ESTIMATIONS OF THE INELASTIC INTERACTION OF NEUTRONS WITH NANOSTRUCTURED MEDIA AT LOW ENERGIES AND TEMPERATURES

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Abstract: We consider peculiarities of the inelastic interaction of cold neutrons with nanostructured media (NSM) of two types: 1) powder of spherical nanoparticles, and 2) irregular composition of cylindrical nanowires. Quantitative estimations allow us concluding that cold NSM of these types do not thermalize cold neutrons but reflect them elastically.

Neutron moderators decrease the energy of fast neutrons close to the moderator thermal energy. Much attention is paid to designing moderators with temperatures 20–100 K [1–11]. Neutrons with a temperature of 1–10 K can also find promising applications in neutron research, for designing sources of very cold neutrons (VCNs), or even for designing self-standing neutron sources [12]. Below 10^2 K, medium structure and peculiarities of the inelastic interaction of neutrons have to be taken into account. We will leave other motivation for the following calculations for a detailed publication in the future but simply consider inelastic interaction of slow neutrons with NSM of the two types mentioned above.

Here, we consider the features of the inelastic interaction of cold neutrons with nanostructured media (NSM) at low temperatures. In the following, matter characteristics correspond to diamond. Temperature T is related to energy ε as $\varepsilon = \hbar$ for phonons and $E = \hbar$ $) = 3k_B T / 2$ for neutrons, where $\hbar$ is the Planck constant, ω is the phonon frequency, k_B is the Boltzmann constant, k is the neutron wave-vector, and m is the neutron mass.

I. A minimum frequency of elastic oscillations for a nanoparticle equals $\omega_{\min} = 2\pi c / \lambda_{\max}$, where $\lambda_{\max}$ is the maximum wavelength of elastic oscillations in the nanoparticle; c is the sound velocity in the nanoparticle medium. Let c_l and c_t be the velocities of propagation of longitudinal and transverse oscillations respectively. For diamond, $c_l = 17.5 \cdot 10^3$ m/s, and $c_t = 12.8 \cdot 10^3$ m/s.

In a spherical nanoparticle of diameter *d*, $\lambda_{\max}^{(S)} = 2\pi d$ and $\lambda_{\max}^{(V)} = 2d$ for surface (*S*) and volume (*V*) elastic oscillations. For diamond nanoparticles, $d = 4.5$ nm (a typical size for detonation nanodiamond), and minimum energies of surface and bulk phonons are: $\hbar$ $1.85 \cdot 10^{-3}$ eV = 22 K and $\hbar$ $5.81 \cdot 10^{-3}$ eV = 69 K. Thus, neutrons with an energy of below 20 K cannot slow down to lower temperatures in a nanopowder consisting of such diamond nanoparticles. Therefore, the nanopowder is ineffective for use as a neutron moderator to temperatures of 1–10 K.

II. Consider another model of the nanostructured medium: an irregular composition of diamond nanowires (nanorods) of diameter $d_t \sim$ 1–20 nm and length $L_t \sim$ 1–100 μm. For the following calculations, we take $d_t = 5$ nm and $L_t = 1$ μm, 5 μm, 10 μm.

We denote $\lambda_{\max}^{(L)} = 2L_t$ the maximum wavelength of the linear (L) elastic waves of the nanowire (L – phonons). Minimum energies of L–phonons are $\hbar$ $2.65 \cdot 10^{-5}$ eV (0.31 K), $5.30 \cdot 10^{-6}$ eV (0.061 K) and $2.65 \cdot 10^{-6}$ eV (0.031 K) for the lengths $L_t = 1$, 5 and 10 μm, respectively. Minimum energies of surface and bulk phonons in the nanowire are $1.67 \cdot 10^{-3}$ eV (20 K) and $5.23 \cdot 10^{-3}$ eV (62 K), respectively.

Basing on these estimations, we conclude that at temperatures below 20 K, the surface and bulk oscillations in the nanowire are practically absent. Elastic linear oscillations (L–phonons) exist in nanowires down to temperatures of 10^{-2}–10^{-1} K. Therefore, a cold neutron has the theoretical possibility of slowing down to low temperatures via inelastic interaction with L–phonons.

At low temperatures, only the low-frequency part of the phonon spectrum of linear elastic vibrations is excited. In this case, the frequency distribution function $g(\omega)$ of the phonon spectrum of L–phonons has the form [13, 14]: $g(\omega) = L_t/(4\pi c_1)$, where $(c_1)^{-1}$ = $(c_l)^{-1}$ + $2(c_t)^{-1}$, c_1 is the effective velocity.

The surface of matter can be considered as its separate region, which has physical properties, different from the matter bulk properties. This near-surface region of matter has a characteristic depth h of 3 to 5 atomic layers, can contain defects and structural atomic disorder. Denote by N_h the number of atoms contained in the near-surface layer of the nanowire material of thickness h. Introduce the value $\delta = N_h / N$, where N is the total number of atoms in the nanowire. Denoting the nanowire cross-section radius $R_t = d_t / 2$, we obtain $\delta = [R_t^2 - (R_t - h)^2] / R_t^2$. The atomic density of diamond $n_C = 17.6 \cdot 10^{28}$ m^{-3} [15], the average distance between carbon atoms is $a_C = (n_C)^{-1/3} = 1.78$ Å. Taking $h = (3-5)a_C$, we get an estimation $\delta = 0.38$–0.59.

III. Consider the inelastic scattering of a cold neutron on a nanowire with loss of energy (exciting one L–phonon). We assume the initial energy of the neutron and the temperature T of the nanowire to be below 20 K.

We introduce the notation: E_0 is the initial energy of the neutron (before scattering); E_i and E_f are initial and final energies of the scatterer, respectively; $\varepsilon = E_i - E_f$ is the change in neutron energy during scattering; $\mathbf{k}_0$ and $\mathbf{k}$ are the neutron wave-vectors before and after the collision with the nanowire, respectively; $\boldsymbol{\kappa} = \mathbf{k}_0 - \mathbf{k}$ is the phonon wave-vector.

We direct the z axis of the orthogonal coordinate system $Oxyz$ along the direction of the neutron initial motion $\mathbf{k}_0$ (Fig. 1). The nanowire is located at an angle θ_t to the direction $\mathbf{k}_0$, and the projection of the nanorod on the xOy plane makes an angle φ_t with the x axis. The phonon wave-vector $\boldsymbol{\kappa}$ is directed along the nanowire. The angle θ is the angle of neutron scattering on the nanowire; the direction of the vector $\mathbf{k}$ after the scattering is given by the angles θ and $(\varphi_t + \pi)$.

Double-differential cross-section of incoherent neutron scattering on a nanostructured sample, referred to one nucleus (atom) of matter is:

$$\left(\frac{d^2\sigma}{d\mathbf{\Omega} d\varepsilon}\right)_{inc} = \frac{k}{k_0}\frac{1}{2\pi\hbar}\frac{1}{r}\sum_{j=1}^{r}\left[(f_{j,inc})^2 + \delta(f_{j,coh})^2\right] e^{-2W_j}\int_{-\infty}^{\infty} dt\ e^{-i\varepsilon t/\hbar}\times$$

$$\exp\{-\frac{1}{N_1}\sum_{\lambda=1}^{3N}\frac{\hbar G(\omega_\lambda)}{2M_j}|\boldsymbol{\kappa}\,\mathbf{e}_j^\lambda|^2\left(e^{-i\omega_\lambda t}e^{\hbar\omega_\lambda/2k_BT} + e^{i\omega_\lambda t}e^{-\hbar\omega_\lambda/2k_BT}\right)\}.$$

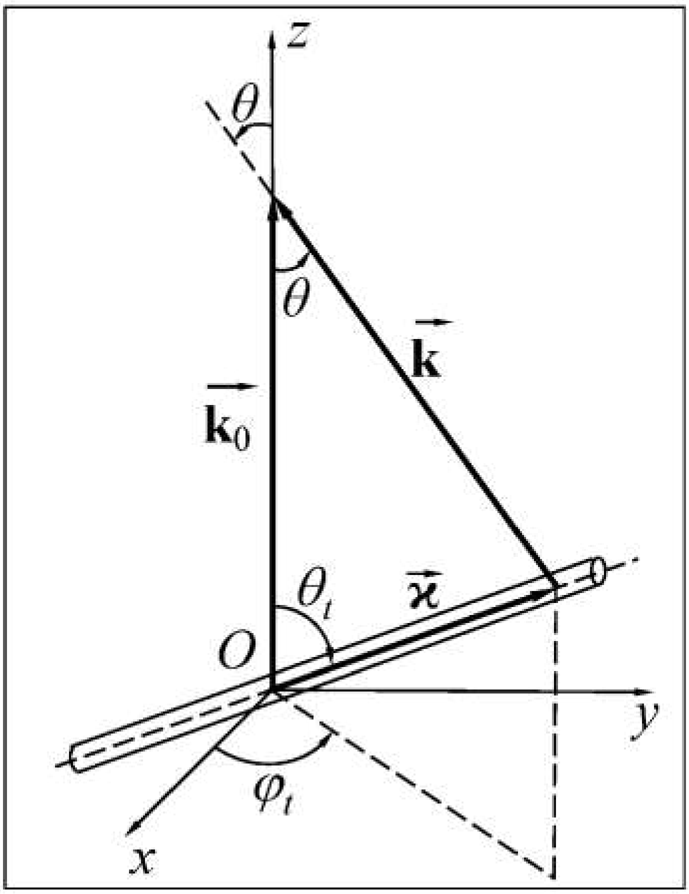

Fig. 1. Scattering of a neutron on a nanowire.

Here, are denoted: $d\mathbf{\Omega}$ is the solid angle element into which the neutron with momentum $\hbar\mathbf{k}$ is scattered; $k_0 = |\mathbf{k}_0|$, $k = |\mathbf{k}|$; N_1 is the number of elementary cells in the sample, r is the number of nuclei (atoms of matter) in one unit cell, $N = rN_1$ is the total number of atoms in the sample; the index j serves to designate the nucleus in the unit cell; $f_j = a_j(A_j+1)/A_j$ is the neutron scattering length on the fixed j-th nucleus, a_j is the neutron scattering length on the free j-th nucleus, A_j is the atomic weight of the j–th atom; M_j is the mass of the nucleus of the j–th atom of the unit cell; $f_j = f_{j,coh} + f_{j,inc}$ the coherent and incoherent neutron scattering lengths at the j–th fixed nucleus, respectively; $\exp(-2W_j)$ is the Debye-Waller factor [16], at low temperatures we shall consider it equal to 1; δ is the relative proportion of structurally disordered atoms in the total number of sample atoms; t is an integration variable having the dimension of time; function $G(\omega) = \omega^{-1}\left[1-\exp\left(-\hbar \quad\right)\right]^{-1}\exp\left(-\hbar \quad\right)$; ω_λ is the frequency of a phonon with a wave-vector $\boldsymbol{\kappa}$ and a polarization index s, the index λ denotes a pair of indexes s and $\boldsymbol{\kappa}$; $\mathbf{e}_j^\lambda$ is the unit polarization vector for the displacement of the nucleus of the j-th atom of the unit cell.

The nanowire has no physically distinguished directions “top” or “bottom”, therefore physically different directions of the orientation of the nanowire in space are limited by the angles $0 \le \theta_t \le \pi/2$ and $0 \le \varphi_t \le \pi$.

At low sample temperatures and neutron energies, the main contribution to inelastic scattering is due to one-phonon scattering. The neutron is inelastically incoherently scattered on individual nuclei. For processes with the excitation of a single phonon, we have $\int_{-\infty}^{\infty} dt \times$

$\exp(-i\varepsilon t/\hbar)\cdot\exp(-i\omega_\lambda t)=2\pi\hbar\cdot\delta(\varepsilon+\omega_\lambda\hbar)$. Therefore, the cross-section for incoherent inelastic scattering of a neutron by a nanowire with excitation of a single phonon, assigned to one nucleus (atom) of matter, is:

$$\left(\frac{d^2\sigma}{d\mathbf{\Omega}d\varepsilon}\right)_{inc}^{1ph+}=\frac{k}{k_0}\frac{1}{r}\sum_{j=1}^{r}\left[(f_{j,inc})^2+\delta(f_{j,coh})^2\right]\frac{1}{N_1}\sum_{\lambda=1}^{3N}\frac{\hbar}{2M_j}|\boldsymbol{\kappa}\mathbf{e}_j^\lambda|^2 e^{\hbar\omega_\lambda/2k_BT}\times$$
$$\delta(\varepsilon+\omega_\lambda\hbar)\cdot\delta(\varphi-\varphi_t-\pi)\cdot\Theta(-\varepsilon-\varepsilon_{\min}).$$

It is taken into account that the L – phonon energy in a nanowire is limited by the quantity $\varepsilon_{\min}=\hbar \quad \cong \pi c_l\hbar$, thus the function $\Theta(x)=\{1, \text{for } x\geq 0 \mid 0, \text{for } x<0\}$ is introduced.

It should be noted that the motion of atoms of nanowire material is spatially three-dimensional: each atom oscillates near its equilibrium position as a three-dimensional object. Only the collective vibrational motion of the atoms of the nanowire substance, interpreted as excitation of the quantum of oscillations (L–phonon), has the form of a one-dimensional quasiparticle (plane wave) due to the limited transverse dimensions of the nanowire and the low temperature of the matter and neutron energy.

We make further simplifications. As a scatterer, we consider a separate nanowire. Let us denote by $\varepsilon=(k_0^2-k^2)\hbar$ >0 the energy of a phonon excited as a result of inelastic neutron scattering. We consider a simple chemical substance possessing cubic symmetry for the Bravais lattices. Summation over normal oscillations is replaced by integration over the phonon spectrum $g(\omega)$ [16]: $\sum_{\lambda=1}^{3N}(\boldsymbol{\kappa}\mathbf{e}^\lambda)^2 f(\omega_\lambda)=(\kappa^2/3)\int_{\omega_{\min}}^{\omega_{\max}}d\omega\, g(\omega)f(\omega)$. We denote f_{inc} and f_{coh} are incoherent and coherent neutron scattering lengths at the bound atomic nucleus of the scatterer substance; N is the total number of atomic nuclei of a substance in one nanowire; M is the mass of the nucleus. After the transformations, we obtain an expression for the cross-section of incoherent inelastic neutron scattering by a substance of one nanowire with the excitation of one phonon assigned to one nucleus (atom) of the nanowire substance:

$$\left(\frac{d^2\sigma}{d\mathbf{\Omega}d\varepsilon}\right)_{inc}^{1ph+}=\frac{k}{k_0}[(f_{inc})^2+\delta(f_{coh})^2]\left(\frac{\kappa^2}{6MN}\right)\frac{\exp(\varepsilon/k_BT)}{[\exp(\varepsilon/k_BT)-1]}\frac{\hbar}{\varepsilon}\cdot g\left(\frac{\varepsilon}{\hbar}\right)\times$$
$$\delta(\varphi-\varphi_t-\pi)\cdot\Theta(\varepsilon-\varepsilon_{\min}).$$

We introduce the notation: $\sigma_1(\theta_t,\varphi_t)=\int\left(\frac{d^2\sigma}{d\mathbf{\Omega}d\varepsilon}\right)_{inc}^{1ph+}d\mathbf{\Omega}d\varepsilon$ – the total cross-section of incoherent inelastic scattering of a neutron with the excitation of a single phonon, attributed to one nucleus (atom) of a nanowire substance, when the nanowire is oriented in space in the direction of the angles θ_t and φ_t (Fig. 1).

The variation of variables in the integration of the inelastic neutron scattering cross-section will be in the range: $k_0\cos\theta_t-\beta\leq\kappa\leq k_0\cos\theta_t+\beta$, where $\beta=\left[k_0^2\cos^2\theta_t-2m\varepsilon/\hbar\right]^{1/2}$; and $\varepsilon_{\min}\leq\varepsilon\leq E_0\cos^2\theta_t=\hbar \quad \theta_t/(2m)$, where $E_0=\hbar \quad$) is the initial neutron energy (before scattering). After the transformations, we get:

$$\sigma_1(\theta_t,\varphi_t)=\iiint d\varepsilon\cdot d\varphi\cdot d\kappa\,\delta(\varphi-\varphi_t-\pi)\left[k_0\kappa/k^2-k_0^2\cos\theta_t/k^2+\cos\theta_t-\kappa^2\cos\theta_t/k^2+\right.$$
$$\left.+k_0\kappa\cos^2\theta_t/k^2\right]\frac{1}{k_0}\left[(f_{inc})^2+\delta(f_{coh})^2\right]\left(\frac{\kappa^2}{6MN}\right)\frac{\exp(\varepsilon/k_BT)}{[\exp(\varepsilon/k_BT)-1]}\frac{\hbar}{\varepsilon}\frac{L_t}{4\pi c_1}.$$

We introduce a new dimensionless variable $\xi = \varepsilon / E_0$, where $\xi_{\min} \le \xi \le \cos^2\theta_t$, $\xi_{\min} = \varepsilon_{\min} / E_0$, and after the transformations we obtain:

$$\sigma_1(\theta_t,\varphi_t) = A_0\,[\,I_1(\theta_t,\varphi_t)+I_2(\theta_t,\varphi_t)+I_3(\theta_t,\varphi_t)+I_4(\theta_t,\varphi_t)+I_5(\theta_t,\varphi_t)\,]. \tag{1}$$

Here, the notations are: $A_0 = [\left(f_{inc}\right)^2 + \delta\left(f_{coh}\right)^2\,]\,L_t\hbar \qquad Nc_1)$;

$$I_1(\theta_t,\varphi_t) = \frac{4mE_0}{\hbar^2}\cos\theta_t \int_{\xi_{\min}}^{\cos^2\theta_t} d\xi\,\sqrt{\cos^2\theta_t-\xi}\,\frac{\left(2\cos^2\theta_t-\xi\right)}{\left(1-\xi\right)\xi}\,\frac{\exp\left(\xi E_0/k_BT\right)}{\left[\exp\left(\xi E_0/k_BT\right)-1\right]};$$

$$I_2(\theta_t,\varphi_t) = -\frac{4mE_0}{3\hbar^2}\cos\theta_t \int_{\xi_{\min}}^{\cos^2\theta_t} d\xi\,\sqrt{\cos^2\theta_t-\xi}\,\frac{\left(4\cos^2\theta_t-\xi\right)}{\left(1-\xi\right)\xi}\,\frac{\exp\left(\xi E_0/k_BT\right)}{\left[\exp\left(\xi E_0/k_BT\right)-1\right]};$$

$$I_3(\theta_t,\varphi_t) = \frac{4mE_0}{3\hbar^2}\cos\theta_t \int_{\xi_{\min}}^{\cos^2\theta_t} d\xi\,\sqrt{\cos^2\theta_t-\xi}\,\frac{\left(4\cos^2\theta_t-\xi\right)}{\xi}\,\frac{\exp\left(\xi E_0/k_BT\right)}{\left[\exp\left(\xi E_0/k_BT\right)-1\right]};$$

$$I_4(\theta_t,\varphi_t) = -\frac{4mE_0}{5\hbar^2}\cos\theta_t \int_{\xi_{\min}}^{\cos^2\theta_t} d\xi\,\sqrt{\cos^2\theta_t-\xi}\,\frac{1}{\left(1-\xi\right)\xi}\,\frac{\exp\left(\xi E_0/k_BT\right)}{\left[\exp\left(\xi E_0/k_BT\right)-1\right]}\times$$
$$[5\cos^4\theta_t + 10\cos^2\theta_t\,(\cos^2\theta_t-\xi) + (\cos^2\theta_t-\xi)^2];$$

$$I_5(\theta_t,\varphi_t) = \frac{4mE_0}{\hbar^2}\cos^3\theta_t \int_{\xi_{\min}}^{\cos^2\theta_t} d\xi\,\sqrt{\cos^2\theta_t-\xi}\,\frac{\left(2\cos^2\theta_t-\xi\right)}{\left(1-\xi\right)\xi}\,\frac{\exp\left(\xi E_0/k_BT\right)}{\left[\exp\left(\xi E_0/k_BT\right)-1\right]}.$$

We denote by $\langle\sigma_1\rangle = \int \sigma_1\left(\theta_t,\varphi_t\right) d\mathbf{\Omega}_t \Big/ \int d\mathbf{\Omega}_t$ the average value of the total cross section of incoherent inelastic neutron scattering by a substance of one nanowire with the excitation of one phonon assigned to one nucleus (atom); averaging is performed for all spatial orientations (for all angles θ_t, φ_t) of the nanowire (Fig. 1); $\langle\sigma_1\rangle = \frac{1}{\pi}\int_0^{\pi} d\varphi_t \int_0^{\pi/2} \sigma_1\left(\theta_t,\varphi_t\right)\sin\theta_t d\theta_t$.

Denote $\varepsilon_\sigma(\theta_t,\varphi_t) = \int \varepsilon\,\left(d^2\sigma/d\mathbf{\Omega}d\varepsilon\right)_{inc}^{1ph+}\,d\mathbf{\Omega}d\varepsilon$. For a nanowire oriented in space in the direction of the angles θ_t and φ_t, the quantity $\langle\varepsilon_1\rangle_t = \varepsilon_\sigma(\theta_t,\varphi_t)/\sigma_1(\theta_t,\varphi_t)$ is the average value of the phonon energy generated in the incoherent inelastic scattering of a neutron on this nanowire.

We denote by $\langle\varepsilon_1\rangle = \left[\pi^{-1}\int\langle\varepsilon_1\rangle_t\,\sigma_1\left(\theta_t,\varphi_t\right) d\mathbf{\Omega}_t\right] \Big/ \left[\pi^{-1}\int\sigma_1\left(\theta_t,\varphi_t\right) d\mathbf{\Omega}_t\right]$ the mean value of the energy of a phonon excited during incoherent inelastic scattering of a neutron on substance of one nanowire; averaging is performed over all spatial orientations of the nanowire; $\langle\varepsilon_1\rangle = \left[\pi\langle\sigma_1\rangle\right]^{-1}\int\varepsilon_\sigma(\theta_t,\varphi_t) d\mathbf{\Omega}_t$.

After the transformations, we get:

$$\varepsilon_\sigma(\theta_t,\varphi_t) = A_0\,[\,B_1(\theta_t,\varphi_t)+B_2(\theta_t,\varphi_t)+B_3(\theta_t,\varphi_t)+B_4(\theta_t,\varphi_t)+B_5(\theta_t,\varphi_t)\,]. \tag{2}$$

Here, we denote:

$$B_1(\theta_t,\varphi_t) = \frac{4m{E_0}^2}{\hbar}\cos\theta_t \int_{\xi_{\min}}^{\cos^2\theta_t} d\xi\,\sqrt{\cos^2\theta_t-\xi}\,\frac{\left(2\cos^2\theta_t-\xi\right)}{\left(1-\xi\right)}\,\frac{\exp\left(\xi E_0/k_BT\right)}{\left[\exp\left(\xi E_0/k_BT\right)-1\right]};$$

$$B_2(\theta_t,\varphi_t) = -\frac{4mE_0^{\,2}}{3\hbar}\cos\theta_t \int\limits_{\xi_{\min}}^{\cos^2\theta_t} d\xi\,\sqrt{\cos^2\theta_t - \xi}\,\frac{\left(4\cos^2\theta_t - \xi\right)}{(1-\xi)}\,\frac{\exp\left(\xi E_0/k_BT\right)}{\left[\exp\left(\xi E_0/k_BT\right)-1\right]};$$

$$B_3(\theta_t,\varphi_t) = \frac{4mE_0^{\,2}}{3\hbar}\cos\theta_t \int\limits_{\xi_{\min}}^{\cos^2\theta_t} d\xi\,\sqrt{\cos^2\theta_t - \xi}\,\left(4\cos^2\theta_t - \xi\right)\frac{\exp\left(\xi E_0/k_BT\right)}{\left[\exp\left(\xi E_0/k_BT\right)-1\right]};$$

$$B_4(\theta_t,\varphi_t) = -\frac{4mE_0^{\,2}}{5\hbar}\cos\theta_t \int\limits_{\xi_{\min}}^{\cos^2\theta_t} d\xi\,\sqrt{\cos^2\theta_t - \xi}\,\frac{1}{(1-\xi)}\,\frac{\exp\left(\xi E_0/k_BT\right)}{\left[\exp\left(\xi E_0/k_BT\right)-1\right]}\times$$
$$[5\cos^4\theta_t + 10\cos^2\theta_t(\cos^2\theta_t - \xi) + (\cos^2\theta_t - \xi)^2];$$

$$B_5(\theta_t,\varphi_t) = \frac{4mE_0^{\,2}}{\hbar}\cos^3\theta_t \int\limits_{\xi_{\min}}^{\cos^2\theta_t} d\xi\,\sqrt{\cos^2\theta_t - \xi}\,\frac{\left(2\cos^2\theta_t - \xi\right)}{(1-\xi)}\,\frac{\exp\left(\xi E_0/k_BT\right)}{\left[\exp\left(\xi E_0/k_BT\right)-1\right]}.$$

Note that the quantities $\sigma_1(\theta_t,\varphi_t)$ and $\varepsilon_\sigma(\theta_t,\varphi_t)$ in (1), (2) do not depend explicitly on the variable φ_t, so we rewrite $\sigma_1(\theta_t) = \sigma_1(\theta_t,\varphi_t)$ and $\varepsilon_\sigma(\theta_t) = \varepsilon_\sigma(\theta_t,\varphi_t)$. The amount of energy transferred from the neutron to the scatterer in an inelastic scattering cannot be smaller than the value $\varepsilon_{\min} = \hbar$, i.e. the minimum L–phonon energy. Therefore $\left(d^2\sigma/d\mathbf{\Omega}d\varepsilon\right)_{inc}^{1ph+} = 0$ when $\varepsilon < \varepsilon_{\min}$; $\sigma_1(\theta_t) = 0$ and $\varepsilon_\sigma(\theta_t) = 0$ for $\cos^2\theta_t < \xi_{\min}$. To simplify the writing of finite computational formulas, we introduce the notation: $y = \cos^2\theta_t$, $C_0 = 4A_0mE_0/\hbar$. After the transformations, we finally get:

$$\langle\sigma_1\rangle = \frac{1}{2}\int\limits_{\xi_{\min}}^{1} dy\cdot\sigma_1(y), \qquad \sigma_1(y) = C_0\, I(y), \tag{3}$$

where $I(y) = \int\limits_{\xi_{\min}}^{y} d\xi\,\frac{\exp\left(\xi E_0/k_BT\right)}{\left[\exp\left(\xi E_0/k_BT\right)-1\right]}\,\frac{\sqrt{y-\xi}}{(1-\xi)\xi}\,\{(2y-\xi) - (4y-\xi)/3 +$
$+ (4y-\xi)(1-\xi)/3 - [5y^2 + 10y(y-\xi) + (y-\xi)^2]/5 + y(2y-\xi)\}$;

$$\langle\varepsilon_1\rangle = \frac{1}{2\langle\sigma_1\rangle}\int\limits_{\xi_{\min}}^{1} dy\cdot\varepsilon_\sigma(y) = E_0\left[\int\limits_{\xi_{\min}}^{1} dy\cdot B(y)\right]\left[\int\limits_{\xi_{\min}}^{1} dy\cdot I(y)\right]^{-1}, \tag{4}$$

where $\varepsilon_\sigma(y) = C_0\,E_0\,B(y)$, $B(y) = \int\limits_{\xi_{\min}}^{y} d\xi\,\frac{\exp\left(\xi E_0/k_BT\right)}{\left[\exp\left(\xi E_0/k_BT\right)-1\right]}\,\frac{\sqrt{y-\xi}}{(1-\xi)}\,\{(2y-\xi) -$
$- (4y-\xi)/3 + (4y-\xi)(1-\xi)/3 -\cdot[5y^2 + 10y(y-\xi) + (y-\xi)^2]/5 + y(2y-\xi)\}$.

The total average cross-section $\langle\sigma\rangle$ of inelastic incoherent neutron scattering on a single nanowire of a simple substance containing N atoms (nuclei), averaged all spatial orientations of the nanowire is equal to $\langle\sigma\rangle = N\langle\sigma_1\rangle$.

IV. We have obtained all the necessary analytical formulas for performing numerical calculations. Let us estimate the characteristic values of the quantities for the following parameters. For convenience of comparison and interpretation of the results, we will use the temperature scale of energy measurements. For the minimum energy of L–phonons $\varepsilon_{\min} = k_BT_{\min}$, we select a characteristic value $T_{\min} = 0.1$ K. The initial energy of a neutron (before collision with a nanowire) $E_0 = 3k_B T_0 / 2$; we select a characteristic value $T_0 = 20$ K. The value $\xi_{\min} = \varepsilon_{\min} / E_0 = 2T_{\min} / (3T_0) \cong 3.33 \cdot 10^{-3}$. The characteristic temperature of the nanowire is $T = 1$ K. Substituting the values of the parameters in (4), after the calculations, we obtain $\langle \varepsilon_1 \rangle$, the average value of the energy of the phonon excited in the incoherent inelastic collision of a neutron with the substance of one nanowire; $\langle \varepsilon_1 \rangle \cong 0.04\, E_0$, which corresponds to an average phonon energy of 1.2 K. In (4), the integration was carried out numerically using the software package Mathcad11. Thus, for one inelastic collision with a nanowire, the cold neutron reduces its energy on average by ~1 K.

Consider NSM, irregular composition of nanowires. We assume that the substance from which the nanowires are made actually takes 5 % of the total volume of NSM, that is, the NSM has a porosity of $p = 0.95$ and a packing factor of $\gamma = 0.05$, respectively ($p + \gamma = 1$). Nanostructured elements in the NSM are nanowires in the form of straight cylinders $L_t =$ 10 μm in length and $d_t = 5$ nm in diameter, the substance is diamond. For diamond, $M / m =$ 12, $(c_1)^{-1} = 2.134 \cdot 10^{-4}$ s/m. The constants are $\hbar$ $1.055 \cdot 10^{-34}$ J·s; $k_B = 1.381 \cdot 10^{-23}$ J·K^{-1}.

For the value $T_0 = 20$ K from (3) after the calculations, we get $\langle \sigma \rangle = N \langle \sigma_1 \rangle = 106.7 \times$ $\left[(f_{inc})^2 + \delta (f_{coh})^2 \right]$. The characteristic value is $\left[(f_{inc})^2 + \delta (f_{coh})^2 \right] = 10^{-28}\,\mathrm{m}^2$. The volume of a single nanowire is $V_t = \pi d_t^2 L_t / 4 = 1.963 \cdot 10^{-22}\,\mathrm{m}^3$. The concentration of nanowires n_t in the volume of NSM will be $n_t = \gamma / V_t = 2.55 \cdot 10^{20}\,\mathrm{m}^{-3}$. The macroscopic cross section of incoherent inelastic scattering of cold neutrons with the excitation of a single phonon in the NSM consisting of nanowires will be $\Sigma^{(1ph+)} = \langle \sigma \rangle\, n_t = 2.72 \cdot 10^{-6}\ \mathrm{m}^{-1}$.

The mean free path $l^{(1ph+)}$ of a cold neutron before an inelastic scattering with the excitation of a single phonon in the NSM consisting of nanowires will be $l^{(1ph+)} = 1 / \Sigma^{(1ph+)} =$ $3.68 \cdot 10^5$ m. The neutron velocity is $v_n = (3k_B T_0 / m)^{1/2} = 704$ m/s. The mean time interval between neutron inelastic collisions is $\Delta t = l^{(1ph+)} / v_n = 523$ s. The neutron lifetime is $\tau_n \cong$ 900 s. Then the mean total number N_{inel} of inelastic collisions of a cold neutron with the NSM does not exceed the value $N_{inel} < \tau_n / \Delta t \cong 1.7$ (even neglecting neutron absorption).

From the estimates obtained, it follows that cold neutrons cannot effectively slow down to lower temperatures in NSM consisting of nanowires. Therefore, the types of NSM considered in this paper are ineffective for use as neutron moderators down to temperatures of 1–10 K but provide excellent properties of the elastic reflection of slow neutrons.

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