1. It is found that the region of deformed nuclei at A>150 the second and the third breaking pairs thresholds are smaller than these thresholds for spherical nuclei. To observe something like that in a superconducting macro system of electron gas is impossible, at least, in the absence of a technique of production of special nanosystem contained combinations of linear nano objects (type of conductor/inculator).

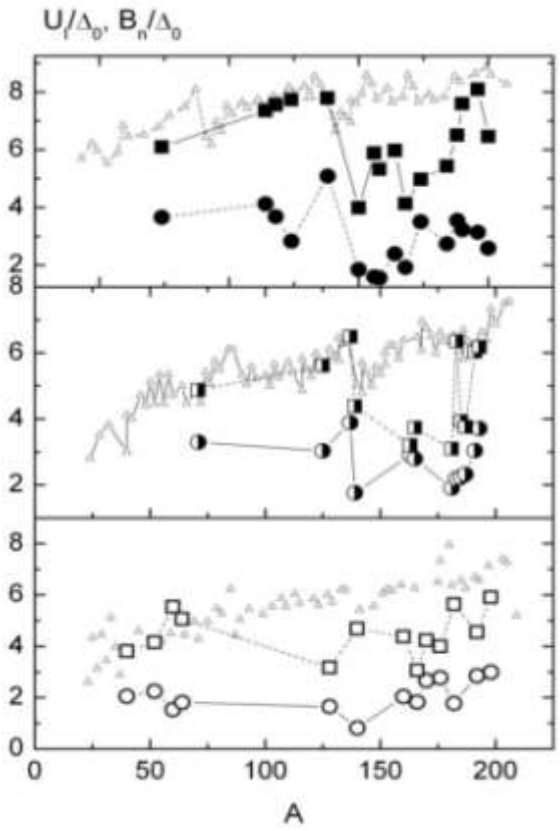

Fig.1. Mass dependencies of ratios of breaking thresholds U_l for the second (points) and the third (squares) Cooper pairs to the approximated average pairing energy Δ_0 for even-even compound nuclei (upper part), for even-odd nuclei (in the middle) and for odd-odd compound nuclei (bottom part). Triangles are known mass dependencies of B_n/Δ_0.

The intensities $I_{\gamma\gamma}(E_1)$ of two-step cascades between neutron resonance (or another compound-state) λ and some group of low-lying nuclear levels f through any intermediate levels i for a fixed energy E_1 of primary transition are written by a system of equations of type:

$$I_{\gamma\gamma}(E_1) = \sum_{\lambda,f}\sum_{i} \frac{\Gamma_{\lambda i}}{\Gamma_{\lambda}} \frac{\Gamma_{if}}{\Gamma_{i}} = \sum_{\lambda,f} \frac{\Gamma_{\lambda i}}{<\Gamma_{\lambda i}>_{m_{\lambda i}}} n_{\lambda i} \frac{\Gamma_{if}}{<\Gamma_{if}>_{m_{if}}}, \qquad (1)$$

where $m_{\lambda i}$ is a number of levels of excited primary gamma-transitions in intervals from the energy of initial level λ to an energy of intermediate level i, m_{if} is a number of levels of excited secondary transitions in intervals from the energy of intermediate level i to energy of the final level f, $n_{\lambda i}$ is a number of intermediate cascade levels in a set of small energy intervals. From the system (1), which connects an unknown level number n (or m) and unknown partial widths, a set of p and q parameters of the model functions $\rho=f(p_1, p_2, ..)$ and $\Gamma=\varphi(q_1,q_2,..)$ with some uncertainty is determined.

The experimental spectrum of the two-step cascade intensity is a sum of infinite set of possible pairs of mirror-symmetrical distributions. Ignoring this circumstance [5, 6] distorts entirely a picture of investigated process [7, 8]. Actually, using nuclear-spectroscopy methods the experimental spectra of cascades between initial and finite levels may be factorized on two mirror-symmetrical distributions as functions of energy of primary and secondary quanta of the cascades [9] with an acceptable systematic error. In a case of approximation of pure experimental spectra [5, 6] (without an execution of above-mentioned procedure) a likelihood function values can't give a reliable result for desired ρ and Γ values.

A necessary condition for determination of reliable ρ and Γ functions is an existence of the experimental data on branching coefficients of partial radiative widths of decay of all possible levels i onto limited group of final low-lying levels. In other words, the experiment is needed for recording all possible cascades connected known initial level λ of the nucleus and finite level f through any intermediate levels i. The required result can be obtained only if to take into account Boson part of nuclear excitations and theoretical (or phenomenological) assumptions of ρ and Γ functions for describing the intensity distributions of all primary quanta of cascades and branching coefficients $B_{\Gamma}(E_2)$ for any secondary quanta. There is a nonrecoverable systematic error of nuclear parameters determined such a way. This error causes are experimental systematic errors and mismatch of models for ρ and Γ to their obtained distributions.

We used the model [10] with varied weight and thermodynamic temperature, which gives a possibility for initialization of Γ functions in a wide range of their initial values. For ρ initialization both the function [12] elaborated for a growing number of quasi-particles of a level density and some phenomenological assumptions [13] were used. Besides, inasmuch as experimental spectra of two-step cascades are measured only for some part of intensities of primary gamma-transitions, an uncertainty of the best ρ and Γ fits gives an addition to inevitable systematic uncertainty.

In spite of the fact that an inevitable systematic uncertainty exists, the calculation results are good enough. To evaluate a quality of the ρ and Γ obtained data is possible by comparison of experimental and calculated gamma-spectra (Fig. 2–6).

Features of the total gamma-spectra at thermal neutron capture

Assumed by us normalization of total gamma-spectrum satisfies the condition $\Sigma I_\gamma E_\gamma = B_n$, where I_γ is an intensity, E_γ is a gamma-quantum energy, and B_n is a neutron binding energy. Consequently, a sum of all possible cascades with any quanta multiplicity doesn't depend on ρ and Γ function types. For determination of shapes of ρ and Γ functions it is need to use only individual cascades ($I_\gamma E_\gamma = \varphi(E_\gamma)$ dependencies). For example, strength function increasing for a part of cascades (Fig.2) is obligatory accompanied by a change both level density and strength functions for the rest cascades.

A distortion of information extracted from the comparison of experimental spectrum with a calculated one is caused by a difference of shapes of these spectra only. And valid information can be obtained if to compare the experiment with two or more calculated spectra. It was done by developed in Dubna practical model of cascade gamma-decay with different representations for the radiative strength functions and for coefficient of vibrational level density enhancement. Two of these representations with various shapes of several (not more than four) local peaks in the functional dependencies on energy of $E1$- and $M1$-primarily transitions of two-step cascades are presented below.

In a framework of quasi-particle model of nucleus authors of [14] calculated a shape of fragmentation of strength of one-particle states at their different deviation from Fermi-surface. Calculated fragmentation sufficiently depended on energies of initial quasi-particle state and of photon excitation. A practical result of these calculations is asymmetry of distribution of strength of fragmented state. If to suppose that local peaks in cascade gamma-spectra (Fig. 2) appear in consequence of defined process we could wait that these peaks are asymmetric. We described each of them by a simple analytical function and added several peaks to a smooth energy dependency expected on a base of modified model [10] for both mutlipolarities with varied thermodynamic temperature T and normalization parameter w:

$$k(E1,E_\gamma),k(M1,E_\gamma) = w\frac{1}{3\pi^2\hbar^2c^2A^{2/3}}\frac{\sigma_G\Gamma_G^2(E_\gamma^2+\kappa 4\pi^2T^2)}{(E_\gamma^2-E_G^2)^2+E_G^2\Gamma_G^2} + \text{ several local peaks.} \qquad (2)$$

Here E_G , Γ_G and σ_G are location of the center, width and cross section in maximum of giant dipole resonance, correspondingly.

In the first variant of calculations each of the local peaks was described as:

$$P\delta^-\exp(\alpha_p(E_\gamma - E_p)) + P\delta^+\exp(\beta_p(E_p - E_\gamma)), \qquad (3)$$

where the first summand is a left slope of peak (energies below maximum) and the second summand is a right slope (energies above maximum). Position E_p, amplitude P and slope parameters α_p, δ– and β_p, δ+ for each peak are determined independently.

In the second case each of the local peaks was described by asymmetric Lorentzian curve:

$$k = W_i\frac{(E_\gamma^2+(\alpha_i(E_i-E_\gamma)/E_\gamma))\Gamma_i^2}{(E_\gamma^2-E_i^2)^2+E_\gamma^2\Gamma_i^2}. \qquad (4)$$

Parameters for each i-th peak are similar to ones in model [KMF]: center position E_i, width Γ_i, amplitude W_i and asymmetry parameter $\alpha_i \sim T^2$. Expression $\alpha_i(E_i–E_\gamma)/E_\gamma$ grows with

increasing B_n–E_i value from zero in the center of peak to maximum at B_n energy and decreases at excitation energy fall. Peaks of $E1$- and $M1$-strength functions are presented by the same expressions.

In Fig. 2 the experimental intensities of two-step cascades and their best fits for 12 nuclei are compared. A quality of measured intensity fitting (χ^2) for all nuclei in cases of (3) and (4) peak shapes is practically the same what gave a possibility to test surely the obtained ρ and Γ values for the total gamma-spectra calculation. If ρ and Γ functions from the statistical model of nucleus are used in calculations there is a mismatch what is seen also in Fig.2. At upper row of Figs. 3–6 the best fits of densities for intermediate cascade levels are presented.

Fractures of level density curves for spherical nuclei correspond to breaking thresholds of the second Cooper pairs, and fitted breaking thresholds of the third pair for these nuclei are near or above neutron binding energy. For deformed and transition nuclei the breaking thresholds of the fourth pair were found also near B_n.

Calculated total gamma-spectra

The fitted data for two chosen forms (2, 3) of the local peaks of strength functions are shown in Figs. 3–6. Fitted ρ values were compared with calculated ones from back shifted Fermi-gas model [11] and from model with taking into account an influence of shell inhomogeneities on a density of single-particle states near Fermi surface [13]. And sums of the strength functions of $E1$- and $M1$-transitions and calculated total gamma-spectra (for 10 nuclei – at capture of thermal neutrons [15]; and for ^{198}Au, ^{128}I [16] – at capture of fast neutrons) are also shown in Figs. 3–6.

For any cascade quantum a fixed threshold 520 keV was chosen because of too complex shape of annihilation line 511 keV and too small cascade primary transition intensities E_1<511 keV. So an intensity of the total experimental gamma-spectrum was compared with a calculated one in $0.52 < E_\gamma < B_n - E_d$ interval of gamma-quanta energy only.

In all cases the total calculated and experimental gamma-spectra were normalized on a sum of $I_\gamma E_\gamma$ products. For a model variant (2) two calculation results of total gamma-spectra were presented in Figs. 3–6. Calculations were done with and without a compensation of local reduction of level density taken into account [1– 4] by corresponding coefficient of increasing of strength functions:

$$M=\rho_{mod}/\rho_{exp}\ , \qquad (5)$$

where ρ_{exp} is a level density obtained from experimental data, and ρ_{mod} is level density from Fermi-gas model. The level densities and radiative strength functions obtained from theoretical representations of statistical model of nucleus and ones calculated using presented model are compared in Figs. 3–6.

Results of all fittings shown in Figs. 3–6 indicate unambiguously that level density and radiative strength functions for correct calculations of any nuclear-physical parameters had to take into account effects of nucleon pairing and existence of the levels with sizeable vibrational components of wave-functions near neutron binding energy and, most likely, at higher energies.

In the Table there are ratios $2(I_{exp}–I_{cal})^2/(I_{exp}+I_{cal})$ of total gamma-spectra intensities in percents for 12 chosen nuclei. In I and II columns are the calculations with use of peak shapes (3) of strengths functions, and calculations in III and IV columns were done using peak shapes (4). In calculations presented in I and III columns it was supposed that level

densities and strength functions are independent (M=1), and in calculations from II and IV columns a compensation (5) was taken into account.

Table 2. $2(S_{exp}-S_{cal})^2/(S_{exp}+S_{cal})$ ratios of total gamma-spectra

Nucleus	I	II	III	IV
^{60}Co	36	34	22	20
^{114}Cd	26	28	24	26
^{128}I	15	9	15	9
^{150}Sm	20	24	17	9.5
^{156}Gd	15	15	19	19
^{158}Gd	20	20	20	20
^{168}Er	32	34	11	17
^{182}Ta	13	16	17	14
^{192}Ir	18	13	26	11
^{196}Pt	22	16	22	15
^{198}Au	20	17	15	8
^{200}Hg	34	28	30	30

Comparing the data of calculations presented in the Table we can do a following resume.

1) The main features of total gamma-spectra (at E_γ =2–3 MeV and some below the neutron binding energy) are reproduced by calculations accurately enough.
2) Description of the local peaks in radiative strength functions by two exponents (3) gives greater distortion between calculated and experimental total gamma-spectra than Lorentzian description (4).
3) In many ways an existence of distortion is caused by insufficient statistical accuracy of data on intensities of measured cascades (changes of level density and radiative strength functions are noticeable if χ^2 of the data from Fig. 2 vary within a few percents). If to use Monte-Carlo method for the system (1) solving the likelihood function always has the same inaccuracy.
4) Practically it is not possible to describe sums of radiative strength functions with a maximal accuracy by smooth functional dependencies because there are peaks caused by influence of structure of wave-functions of nuclear fragmented state on matrix elements of all cascade transitions. There is no any reason of an absence of similar dependence in cascades with multiplicity of 3 quanta or more. This proposition means that to describe exactly a total gamma-spectrum only by nuclear parameters obtained from $I_{\gamma\gamma}$ fittings (Fig.2) is unachievable.
5) An evaluation of systematic errors of calculated total gamma-spectra allows to wait an accuracy of the practical model of some percents for calculating the spectra of nuclear reaction products. It may be achieved if a statistic accuracy of an experiment on cascade intensity measuring will be, at least, 3–10 times more than now. In order to increase the practical model accuracy it is need also to develop a theoretical model of vibrational level density with taking into account both sequential breaking of Cooper pairs of neutrons and protons (an appearance of mixed neutron-proton pairs may be possible at some excitation energies also) and corresponding change of quasi-particle level density.

Conclusion

A comparison of results obtained in different variants of Dubna model with an available set of experimental $I_{\gamma\gamma}$ data shows that determination of breaking threshold of the second Cooper nucleon pair was done with an excellent accuracy. It isn't possible to determine from the $I_{\gamma\gamma}$ data the breaking threshold of the first Cooper pair because density of low-lying levels is small. But it is need to take its existence into account in $I_{\gamma\gamma}$ analysis so the condition of equality of fitted and experimental level densities had to be kept in the point E_d of transition from discrete individual levels to a range of unresolved ones.

An uncertainty of the breaking threshold determination for each consequent pair grows because number of quasi-particles (and an appropriate derivative $d\rho/dE_{ex}$ [Str]) quickly increase. In addition, an increase of correlation between the breaking threshold of consequent Cooper pairs and a coefficient of vibrational level density inhancement may give the similar effect.

An existence of the sources of uncertainties of the sought ρ and Γ functions is a fundamental problem and it is inevitable for any nuclear model used for experimental data analysis and for prediction of spectra and cross sections.

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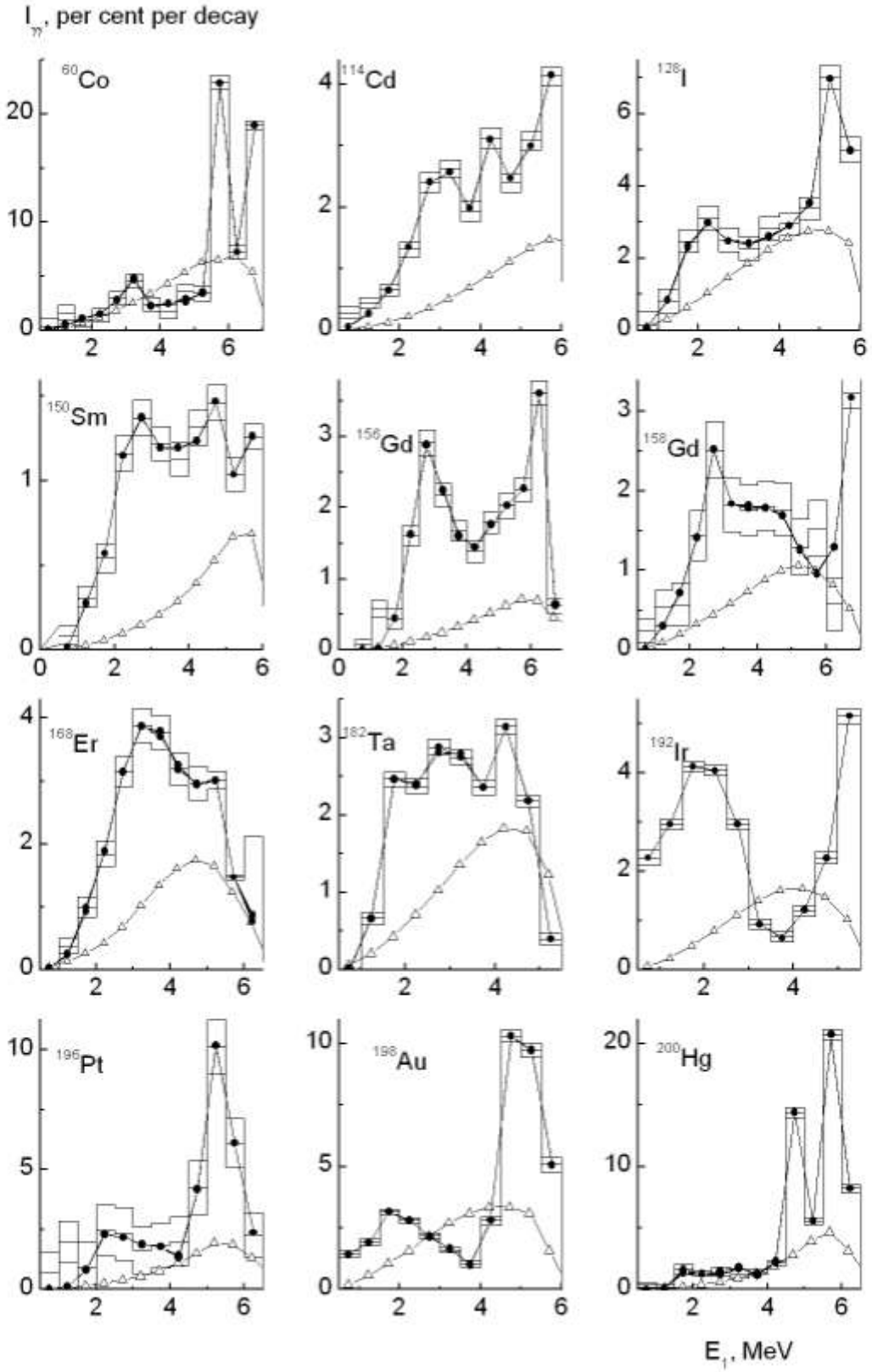

Fig.2. Fitting the experimental intensities $I_{\gamma\gamma}$ for investigated nuclei. Histogram is experiment with its errors, black points are the best fitted values, triangles are calculations by models [10, 11].

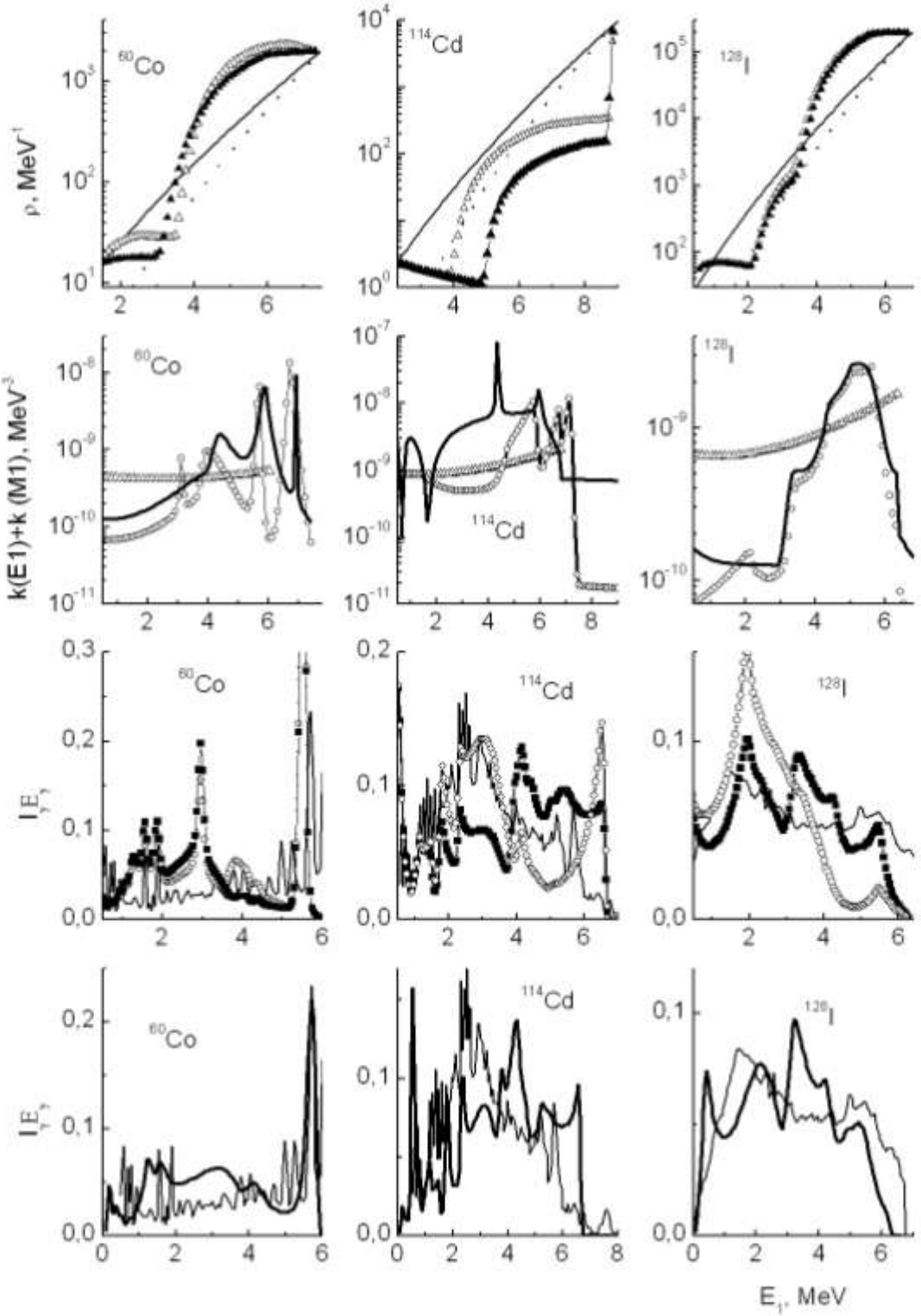

Fig.3. Upper row: level densities calculated with use of function (3) (open triangles), with function (4) (black triangles), and model calculations (solid line – model [11], dashed line – model [13]). Second row: strength functions with local peaks described by exponents (3) (open points), by asymmetric Lorentzian curve (4) (black points), and calculation by model [10] in a sum with k(M1)=const (triangles). Third row: the best fits of the total gamma spectra if local peaks described as (3) (open points) and as (4) (squares), and experimental one (bottom line). Down row: the total gamma spectra calculated using function (4) and condition (5) (bold solid line) and experimental one (solid line).

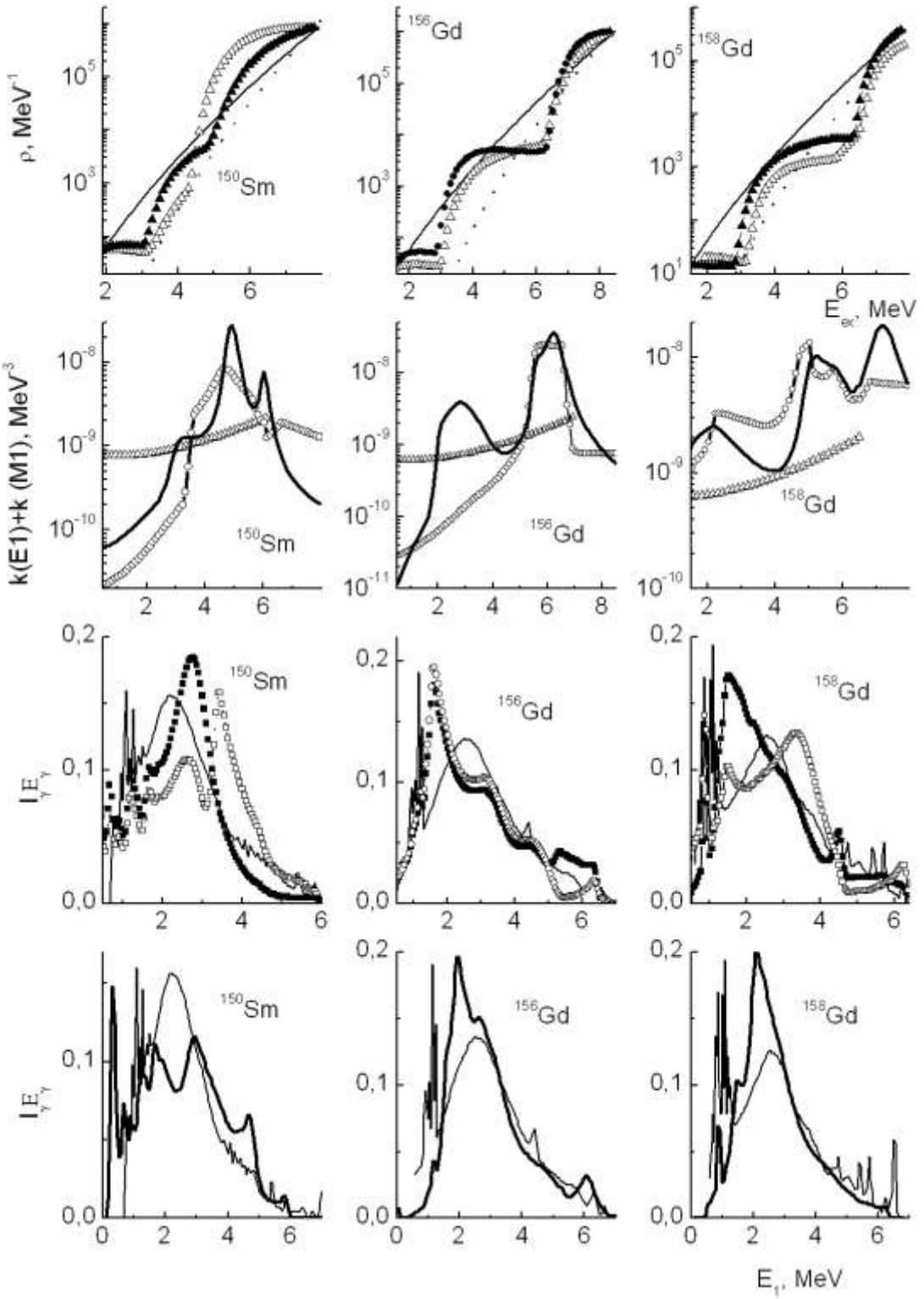

Fig. 4. The same as in Fig.3 for ^{150}Sm and ^{156,158}Gd.

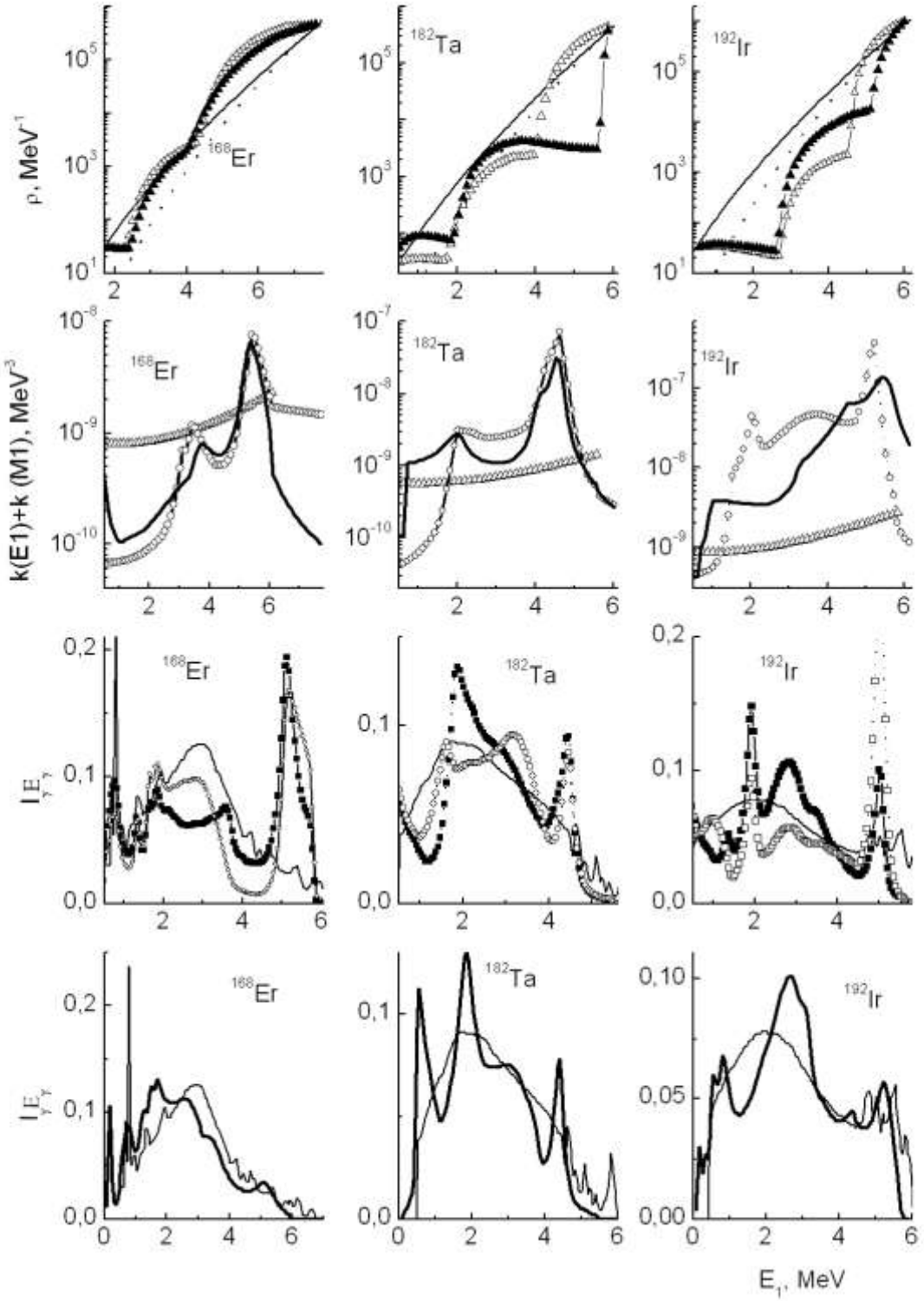

Fig. 5. The same as in Fig. 3 for ^{168}Er, ^{182}Ta, and ^{192}Ir.

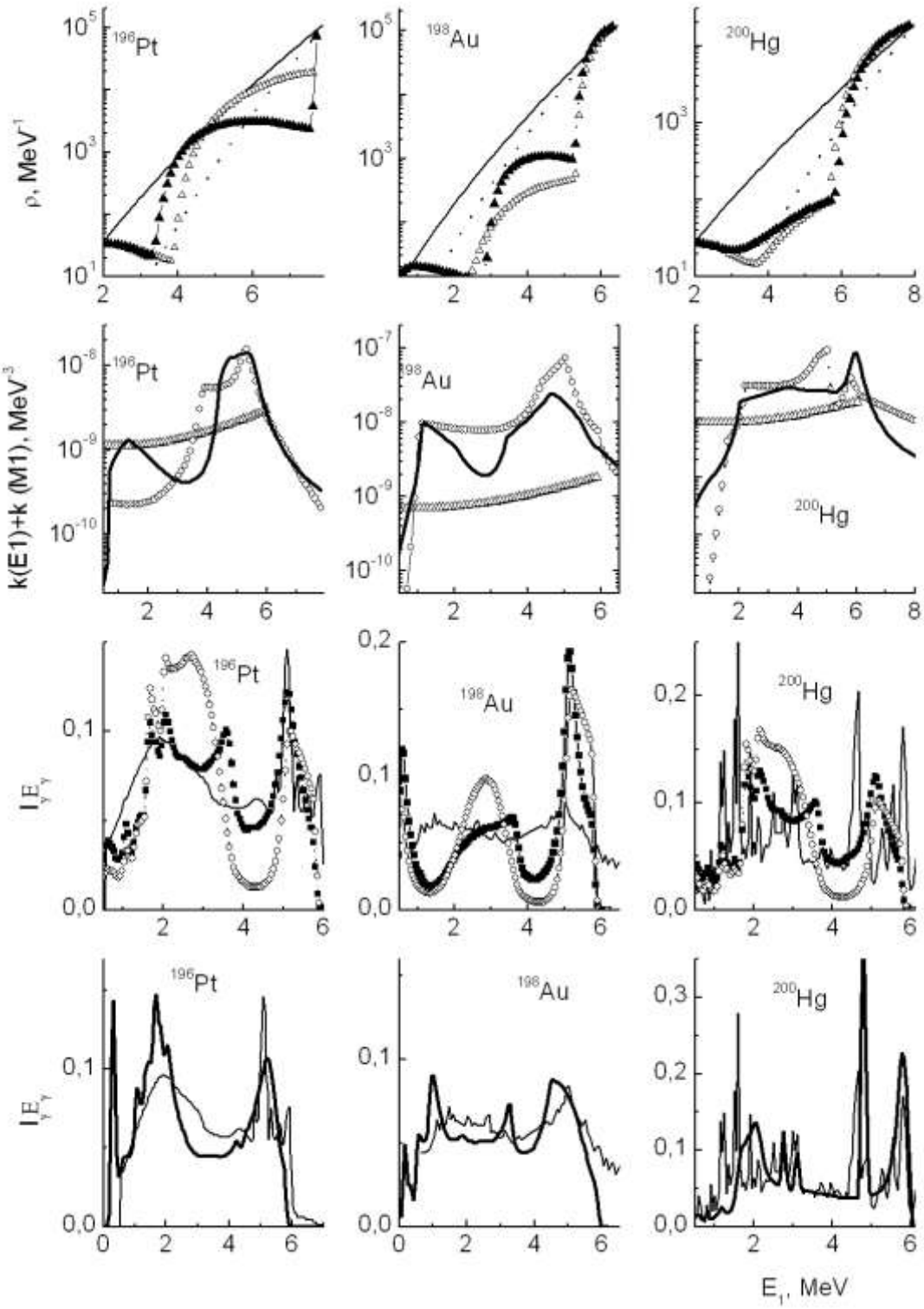

Fig. 6. The same as in Fig. 3 for ^{196}Pt, ^{198}Au, and ^{200}Hg.