THE QUANTUM-MECHANICAL NATURE OF TRI- AND ROT-ASYMMETRIES IN THE REACTIONS OF THE TERNARY FISSION OF NONORIENTED TARGETS-NUCLEI BY THE COLD POLARIZED NEUTRONS

S.G. Kadmensky[1], V.E. Bunakov[2]

[1]*Voronezh State University, Voronezh, Russia*
[2] *Peterburg Nuclear Physics Institute of Natioanal Research Centre "Kurchatov Institute", Gatchina, Russia*
e-mail: kadmensky@phys.vsu.ru

ABSRACT

It has been analyzed the deficiencies of the approaches to the description of T-odd ROT-asymmetries in the differential cross sections of ternary fission reactions of nonoriented target-nuclei by cold polarized neutrons with the emission of prescission α-particles and evaporational γ-quanta and neutrons based on taking into account the influence of the compound fissile system rotation on fission products angular distributions with the usage of the classical trajectory calculations method, in which the interference of fission widths amplitudes for various neutron resonance states sJ_s and $s'J_{s'}$ formed in the first well of the compound fissile nucleus deformation potential are absent. It has been demonstrated that the method of construction of T-odd TRI-asymmetries for prescission α-particles, based on taking into account collective bending-vibrations of compound fissile nucleus at rupture of it's scission point, contradict to the traditional representations of the generalized model of nuclei and to methods of description of these collective vibrations.

Within framework of the quantum fission theory it has been proposed the unified mechanism for the description of T-odd TRI- and ROT-asymmetries characteristics of analyzed ternary fission reactions, based on taking into account the influence of the quantum rotation of the compound fissile system on angular distribution as fission fragments, as well third prescission and evaporated particles. It has been shown that taking into account the interference of amplitudes of fission widths for named above neutron resonance states allows to use the proposed approach for the description of differences in signs of T-odd ROT-asymmetry coefficients for emission of prescission α-particles and evaporational γ-quanta and neutrons.

1. INTRODUCTION

The T-odd P-even asymmetries in the differential cross sections $\frac{d\sigma_{n,f}}{d\Omega_3}$ of reactions of the ternary fission of the compound fissile nuclei (CFN) formed by the capture of cold longitudinally polarized neutrons by target-nuclei $^{233}\mathrm{U}$, $^{235}\mathrm{U}$, $^{239}\mathrm{Pu}$ and $^{241}\mathrm{Pu}$ with the flight of the prescission α-particle as third particle were experimentally investigated in papers [1-7]. The values $\frac{d\sigma_{n,f}}{d\Omega_3}$ were analyzed in laboratory coordinate system (LCS), where axes Z and Y were chosen along the directions of the asymptotic light fission fragment wave vector $\mathbf{k}_{LF}$ and of the polarization vector of the incident neutron $\mathbf{s}_\mathrm{n}$ correspondently. The coefficients of investigated T-odd asymmetries $D(\Omega_3)$ were calculated by the formula:

$$D(\Omega_3)=\left(\frac{d\sigma^{(+)}_{n,f}}{d\Omega_3}-\frac{d\sigma^{(-)}_{n,f}}{d\Omega_3}\right)\Bigg/\left(\frac{d\sigma^{(+)}_{n,f}}{d\Omega_3}+\frac{d\sigma^{(-)}_{n,f}}{d\Omega_3}\right), \quad (1)$$

where the signs (+/–) correspond to the two opposite directions of the neutron polarization vector $\mathbf{s}_\mathrm{n}$.

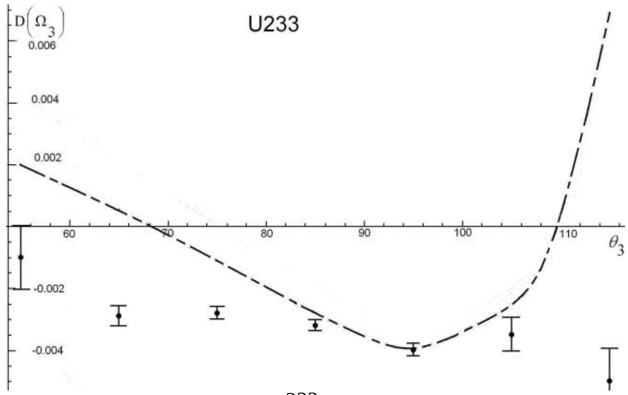

Fig. 1. T-odd asymmetry coefficients for ^{233}U, taking into account the error in the experimental determination of the emission angle of the light fission fragment.

As can be seen on Fig. 1, taken from [1, 7] for the target-nucleus $^{233}\mathrm{U}$, the coefficient $D(\Omega_3)$ does not change sign and is approximately constant in the wide range of angles θ_3. This asymmetry was called in [1] as T-odd TRI (Time Reversal Invariance)-asymmetry, because it was assumed that the named above asymmetry is connected with the representation about the violation of T-invariance in investigated nuclear reactions, which was not confined later. The mechanism of the investigated TRI-asymmetry appearance was related [6] with the influence on the formation kinematic characteristics of the emitted α-particle by classical catapult forces caused by decrease in the time of the collective rotation frequency of the fissile system after the scission of CFN, and later [7] with the influence on the these characteristics of the transverse bending-vibrations of CFN in the vicinity of it's scission point.

Later in the papers [3-7] it has been demonstrated that (see Fig. 2, 3, 4) the experimental coefficients $D(\Omega_3)$ for target-nuclei ^{235}U, ^{239}Pu and ^{241}Pu change sign in the vicinity of the angle $\theta_3 \approx 90^\circ$ corresponding to the maximum angular distribution of α-particles emitted during fission of the same nuclei by cold nonpolarized neutrons [8].

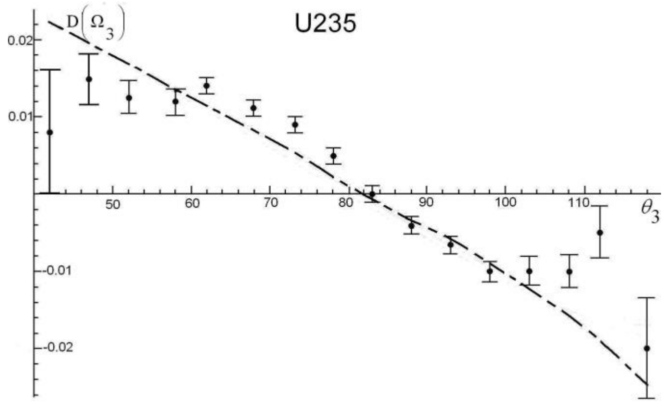

Fig. 2. T-odd asymmetry coefficients for ^{235}U, taking into account the error in the experimental determination of the emission angle of the light fission fragment.

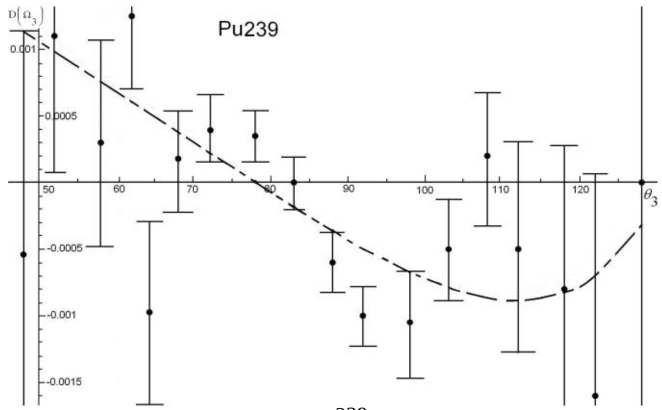

Fig. 3. T-odd asymmetry coefficients for ^{239}Pu, taking into account the error in the experimental determination of the emission angle of the light fission fragment.

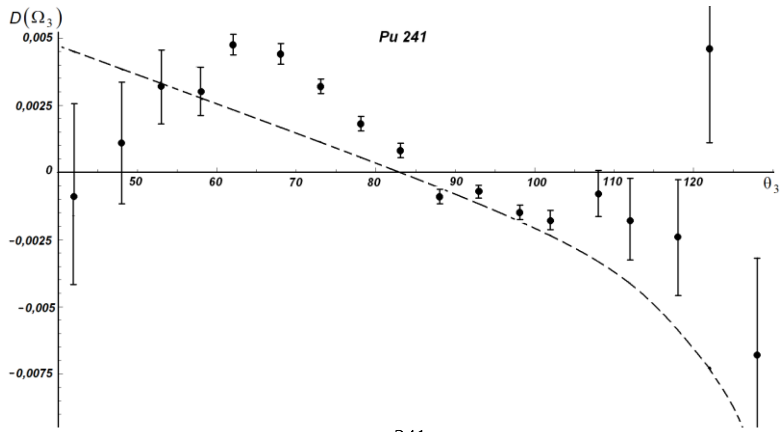

Fig. 4. T-odd asymmetry coefficients for ^{241}Pu, taking into account the error in the experimental determination of the emission angle of the light fission fragment.

Such an asymmetry in [3-7] was connected with the prevailing role of the new T-odd asymmetry, named by ROT (Rotation)-asymmetry, which was related with taking into account of the influence of the collective rotation of the polarized CFN on the angular distributions of the emitted fission fragments and α-particles on the base of the classical scheme of trajectory calculations [3, 5].

In the general case the T-odd asymmetry coefficients for the reactions of the ternary nuclear fission by cold polarized neutrons were presented [1-7] as

$$D(\Omega_3) = D_{ROT}(\Omega_3) + D_{TRI}(\Omega_3)\,, \qquad (2)$$

where the coefficients $D_{ROT}(\Omega_3)$ and $D_{TRI}(\Omega_3)$ correspond to the T-odd ROT- and TRI-asymmetries correspondently and are constructed with taking into account named above mechanism of their appearance as

$$D_{ROT}(\Omega_3) = 2\Delta \frac{W'(\Omega_3)}{2W(\Omega_3)}\,,\ D_{TRI}(\Omega_3) = const\,, \qquad (3)$$

where Δ is the difference between the turn angles of the light fission fragment and the α-particle with taking into account the influence of CFN rotation, $W(\Omega_3)$ is the angular distribution of the α-particle for the ternary nuclear fission by nonpolarized cold neutrons.

Somewhat later in [9-12] in the differential cross sections $\frac{d\sigma_{n,f}}{d\Omega_3}$ of the fission reaction for target-nuclei ^{233}U, ^{235}U by cold polarized neutrons with evaporated by fission fragments instantaneous neutrons and γ-quanta as third particles the T-odd asymmetries were found, for which the coefficients $D(\Omega_3)$ correspond to the discussed above ROT-asymmetry (see Fig. 5 taken from [11] for evaporative γ-quanta).

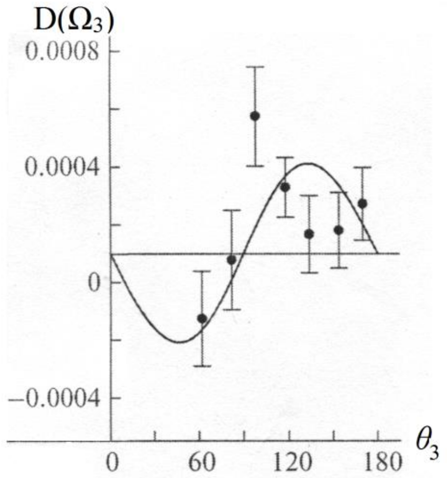

Fig. 5. T-odd asymmetry coefficients for ^{233}U taking into account the error in the experimental determination of the emission angle of evaporative γ-quanta.

At present time, the mechanisms of the appearance of observable T-odd asymmetries are actively searched. The problem consists in finding of the answer to the question - why the target-nuclei ^{233}U, ^{235}U, ^{239}Pu and ^{241}Pu having close masses, charges, angular and energy distributions of third particles $W(\Omega_3)$ for ternary fission by cold nonpolarized neutrons can have so much different forms (see Fig. 1-4) of the coefficients of the discussed T-odd asymmetries in the ternary fission of these nuclei by cold polarized neutrons.

The aim of this paper is to demonstrate that all the discussed T-odd asymmetries for ternary fission reactions of nonoriented target-nuclei by cold polarized neutrons can be explained in principle within the framework of quantum fission theory [13-24] taking into account the interference of the fission amplitudes of the neutron resonance states formed in compound fissile nucleus in the it's first potential well.

2. CHARACTERISTICS OF THE LOW-ENERGY BINARY AND TERNARY NUCLEAR FISSION

In this paper for the description of the binary and ternary low-energy nuclear fission it will be used the following representations reasonably well-founded within the framework of the quantum fission theory

1. the conservation of the axial symmetry at all stages of deformation motion of CFN [25] towards to it's scission point;
2. the coldness [26] of CFN at all fission stages after it's passage of the second fission barrier towards to it's scission point;

3. connected with named above coldness the conservation of the projection K of the spin J of CFN on the it's symmetry axis [26, 13, 16, 18, 20];

4. connected with named above coldness the necessary of taking into account only zero wriggling- and bending-vibrations [27] for the formation of angular and spin fission fragments distributions [23, 24];

5. the nonevoporational and nonadiabatical mechanism [28, 29] of the flight of prescission third particles;

6. for P-even T-odd asymmetries in angular distributions of prescission third particles for the ternary fission of nonoriented target-nuclei by cold polarized neutrons the spin density matrix of CFN must be constructed in the form, taking into account the interference of fission amplitudes of two different named above s-neutron resonances of the fissile compound nucleus with same and different spins J , J' and it's projections M, M' [15]:

$$\rho_{MM'}^{JJ'} = \frac{1}{2(2I+1)}\delta_{J,J'}\delta_{M,M'} + \frac{is_n}{2(2I+1)}A(J,J')\left[C_{J1M1}^{J'M'} + C_{J1M-1}^{J'M'}\right], \tag{4}$$

where $A(J,J')$ is defined as

$$A(J,J') = \delta_{J,J'}\left(\sqrt{\frac{J}{2(J+1)}}\delta_{J,J_<} - \sqrt{\frac{J+1}{2J}}\delta_{J,J_>}\right) - \sqrt{\frac{2J+1}{2J}}\delta_{J,J'+1} + \sqrt{\frac{2J+1}{2(J+1)}}\delta_{J,J'-1}. \tag{5}$$

7. the amplitude of nonperturbated by CFN rotation angular distribution of α-particle $W(\Omega_3)$ in the internal coordinate system (ICS) can be represented as

$$A(\theta') = \sum_l d_l Y_{l0}(\theta') = \sum_l \{d_l\} e^{i\delta_l} Y_{l0}(\theta'), \tag{6}$$

where θ' is angle between the direction of α-particle flight and CFN symmetry axis, $\{d_l\}$ and δ_l is the real main value and the phase of quantity d_l. Because the Coulomb parameter $\eta = \frac{2Ze^2}{\hbar v_\alpha} \gg l$, where quantity Z is connected with charges of light and heavy fission fragments, v_α is the α-particle relative velocity and l – the relative orbital moment of α-particle, it can be used the quasi-classical approximation, in which phase δ_l is independent from the α-particle orbital momentum l and has the form $\delta_l = \delta_0$. Then the amplitude $A(\theta')$ can be represented as

$$A(\theta') = e^{i\delta_0}\{A(\theta')\} = e^{i\delta_0}\sum_l \{d_l\} Y_{l0}(\theta'), \tag{7}$$

and the α-particle angular distribution $P(\theta')$ is defined as

$$P(\theta') = |A(\theta')|^2 = \{A(\theta')\}^2 \tag{8}$$

Then representing the amplitude $A(\theta')$ (7) as

$$A(\theta') = A^{ev}(\theta') + A^{odd}(\theta') \tag{9}$$

where the amplitude $A^{ev}(\theta')$ ($A^{odd}(\theta')$) is defined by the formula (7) with only even (odd) orbital moments. Since at the inversion of the α-particle wave vector $\mathbf{k}_3$ the distribution $P(\theta')$ transits to distribution $P(\pi-\theta')$, with the usage of formulae (8-9) it can be found:

$$\{A^{ev}(\theta')\} = \frac{1}{2}\left[\sqrt{P(\theta')} + \sqrt{P(\pi-\theta')}\right]; \quad \{A^{odd}(\theta')\} = \frac{1}{2}\left[\sqrt{P(\theta')} - \sqrt{P(\pi-\theta')}\right]$$
$$\{A(\theta')\} = \sqrt{P(\theta')}. \tag{10}$$

Using the normalized experimental angular distribution of α-particle $P(\theta')$ for ternary fission of target-nuclei ^{235}U by cold nonpolarized neutrons [8]:

$$P(\theta') = A\exp\left(-\frac{1}{2}\left(\frac{\theta'-\theta_0}{w}\right)^2\right), \quad (11)$$

where θ_0 = 81.65°, w = 11.57° , it can be calculated amplitudes $\{A^{ev}(\theta')\}$ and $\{A^{odd}(\theta')\}$, which presented on the Fig. 6, where lines 1, 2 and 3 correspond to $\{A(\theta')\}$, $\{A^{ev}(\theta')\}$ and $\{A^{odd}(\theta')\}$.

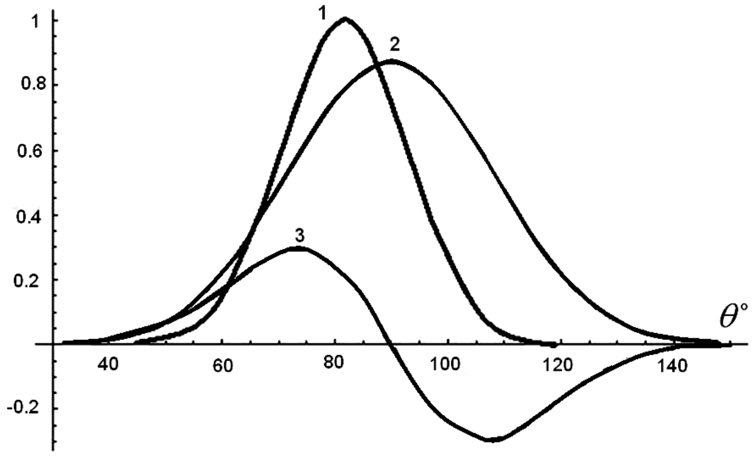

Fig. 6. Dependence of the amplitudes $A(\theta_\alpha)$ (line 1), $A_0^{ev}(\theta_\alpha)$ (line 2) и $A_0^{odd}(\theta_\alpha)$ (line 3) on the emission angle of the third particle.

From Fig. 6 it can be seen that the amplitude $|A^{ev}(\theta')|$ has the positive sign and it's maximal value prevailing the maximal value of quantity $|A^{odd}(\theta')|$ by factor 3.

3. T-ODD TRI- AND ROT-ASYMMETRIES FOR TERNARY FISSION AND CORIOLIS INTERACTION

The appearance of all types of T-odd asymmetries in the reactions of ternary fission of nonoriented nuclei-targets by cold polarized neutrons with emission of prescission α-particles can be connected [15, 19, 21-22] by the influence of the rotation CFN on the angular distributions of ternary fission products through the Hamiltonian H_{cor} of the Coriolis interaction consisting of two terms: the first term associated with the interaction of the total spin **J** of CFN with the relative orbital momentum **L** fission fragments and the second term associated with the interaction of the total spin **J** with the relative orbital momentum **l** of the third particle:

$$H_{cor} = -\frac{\hbar^2}{2\Im_\perp}\left(\left[J_+L_- + J_-L_+\right] + \left[J_+l_- + J_-l_+\right]\right), \quad (12)$$

where operators $J_\pm$, $L_\pm$ and $l_\pm$ are defined in ICS of CFN as

$$J_\pm = J_1 \pm iJ_2\,;\; L_\pm = L_1 \pm iL_2\,,\; l_\pm = l_1 \pm il_2 \quad (13)$$

and $\Im_\perp$ is the moment of inertia of the axial-symmetrical CFN for it's rotation around an axis perpendicular to CFN symmetry axis.

The action of the operators $J_\pm$, $L_\pm$ and $l_\pm$ on the functions $D^J_{MK}(\omega)$, $Y_{LK_L}(\Omega'_{LF})$ and $Y_{lK_l}(\Omega'_\alpha)$ is defined as:

$$\begin{aligned} J_\pm D^{J_s}_{M_s K_s}(\omega) &= \left[(J_s \pm K_s)(J_s \mp K_s + 1)\right]^{1/2} D^{J_s}_{M_s (K_s \mp 1)}(\omega); \\ L_\pm Y_{LK_L}(\Omega'_{LF}) &= \left[(L \mp K_L)(L \pm K_L + 1)\right]^{1/2} Y_{L(K_L \pm 1)}(\Omega'_{LF}); \\ l_\pm Y_{lK_l}(\Omega'_\alpha) &= \left[(l \mp K_l)(l \pm K_l + 1)\right]^{1/2} Y_{l(K_l \pm 1)}(\Omega'_\alpha). \end{aligned} \tag{14}$$

Since the Coriolis interaction is small, it can be taken into account in the first order of perturbation theory.
Using the formula:

$$Y_{L0}\left(\theta'_{LF_0,k_\alpha}\right) = \sqrt{\frac{4\pi}{2L+1}} \sum_{K_L} Y_{LK_L}\left(\theta'_{LF_0}\right) Y^*_{LK_L}\left(\Omega'_{k_\alpha}\right) \tag{15}$$

and the adiabatic approximation associated with the slowness of the rotation motion of CFN in comparison with it's nucleons internal motion the wave functions $\Psi^{J_s M_s}_{K_s}$ of the transitional fission states of CFN can be represented as

$$\psi^{J_s M_s}_{K_s} = \sum_c \sqrt{\frac{2J_s+1}{16\pi^2}} \sqrt{\frac{\Gamma_{cK}}{\hbar \upsilon_c}} \frac{e^{ik_c\rho}}{\rho^{5/2}} \left\{ D^{J_s}_{M_s K_s}(\omega)\chi_{K_s} + (-1)^{J_s+K_s} D^{J_s}_{M_s -K_s}(\omega)\chi_{\overline{K_s}} \right\} B^{(0)}(\Omega'_{LF}) A^{(0)}(\Omega'_3). \tag{16}$$

Because of the effect of the pumping of large values of fission fragments relative orbital moments associated with zero wriggling-vibrations of CFN the amplitude of the normalized fission fragments angular distribution in formula (16) coincides with the high degree of accuracy with the amplitude of the "spreaded" δ-function:

$$B^{(0)}(\Omega'_{LF}) = \frac{1}{\sqrt{2\pi}} \delta^{1/2}(\cos\theta'_{LF} - 1) = \sum_{L=0}^{L_m} b_l Y_{L0}(\Omega'_{LF}), \tag{17}$$

where

$$b_l = \left\{ \sum_{L=0}^{L_m} (2L+1) \right\}^{-1/2} \sqrt{2L+1}.$$

The influence of the polarized CFN rotation on the fission fragments angular distributions is defined by the term of the Hamiltonian of the Coriolis interaction H_{cor} (12), depending from orbital momentum *L*, and with the usage of formulae (15), (18) leads to the appearance of the amplitude $B^{(\text{cor})}(\Omega'_{LF})$ of fission fragments angular distribution in the first order of perturbation theory for H_{cor} having the form:

$$\begin{aligned} B^{(cor)}(\Omega'_{LF}) &= b(L_m) \sum_L \sqrt{L(L+1)} \left[Y_{L(-1)}(\Omega'_{LF}) - Y_{L1}(\Omega'_{LF}) \right] \sqrt{(2L+1)/4\pi} = \\ &= -2\cos\varphi'_{LF} \frac{d\delta^{1/2}(\cos\theta'_{LF} - 1)}{d\theta'_{LF}} \end{aligned} \tag{18}$$

Taking into account the similar transformations for the amplitudes $A^{(0)}(\Omega'_3)$ of the third particle angular distribution it can be obtain:

$$A^{cor}(\Omega'_3) = \sum_{l\geq 1} b_l \sqrt{l(l+1)} \left[Y_{l(-1)}(\Omega'_3) - Y_{l1}(\Omega'_3) \right] = -2\cos\varphi'_3 \sum_{l\geq 1} b_l \frac{dY_{l0}(\theta'_3)}{d\theta'_3}. \tag{19}$$

Then the components of the asymmetry coefficient (2) can be represented [24] as

$$D_{ROT}\left(\theta,\varphi\right)=\alpha_{ROT}\,\frac{d\left\{A^{(0)ev}\left(\theta\right)\right\}}{d\theta}\frac{\cos\varphi}{\left\{A^{(0)}\right\}}\Delta\theta, \quad (20)$$

$$D_{TRI}\left(\theta,\varphi\right)=\alpha_{TRI}\,\frac{d\left\{A^{(0)\mathrm{odd}}\left(\theta\right)\right\}}{d\theta}\frac{\cos\varphi}{\left\{A^{(0)}\right\}}\Delta\theta, \quad (21)$$

where

$$\alpha_{ROT}=\left(\sin\left(\delta_{sJ_s s'J_{s'}}+\bar{\delta}_{ev}-\delta^0\right)k^{ev}-\sin(\delta_{sJ_s s'J_{s'}})\right), \quad (22)$$

$$\alpha_{TRI}=\left(\sin(\delta_{sJ_s s'J_{s'}}+\bar{\delta}_{odd}-\delta^0)k^{odd}-\sin(\delta_{sJ_s s'J_{s'}})\right), \quad (23)$$

and the angle of turn out $\Delta\theta$ is represented as

$$\Delta\theta=-\omega\left(K_s,J_s,J_{s'}\right)\tau. \quad (24)$$

The effective angular velocity of rotation $\omega\left(K_s,J_s,J_{s'}\right)$ is determined by the relation of paper [24]:

$$\omega\left(K_s,J_s,J_{s'}\right)=\frac{\hbar p_n}{2\Im_0}g\left(K_s,J_s,J_{s'}\right), \quad (25)$$

where

$$g\left(K_s,J_s,J_{s'}\right)=\begin{cases}\dfrac{J_s\left(J_s+1\right)-K_s^2}{J_s} & \text{для } J_s=I+1/2\equiv J_>\\[2ex] -\dfrac{J_s\left(J_s+1\right)-K_s^2}{J_s+1} & \text{для } J_s=I-1/2\equiv J_<\end{cases}$$

$$g\left(K_s,J_<,J_>\right)=g\left(K_s,J_>,J_<\right)=\frac{K_s\sqrt{J_s^2-K_s^2}}{J_>} \quad (26)$$

On the basis of χ^2-method the formulae (20) – (21) were used in [21] for analysis of the angular dependences of the coefficients $D^{\exp}\left(\Omega_\alpha\right)$ for ternary fission reactions for target-nuclei ^{233}U, ^{235}U, ^{239}Pu and ^{241}Pu by cold polarized neutrons [2-5] and to calculate the values of D_{ROT} and D_{TRI} in (2). As can be seen from Figs. 2, 3 and 4 the angular dependences of experimental and theoretical coefficients $D^{\exp}\left(\Omega_\alpha\right)$ and $D\left(\Omega_\alpha\right)$ at the angle θ_3 are in good agreement one with another for the target-nuclei ^{235}U, ^{239}Pu and ^{241}Pu.

At the same time the angular dependence of the theoretical coefficient $D\left(\Omega_\alpha\right)$ for the target nucleus ^{233}U (see Fig. 1) corresponds to the similar dependence of experimental coefficient $D^{\exp}\left(\Omega_\alpha\right)$ in the $60° < \theta_3 < 110°$ range of angles, where the value of undisturbed experimental amplitude $A^0\left(\theta_3\right)$ is significantly differ from zero. The noticeable discrepancy between these coefficients is observed at ranges of angles $\theta_3 < 60°$ and $\theta_3 > 110°$. It is not excluded that this discrepancy is associated, on the one hand, with the inaccuracy of the definition [21-22] of the theoretical amplitudes $\left\{A_{ev}^{cor}\left(\theta_3\right)\right\},\left\{A_{odd}^{cor}\left(\theta_3\right)\right\}$ though amplitudes

$\frac{d\left\{A_{ev}^{0}\left(\theta_3\right)\right\}}{d\theta_3}$, $\frac{d\left\{A_{odd}^{0}\left(\theta_3\right)\right\}}{d\theta_3}$ and on the other hand, with possible inaccuracy in the definition experimental values of $D^{\exp}\left(\Omega_\alpha\right)$.

4. CONCLUSION

In the present paper the consideration of the T-odd asymmetries for prescission and evaporative third particles appearing in the ternary fission of the actinide nuclei by cold polarized neutrons has allowed to concretize the basic dynamic mechanisms for the binary and ternary nuclear fission, which determinate at the formation of these asymmetries. It has been shown that the considered T-odd asymmetries can be classified by taking into account the effects connected with interference of the fissile amplitudes of neutron resonances and the different influence in the common case of the appearance of the even and odd orbital moments of the third particles.

REFERENCES

1. P. Jessinger *et al.*, Nucl. Instrum. Methods A **440**, 618 (2000)
2. P. Jessinger *et al.*,, Nucl. At. Phys. **65**, 662 (2002)
3. A. M. Gagarski *et al.*, *Proceedings of the ISINN-14, Dubna, Russia,* 2006, 93 (JINR, Dubna, 2007)
4. F. Gonnenwein *et al.* Phys. Lett. B **652**, 13 (2007)
5. A. M. Gagarski *et al.*, *Proceedings of the ISINN-16, Dubna, Russia, 2008*, 356 (JINR, Dubna, 2009)
6. A. Gagarski *et. al.*, *Proceeding of 4-Internal Workshop*, 323 (Cadarache, France, 2012)
7. A. Gagarski *et. al.*, Phys. Rev. C **93**, 054619 (2016)
8. M. Mutterer, J.P. Theobald, *Nuclear Decay Modes* (Bristol: Inst Publ., 1996)
9. G. V. Danilyan, J. Klenke, V.A. Krakhotin, Phys. At. Nucl. **72**, 1872 (2009)
10. G. V. Danilyan, P. Granz, V.A. Krakhotin, Phys. Lett. B **679**, 25 (2009)
11. G. V. Danilyan, J. Klenke, V.A. Krakhotin, Phys. At. Nucl. **73**, 73 (2010)
12. G. V. Danilyan, J. Klenke, Yu. N. Kopach, Phys. At. Nucl. **77**, 715 (2014)
13. S.G. Kadmensky, Phys. At. Nucl. **65**, 1390 (2002)
14. S.G. Kadmensky, Phys. At. Nucl. **66**, 1846 (2003)
15. V. E. Bunakov, S. G. Kadmensky, Phys. At. Nucl. **66**, 1846 (2003)
16. S.G. Kadmensky, L.V. Rodionova, Phys. At. Nucl. **66**, 1219 (2003)
17. S.G. Kadmensky, L.V. Rodionova, Bull. Russ. Sci. Phys. **69**, 751 (2004)
18. S.G. Kadmensky, Phys. At. Nucl. **68**, 1968 (2005)
19. V. E.Bunakov, S. G. Kadmensky, S. S. Kadmensky, Phys. At. Nucl., **71**, 1887 (2008)
20. S. G. Kadmensky, L. V. Titova, Phys. At. Nucl. **72**, 1738 (2009)
21. D. E. Lyubashevsky, S. G. Kadmensky Bull. Russ. Acad. Sci.: Phys. **74**, 791 (2010)
22. S. G. Kadmensky, D. E. Lyubashevsky, L. V. Titova, Bull. Russ. Acad. Sci.: Phys. **75**, 989 (2011)
23. S. G. Kadmensky, D. E. Lyubashevsky, L. V. Titova, Bull. Russ. Acad. Sci.: Phys. **79**, 975 (2015)
24. S. G. Kadmensky, D. E. Lyubashevsky, V.E. Bunakov, Phys. At. Nucl., **77**, 198 (2015)
25. M. Brack, *et al.*, Rev. Mod. Phys. **44**, 320 (1972)
26. A. Bohr, and B. Mottelson, *Nuclear Structure* (NY, Amsterdam, 1969, 1975) V. 1, 2

27. J.R. Nix, W.J. Swiatecki, Nucl. Phys. A 71, 1(1965)
28. O. Tanimura, T. Fleshbach, Z. Phys. A **328**, 975 (1987)
29. S. G. Kadmensky, L. V. Titova, A. O. Bulychev, Phys. At. Nucl. **78**, 716 (2015).