THE ANGULAR AND SPIN DISTRIBUTIONS OF THE BINARY AND TERNARY LOW-ENERGY NUCLEAR FISSION PRODUCTS AND THE TRANSVERSE WRIGGLING- AND BENDING-VIBRATIONS OF THE COMPOUND FISSILE NUCLEI

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Abstract
It has been demonstrated, that the angular and spin distributions of products of the binary and ternary low-energy nuclear fission can been successfully described with taking into account the influence of the zero-point transverse wriggling- and bending-vibrations of the compound fissile nucleus near it's scission point with the domination of the wriggling-vibrations.

1. INTRODUCTION

The normalized light fission fragments angular distribution $T^J_{MK}(\Omega)$ in the laboratory coordinate system (LCS) for the low-energy binary nuclear fission from the transition fission state (TFS) [1] *JMK* of compound fissile nucleus (CFN) can be represented [2-4] through the analogous distribution $T(\Omega')$ in the internal coordinate system (ICS):

$$T^J_{MK}(\Omega) = \frac{2J+1}{16\pi^2}\int d\omega\left[\left|D^J_{MK}(\omega)\right|^2 + \left|D^J_{M-K}(\omega)\right|^2\right]T(\Omega'), \quad (1)$$

where the solid angles Ω, Ω' are defined by the fragment emission angles φ, θ and φ', θ' in LCS and ICS correspondingly, $D^J_{MK}(\omega)$ is the generalized spherical function depending from Euler angles $\omega = (\alpha, \beta, \gamma)$, which define the orientation of the axial symmetrical fissile nucleus axes relatively to the axes LCS.

As a rule, the distribution $T(\Omega')$ is constructed with the usage of A. Bohr's hypothesis [1]: $T(\Omega') = T_0(\Omega')$, where the fission fragments distribution $T_0(\Omega')$ has a δ-function character [2-4]:

$$T_0(\Omega') = \frac{1}{2\pi}\left[\delta(\xi'-1)\right], \quad (2)$$

where $\xi' = \cos\theta'$. Then the distribution $T^J_{MK}(\Omega)$ (1) is determined by the distribution $\left(T^J_{MK}\right)_0(\Omega)$:

$$\left(T^J_{MK}\right)_0(\Omega) = \frac{2J+1}{8\pi^2}\left[\left|D^J_{MK}(\omega)\right|^2 + \left|D^J_{M-K}(\omega)\right|^2\right]_{\alpha=\varphi,\beta=\theta,\gamma=0}, \quad (3)$$

coinciding with the probability of the orientations of ICS axes in LCS defined by Euler angles $\alpha = \varphi$, $\beta = \theta$, $\gamma = 0$.

Because of the angular distribution (2) differs from zero for a fixed value of the angle $\theta' = 0$, from the quantum-mechanical uncertainty relation [5-6] for ΔL and $\Delta\theta'$:

$$(\Delta L)^2(\Delta\theta')^2 \geq \hbar^2/4,$$

it follows that the distribution (2) corresponds to the case of complete uncertainty $\Delta L = \infty$ in the values of the relative orbital moment of fission fragments L. Therefore for real experimental situations it is necessary instead of distribution (2) to use [2-4] the physically close distribution $T(\Omega')$, which differs from zero in a small vicinity of the angle $\theta' = 0$ at $\Delta\theta' \ll 1$:

$$T(\Omega') = \left|A(\Omega')\right|^2 . \tag{4}$$

The amplitude $A(\Omega')$ of the named above distribution is represented by the formula:

$$A(\Omega') = \sum_L \psi_L Y_{L0}(\Omega') , \tag{5}$$

where the wave function ψ_L defines the L-distribution $W(L) = \left|\psi(L)\right|^2$ of fission fragments in the vicinity of the scission point of the CFN. Because of the approximate validity of formula (2) characteristic orbital moments L in the sum (5) will have large values in comparison with the value of the compound fissile nucleus spin J. Then from the law of compound nucleus total spin conservation $\mathbf{J} = \left(\mathbf{F} + \mathbf{L}\right)$, where $\mathbf{F} = \left(\mathbf{J}_1 + \mathbf{J}_2\right)$ is the summary fission fragments spin, it follows $\mathbf{F} \approx -\mathbf{L}$ and for $L \gg J$ the summary spin F must have the large values $F \gg J$.

The experimental multiplicities, energy and angular distributions of instantaneous neutrons and γ-quanta, evaporated from thermalized fragments of low-energy fission, isomeric ratios of yields of final fragments, as well as characteristics of delayed neutrons emitted during the β-decay of named above final fragments are consistent with the fact [7-15] of the appearance of large values of fission fragments spins $\mathbf{J}_1$, $\mathbf{J}_2$, which are oriented perpendicularly to the direction of CFN symmetry axis.

The question arises about mechanisms of appearance of the large values of the relative orbital moments L and spins J_1, J_2 fission fragments. Attempts of the explanation of these facts through the Coulomb interaction of strongly deformed primary fission fragments were unsatisfactory, since this interaction can change [8-9] the average values of spins and orbital moments of fission fragments only on small quantities $\Delta\bar{L}$, $\Delta\bar{J}_1$, $\Delta\bar{J}_2 \leq 2$. The answer to this question can be obtained on the base of the development the representations of the quantum theory fission [1-4] in papers [17-20], allowing simultaneously to take into account effects of transverse wriggling- and bending-vibrations of the compound fissile nucleus with the defining role of wriggling-vibrations.

2. CHARACTERISTICS OF THE LOW-ENERGY BINARY AND TERNARY NUCLEAR FISSION.

In the description of the binary and ternary low-energy nuclear fission it is used the following predictions:

1. the conservation of the direction of the symmetry axis of the fissile nuclear system at all stages of it's evolution including the internal collective deformation motion of CFN [21] to scission point of this nucleus to the fission fragments;
2. the coldness [1] of fissile nucleus at all fission stages after passage of the first fission barrier up to it's scission point;
3. connected with named above coldness the conservation of the projection K of the spin J of fissile nucleus on the symmetry axis [1-4, 22-23];

A modern understanding of the nature of the appearance of large values of the relative orbital moments and spins of fission fragments is based [17-20] on the account of the two types of collective transverse vibrations of the CFN introduced in paper [16] in the vicinity of it's scission point, which must have the character of zero-point vibrations due to named above coldness of CFN.

The first type includes bending-vibrations associated with rotations of the symmetry axes of two strongly deformed fission fragments that touch their vertices in the neck region of a strongly deformed CFN and pass after the scission of named above neck into primary fission fragments. These rotations occur in opposite directions around axes perpendicular to the symmetry axis of CFN. Because of the law of conservation of the total spin of CFN the spins $\mathbf{J}_{b1}$ and $\mathbf{J}_{b2}$ of fission fragments connected with the bending-vibrations satisfy the condition $\mathbf{J}_{b1} = -\mathbf{J}_{b2}$.

The second type of transverse vibrations corresponds to the wriggling-vibrations of CFN, which are associated with rotations of the symmetry axes of the analogous fission fragments in the unit direction around axes perpendicular to the symmetry axis of CFN. which lead to the appearance of equally directed and large in magnitudes spins of the these fission fragments $\mathbf{J}_{w1}$ and $\mathbf{J}_{w2}$. In order to compensate for the nonzero total spin of these fragments $\mathbf{J}_w = \left(\mathbf{J}_{w1} + \mathbf{J}_{w2}\right)$, CFN rotates about the axis parallel to the axes of rotation of the fission fragments in the opposite direction, that leads to the appearance of relative orbital momenta of the named above fission fragments $\mathbf{L}$, which have values $\mathbf{L} = -\mathbf{J}_w$ because of the law of conservation of the total spin of CFN.

Both types of transverse vibrations contribute to the values of the spins J_1 and J_2 of the emitted fission fragments, the average values of which are larger than the usually observed values of the spin *J* of the CNF. But only wriggling-vibrations define the distribution of the orbital moments L of the fission fragments, which have characteristic values, as will be shown below, significantly exceeding the values of *J*.

The wave functions of zero-point wriggling- and bending-vibrations in the orbital momentum representation $\Psi_0(J_{w_x})$, $\Psi_0(J_{w_y})$ and $\Psi_0(J_{b_x})$, $\Psi_0(J_{b_y})$ depend from the spin projections for wriggling- and bending-vibrations J_{w_x}, J_{w_y} and J_{b_x}, J_{b_y} [16], which are related with projections of the spins $\mathbf{J}_1$ and $\mathbf{J}_2$ fission fragments on the axis *X*, *Y* perpendicular to the symmetry axis of CFN as

$$J_{w_x} = J_{1x} + J_{2x},\ J_{w_y} = J_{1y} + J_{2y};\ J_{b_x} = J_{1x} - J_{2x},\ J_{b_y} = J_{1y} - J_{2y};\ J_1^2 = J_{1x}^2 + J_{1y}^2,\ J_2^2 = J_{2x}^2 + J_{2y}^2. \quad (6)$$

As a result $\Psi_0(J_{w_x})$ and $\Psi_0(J_{b_x})$ can be represented in the forms [30]:

$$\Psi_0(J_{w_x}) = \left(\pi C_w\right)^{-1/4} \exp\left(-\frac{J_{w_x}^2}{4C_w}\right); \qquad \Psi_0(J_{b_x}) = \left(\pi C_b\right)^{-1/4} \exp\left(-\frac{J_{b_x}^2}{4C_b}\right), \quad (7)$$

where $C_w = M_w \hbar\omega_w$, $C_b = M_b \hbar\omega_b$ and frequencies ω_w and ω_b of both wriggling- and bending-vibrations are determined by the classical formulas $\omega_w = \sqrt{K_w / M_w}$ and $\omega_b = \sqrt{K_b / M_b}$, where *K* is the stiffness parameter, and *M* is the mass parameter. Expressing the spins distribution function $W\left(\mathbf{J}_1, \mathbf{J}_2\right)$ of the fission fragments as

$$W\left(\mathbf{J}_1,\mathbf{J}_2\right)=\left|\Psi_0(J_{w_x})\right|^2\left|\Psi_0(J_{w_y})\right|^2\left|\Psi_0(J_{b_x})\right|^2\left|\Psi_0(J_{b_y})\right|^2, \tag{8}$$

it can obtain [16]:

$$W\left(\mathbf{J}_1,\mathbf{J}_2\right)=\frac{4J_1J_2}{\pi C_b C_w}\exp\left[-\frac{1}{2}\left(\frac{1}{C_b}+\frac{1}{C_w}\right)\left(J_1^2+J_2^2\right)+\left(\frac{1}{C_b}-\frac{1}{C_w}\right)J_1 J_2\cos\phi\right], \tag{9}$$

where $\phi\left(0\le\phi\le 2\pi\right)$ is the angle between the two-dimensional spin vectors of the fragments $\mathbf{J}_1$ and $\mathbf{J}_2$ lying in the plane *xy*. By integrating formula (9) with respect to the variables J_2 and ϕ, one can obtain [16] the spin distribution $W(J_1)$ of one from initial fission fragment:

$$W(J_1)=\frac{4J_1}{C_b+C_w}\exp\left[-\frac{2J_1^2}{C_b+C_w}\right]. \tag{10}$$

Then the average spin $\bar{J}_1$ of this fragment has the value:

$$\bar{J}_1=\int_0^\infty J_1 W(J_1)\,dJ_1=\frac{1}{2}\sqrt{\frac{\pi}{2}}\left(C_b+C_w\right)^{1/2}. \tag{11}$$

From the estimates of [16] for fissile actinide-nuclei it can be obtained values: M_w = 1.6·10^6 MeV·Fm^2·s^2; M_b = 2.0·10^6 MeV·Fm^2·s^2; K_w = 295 MeV·rad^{-2}; K_b = 52 MeV·rad^{-2}; $\hbar\omega_w$ = 2.3 MeV; $\hbar\omega_b$ = 0.9 MeV; $C_w=132$ and $C_b=57$, from which it follows that the stiffness parameters, the quantum energies and the coefficients for the wriggling-vibrations turn out to be noticeably larger than the analogous values for the bending-vibrations . Then the quantity $\left(C_b+C_w\right)/2$ determining the character of the distribution (10) turns out to be ≈ 95, which leads to the average value (11) of the spin of the fission fragment $\bar{J}_1$ ≈ 8.6. At the same time, if the value $C_b=57$ is neglected in the comparison with value $C_w=132$ in the formula $\left(C_b+C_w\right)/2$, the average value of the spin of the fission fragment turns out to be equal to 6, which differs by a factor of 1.5 from the spin value obtained above, while taking into account the wriggling- and bending-vibrations. Hence, the conclusion is drawn that the wriggling-vibrations play a predominant role in comparison with bending-vibrations in the formation of the distribution of the spins of fission fragments.

The approach developed above to the description of the spin distribution of fission fragments based on the concept of the coldness of CFN at the scission point and taking into account the zero-point transverse vibrations of CFN is fundamentally different from the approach of [24, 8-11], in which the assumption of appreciable thermalization of fission fragments near scission point of the fissile nucleus, when the temperature *T* of the fission fragments exceeds 1 MeV. In this case, due to the significantly lower energy of the $\hbar\omega_b$ quantum of bending-vibrations compared to the analogous energy $\hbar\omega_w$ of the quantum of wriggling-vibrations (for example, for the nucleus), the main role in the temperature distribution of the fission fragments in terms of the number n_b and n_w quanta of bending- and wriggling- vibrations is played by the bending-vibrations. But, since the fissile nucleus remains in the cold state near it's scission point to fragments of fission, the representation of papers [24, 8-11] are not realized, and the formation of spin distributions of fission fragments is determined by zero wriggling- and bending- vibrations of CFN with the dominant role of wriggling-vibrations.

For construction of the distribution $W(L)$ of the relative orbital moments of the fission fragments the spin distribution of the fission fragments (8) can be transformed to the form:

$$W\left(\mathbf{L},\mathbf{J}'\right)=\frac{1}{\pi^2 C_w C_b}\exp\left[-\frac{\mathbf{L}^2}{2C_w}-\frac{\mathbf{J}'^2}{2C_b}\right], \tag{12}$$

where the relative orbital momentum $\mathbf{L}$ and the relative spin $\mathbf{J}'$ of the fission fragments can be represented as

$$\mathbf{L}=-(\mathbf{J}_1+\mathbf{J}_2),\quad \mathbf{J}'=\left(\mathbf{J}_1-\mathbf{J}_2\right)/2; \tag{13}$$

$$\mathbf{J}_1=-\mathbf{L}/2+\mathbf{J}',\quad \mathbf{J}_2=-\mathbf{L}/2-\mathbf{J}', \tag{14}$$

with the Jacobian of the change in the transition from the phase volume element $d\mathbf{J}_1 d\mathbf{J}_2$ to the element $d\mathbf{L}d\mathbf{J}'$ is equal to 1. Taking into account that the elements of the phase volume $d\mathbf{L}$, $d\mathbf{J}'$ for two-dimensional vectors $\mathbf{L}$, $\mathbf{J}'$ can be represented in the cylindrical coordinate system as

$$d\mathbf{L}=LdLd\varphi_{\mathbf{L}}\ ,\ d\mathbf{J}'=J'dJ'd\varphi_{\mathbf{J}'} \tag{15}$$

and integrating the distribution (12) with respect to $dJ', d\varphi_{\mathbf{J}'}, d\varphi_{\mathbf{L}}$ can obtain the distribution normalized by integration with respect to unity:

$$W(L)=\frac{L}{C_w}\exp\left[-\frac{L^2}{2C_w}\right]. \tag{16}$$

As expected, the obtained distribution $W(L)$ is determined only by a constant C_w for wriggling-vibrations. Then the average value $\bar{L}$ of the relative orbital momentum L of the fission fragments is defined as

$$\bar{L}=\int_0^\infty L\left|\psi(L)\right|^2 dL=\frac{1}{C_w}\int_0^\infty L^2\exp\left(-\frac{L^2}{2C_w}\right)dL=\sqrt{\frac{\pi}{2}}(C_w)^{1/2}. \tag{17}$$

With usage of the found above value $C_w=132$ for the nucleus ^{236}U the value $\bar{L}$ is equal 14.4 and significantly exceeds the values of the spin J of CFN.

4. THE DESCRIPTION OF SPIN DISTRIBUTIONS OF FISSION FRAGMENTS

After the scission point of CFN initially cold i-th fission fragment (i = 1, 2) in a short time (of the order of 10^{-21}s) go over to equilibrium state, for which the distribution of excitation energy E_i^* and spin J_i in [1] is built as Gibb's distribution $\rho_i(E_i^*, J_i)$:

$$\rho_i(E_i^*,J_i)=\rho_i(E_i^*)\rho_i(J_i), \tag{18}$$

where the energy $\rho_i(E_i^*)$ and spin $\rho_i(J_i)$ distributions with the temperature T_i and the fission fragment moment of inertia $\Im_i$ have forms:

$$\rho_i(E_i^*)\sim\exp(-E_i^*/kT_i), \tag{19}$$

$$\rho_i(J_i)\sim(2J_i+1)\exp[-\hbar^2 J_i(J_i+1)/\Im_i kT_i]. \tag{20}$$

In later papers [8, 12-13] it was used the representation according to which the statistical equilibrium in thermalized fission fragment arises only for their excitation energies, but the spin distribution $\rho_i(J_i)$ of fission fragment is nonequilibrium, since it forms at the scission point of the cold CFN and is not associated with the temperature of the fission fragment after it's thermalization. In this case $\rho_i(J_i)$ it is presented in the "standard" form [12]:

$$\rho_i(J_i) \sim (2J_i+1)\exp[-J_i(J_i+1)/B^2], \tag{21}$$

where the value B^2 can differ markedly from the value $\Im_i kT_i/\hbar^2$ in formula (20). Based on the statistical model of nuclear reactions [27-28] and the cascade-evaporation model [29], using energy (19) and spin (21) distributions of thermalized fragments of spontaneous and low-energy fission of actinide nuclei, multiplicities, energy and angular distributions instantaneous neutrons [11] and gamma quanta [8, 13], evaporated from thermalized fragments, relative yields of the ground and isomeric states of the final fission fragments [8, 13], as well as the characteristics of the delayed neutrons emitted during the β-decay of these fragments. The parameter B^2 in formula (21) was assumed to be (80-120), which leads to the average values of the spins of the fission fragments $\bar{J}_1$ in the range (7-9), close to their experimental values.

The spin distribution of the fission fragments (21) coincides in form with the introduced above analogous distribution (10) caused by transverse wriggling- and bending-vibrations of CFN near it's scission point, for which the constant B^2 has the form:

$$B^2 = \frac{C_b + C_w}{2}. \tag{22}$$

The presented above values of constant $(C_b + C_w)/2 \approx 95$ and average spin of fission fragment $\bar{J}_1 \approx 8.6$ from (10) are clearly correlated with the range of values (80-120) for B^2 and (7-9) for the average spin of the fission fragment $\bar{J}_1$ calculated in [12-13].

5. THE ANGLE DISTRIBUTIONS OF FRAGMENTS OF THE LOW-ENERGY PHOTOFISSION

The angular distribution $W(\Omega)$ of the fragments of the photofission reaction of even-even target nucleus can be represented with the usage of the formalism [1] as

$$W(\Omega) \equiv \frac{d\sigma_{\gamma f}(\theta)}{d\Omega} = \sum_J \sum_M \sigma(E_\gamma, JM) \sum_{K=0}^{J} \frac{\Gamma_f(JK)}{\Gamma(J)} T^J_{MK}(\Omega). \tag{23}$$

where $\sigma(E_\gamma, JM)$ is the cross section for the formation of CFN with spin J ($J=1$ for electric dipole and $J=2$ for electric quadrupole photons) and it's projection M on the direction of the incident beam of photons with energy E_γ, $\Gamma_f(JK)$ and $\Gamma(J)$ are the fission and total widths of the transition fission state of CFN. Using the formulas (1, 4-5) coefficient $T^J_{MK}(\Omega)$ can be represented [20] by formula:

$$T^J_{MK}(\Omega) = \sum_l B_{JKMl} P_l(\cos\theta), \tag{24}$$

where

$$B_{JKMl} = \frac{2J+1}{4\pi} \sum_{LL'} \psi_L \psi_{L'} \sqrt{(2L+1)(2L'+1)} \sum_{jl} (-1)^{j+J+l} \frac{\sqrt{2l+1}}{\sqrt{2J+1}} C^{jK}_{JLK0} C^{jK}_{JL'K0} C^{l0}_{LL'00} C^{JM}_{JlM0} \begin{Bmatrix} L & j & J \\ J & l & L' \end{Bmatrix}, \tag{25}$$

at that for $l=0$ value $B_{JKM0} = 1/4\pi$.

Using the formulas (1, 4-5) and the wave function ψ_L associated with the distribution $W(L)$ (16) as $\psi_L = \sqrt{W(L)}$, it can be obtained [20] for the anisotropy of the angular distribution (23):

$$\frac{d\,\sigma_{\gamma f}\left(\theta\right)/d\Omega}{d\sigma_{\gamma f}\left(90^{\circ}\right)/d\Omega}=a+b\sin^{2}\theta+c\sin^{2}\left(2\theta\right), \tag{26}$$

The coefficients *a*, *b*, *c* in formula (26) are defined in [20].

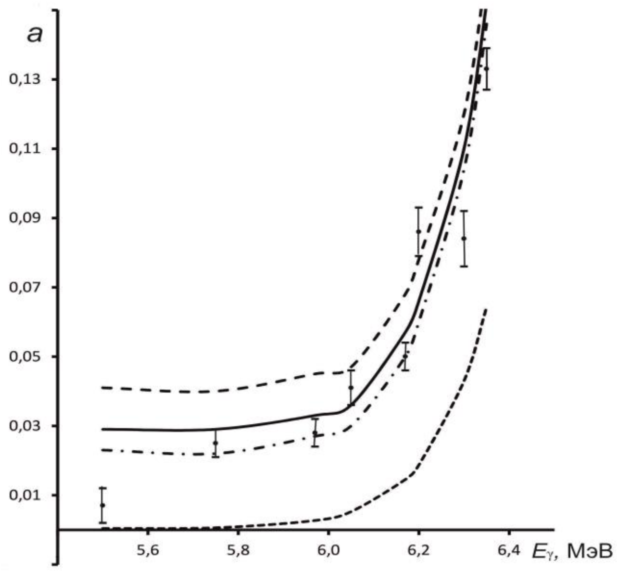

Fig. 1. Approximation of the energy dependence of the angular distribution coefficients of fragments of the subthreshold photofission ^{234}U for different values of the wriggling-vibrations parameter.
\- - - - - - $C_w = 70$; ———— $C_w = 110$; -.-.-.-.-.- $C_w = 140$; ---------- $C_w \to \infty$ (A. Bohr limit).

From Fig. 1, where dependences from γ-quanta energy E_γ of the experimental and theoretical values of asymmetry coefficient *a* for different values of parameter C_w are represented, it is seen that values of the parameter C_w lie in the range $C_w = 130 \pm 40$ and are in good agreement with the estimate of the wriggling-vibrations parameter $C_w = 132$ for ^{234}U obtained in [16] and essentially different from A. Bohr limit $C_w \to \infty$. Similar values of the parameter C_w are optimal [20] for comparison of the theoretical and experimental values of the asymmetry coefficients *a* for the photofission of the ^{236}U and ^{238}U nuclei.

6. CONCLUSION.

The successful description of the angular distributions fission fragments of low-energy nuclear, as well as the characteristics of the instantaneous neutrons and gamma quanta, evaporated from fission fragments after their thermalization, is based on the use of introduced above three basic representations. The first of these representation introduced at the first by A. Bohr is related to the coldness of the CFN at it's scission point. The second representation is based on taking into account the transverse bending- and wriggling-vibrations of this CNF [16], which because of named above coldness of CFN have the zero-point character and lead to the appearance of large values of the spins and relative orbital momentum of the fission fragments oriented perpendicular to the symmetry axis of CFN at the moment of its rupture. And, finally, the third representation uses of the generalized cascade-evaporation model, taking into account the nonequilibrium character of the distributions of spins and relative orbital moments of fission fragments due to considered above bending- and wriggling-vibrations with the domination role of wriggling-vibrations.

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