COAXIAL FISSION FOR THREE COMPARABLE MASS FRAGMENTS AS A CONSEQUENCE OF THE COLLECTIVE MODEL OF NUCLEUS

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Abstract

The trajectory calculations for the fragments of spontaneous true ternary fission of the ^{252}Cf nuclei are performed, aimed at studying the effect of the collective rotation of the fragments on their angular distribution. The rotation arises at the moment of scission due to formation of spins in the fragments, in spite of conservation of zero total angular momentum. The conclusion is that the collinear flight of all the fragments of the spontaneous true ternary fission of ^{252}Cf, which is in the model on the prescission stage, survives at the final stages of spreading of the fragments. The results prove the experimental data obtained.

Introduction

The question of fission into three comparable fragments has a long, challenging and fascinating story. As distinct from traditional ternary fission, where emission of two massive fragments is accompanied with a ternary light particle, like an α particle, sometimes it is called true ternary fission (TTF). Strutinsky *et al.* were the first who proposed search for this process [1]. Attempts of creating a theory of TTF were undertaken by many theorists. Proceeding from typical initial conditions on the top of usual fission barrier, within the framework of the liquid drop model, Nix [2] demonstrated formation of a third very light fragment which arose between two other massive fragments in the case of very heavy fissioning systems with $A \gtrsim 300$. Family of shapes leading to fission into three massive fragments was deduced in Ref. [3, 4]. Legendre-polynomial expansion up to tenth order and more was usually exploited. In some papers, though, the role of hexadecapole deformation was underlined. The idea of generic mechanism of TTF was expressed in Refs. [5, 6, 7], in contrast to the consecutive one. It is suggested that TTF develops along a special dynamical path to which the nucleus enters at the very beginning of fission. The predetermining role belongs to the hexadecapole deformation, as the quadrupole deformation plays the leading role in habitual binary fission.

First experimental studies were undertaken, searching for this mode in fission of actinide nuclei by thermal neutrons [8] and α particles [9], in heavy-ion collisions [10], or spontaneous fission of ^{252}Cf [11]. Only upper limits of the probability of the processes were established at the level of 10^{-4} – 10^{-8} . It is worthy of noting that there was a tacit contradiction between theory and experimental search. From the theoretical point of view, linear form of a fissile nucleus is more favourable than a clover-leaf shape (*e. g.*

[5] and refs. cited therein). However, experimental efforts were mainly aimed at detecting fragments at approximately similar angles, i.e., $\sim 120°$. Based on general considerations, the experimenters likely believed that the mutual electrostatic repulsion could align the spreading angles. Solyakin et al. proposed the collinear mode of tripartition [12], when searching for TTF of ^{238}U by 1-GeV protons. This concept was most successfully realized in JINR experiments on FOBOS and mini-FOBOS setup [13, 14, 15]. Use of the missing-mass method in combination with net detectors led to conclusion that the collinear mode of TTF of ^{235}U and spontaneous fission of ^{252}Cf may be at the level of up to $10^{-4} - 10^{-5}$. At first sight, this mode is in contrast with ordinary ternary fission, where α particles or protons are emitted approximately perpendicularly to the fission axis. At the same time, there is a small fraction ∼10 percent of polar alpha particles, emitted along the fission axis (e.g. [16] and refs. cited therein). In Ref. [17] the authors doubted a possibility of a "perfectly" collinear flight in the case of three massive fragments. It follows from this qualitative consideration that the question of angular distribution of the fragments is of primordial interest. As we will see, the collinear character of fragment flight off follows the same principles the collective model of nuclear motion is founded on. As a result, the description turns out to be completely different in the cases of usual ternary fission accompanied by emission of an alpha particle, and TTF. In the first case, the two nascent massive fragments form an axially-symmetric core, in the field of which the alpha particle is formed and emitted. In the latter case, all three nascent fragments form the core, which may be axially-symmetric, if all three fragments are moving coaxially before separation, or not to be. These two possibilities have drastic consequences, as, according to the first principles, the axially-symmetric shape replies to the projection of the core angular momentum on the fission axis $K = 0$. Then the nascent fragments move co-axially till scission. Broken axial symmetry is associated with $K > 0$. In order to realize this possibility, some energy is needed for collective rotation around the fission axis. This leads to an effective increase of the fission barrier and related suppression of the fission probability. Actually, this consideration is founded on the same ground as the known Bohr's hypothesis [18, 19] about the predominance of a certain channel in photofission of ^{238}U. In the case of spontaneous fission of spinless nuclei of ^{252}Cf, considered herein, the condition $K = 0$ undoubtedly holds up to the first scission. Therefore, the nascent fragments move strictly co-axially on the fission axis.

In principle, the equality $K = 0$ does not forbid rotation in the plane of symmetry. Arising rotation of the fission axis perpendicular to the fission direction brakes picture of the co-axial fragment flight after scission, as this is shown below. This kind of the collective rotation is also absent in our case of spontaneous fission of ^{252}Cf yet. However, also in this case, the co-axial symmetry is broken at scission [20]. In the case of binary fission of actinide nuclei, the mean value of the fragment spin is ~ 7 to 8 [21, 22]. Moreover, the arising rotational moment is likely directed perpendicularly to the fission axis [20]. Therefore, one may expect that the total angular momentum of the relative motion of all three fragments of TTF may achieve as much as $L \sim 10 - 20$ and more. Conserving after scission, such a rotation might brake the collinear scenario of fragment flight off.

A similar effect is known in fission of nuclei with spins different from zero. Then the angular momentum of such a collective rotation before scission goes over the transverse velocity of the fission fragments after scission. In the case of fission with polarized neutrons, this gives rise to the known ROT effect [23]. It manifests as correlation of the direction of emission of alpha particle with the directions of heavy fragment and the spin of the fissile nucleus. The ROT effect is observed by the difference method in fission by polarized neutrons [23, 24]. As follows from the above consideration, the collective angular momentum arising at scission may be several times more that the spin of the fissile nucleus. However, it does not contribute to the ROT effect because of angular averaging: there is no correlation of the L direction with the spin of the fissile nucleus. But in TTF, where each event is detected independently of others, the presence of the collective moment L could easily prove itself in violation of the collinearity of the scattering of the fragments. We will proceed from a strongest assumption that the transverse spin of each fragment may be as high as 7 and even more, so that the total transverse collective spin after scission may reach values of $L \approx 20$. Such big conceivable values of the collective spin are compensated by the sum of the spins of each of the fragments, which is of the opposite sign. Of course, if the transverse collective spin is smaller, or even close to zero, all the more the trajectories of the fragments hold collinear. Let us consider the question in more detail.

Calculation formulas

Numerical simulation of trajectories of representative fragments is a classical method. Its applicability follows a known fact that wavelength is much smaller than the nuclear fragment size. Such calculations were found to work well for description of α data in ternary fission (e. g.[25]), specifically, of the ROT effect [26]. Representative trajectories are simulated in the next section by solving the Newton equations of motion with initial conditions of position and velocity for each fragment at scission. We consider the generic mechanism of the TTF, when the both scissions occure nearly simultaneously within a narrow zone comparable to R_0. For simplicity, it is assumed a spherical shape of the fragments. The choice of the initial conditions is presented in Fig. 1. Let the fission axis coincide with the quantization axis z at the moment of scission. Denote the extreme side fragments with indices 1 and 2, and the mean fragment as No. 3. In view of the axial symmetry of the problem, let x be the transverse direction axis. The atomic and mass numbers of the fragments are assumed to be Z_i and A_i, respectively, $i = 1$, 2 and 3, with the distances r_{12}, r_{23} and r_{13} between the fragments. The positions of the fragments must be selected, baring in mind their future total kinetic energy (TKE), which must not exceed reaction heat Q. For the parameterization purposes, the total Coulomb energy of the fragments is minimized, based on the position of the second fragment at fixed distance $D = r_{12}$ between the extreme fragments:

$$r_{23} = D\frac{\sqrt{Z_2}}{\sqrt{Z_1} + \sqrt{Z_2}}\,. \qquad (1)$$

Hence, the initial positions of all three fragments are fixed by the single parameter D defined by the TKE value T, $T \leq Q$:

$$T = (\frac{Z_1 Z_2}{r_{12}} + \frac{Z_1 Z_3}{r_{13}} + \frac{Z_2 Z_3}{r_{23}})e^2\,. \qquad (2)$$

Initial conditions for the trajectory simulations. V_1, V_2 and V_3 are the transverse velocities to the

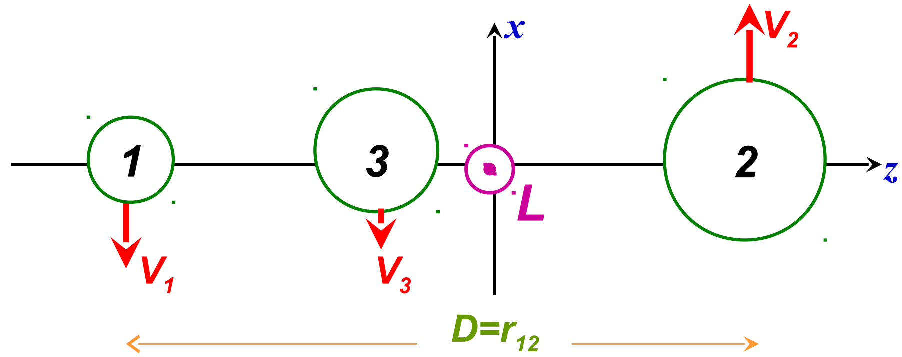

Figure 1: Initial conditions for the trajectory simulations. V_1, V_2 and V_3 are the transverse velocities to the fission axis of the fragments $1-3$, which comprise the total relative angular momentum of the collective rotation of the fragments (directed towards us). D is the distance between extreme fragments.

fission axis of the fragments $1-3$, which comprise the total relative angular momentum of the collective rotation of the fragments (directed towards us). D is the distance between extreme fragments. A small possible initial velocity of the fragments in the z direction is not important for the present purposes. In accordance with what is said in the Introduction, in order to calculate the initial velocity of the fragments in the transverse direction, we use the value of their assumed total angular momentum and the position of each fragment on the axis relative to their center of gravity. Let us designate the masses of the fragments and their positions along the axis of fission as M_1, z_1, M_2, z_2 and M_3, z_3, respectively. Center of gravity of the fragments, determined during fission, is set as

$$\zeta = (M_1 z_1 + M_2 z_2 + M_3 z_3)/M \,, \tag{3}$$

where $M = M_1 + M_2 + M_3$. Total angular momentum of the fragments is described by the equation

$$\omega \left[M_1 (z_1 - \zeta)^2 + M_2 (z_2 - \zeta)^2 + M_3 (z_3 - \zeta)^2 \right] = L\hbar \,, \tag{4}$$

and the initial transverse velocity of fragment i is

$$V_i = \omega (z_i - \zeta) \,. \tag{5}$$

The macroscopic—microscopic landscape of the potential deformation energy was calculated in Ref. [27] for the case of TTF of ^{252}Cf. It suggests the following mode as a likely candidate:

$$^{252}\mathrm{Cf} \to {}^{132}\mathrm{Sn} + {}^{48}\mathrm{Ca} + {}^{72}\mathrm{Ni}, \qquad Q = 251\,\mathrm{MeV} \,. \tag{6}$$

The Q value in fission (6) was calculated, using AME2012 atomic mass evaluation [28]. The presence of two magic or semimagic fragments in the final state provide a great released energy Q. The situation is like in three-partition of the atomic clusters of $_{27}Na^{+++} \to 3\ {}_9Na^+$ into three magic clusters of $_9Na^+$ [5]. The final TKE values of the fragments depend on the scission configuration: position of the fragments, thickness of the necks. Deformation of the fragments takes a part of the released energy, which is subtracted from the Q value. We will consider various representative TKE, and total angular momenta L. We will differ the extreme fragments as light and heavy ones, and the smallest fragment which we put in the middle we will call the ternary one.

Calculation results

In the landscape of the potential energy in Ref. [27], pronounced valleys favorable for ternary fission were found. One of they, which may be related with channel (6), lies after a saddle point at $R_{12} \approx 3R_0 = 22$ fm, where R_0 is the radius of the mother nucleus. At this distance, formation of the final fragment starts. This is close to the scission range in the case of binary fission, where the scission distance is approximately twice as large as the total radius of the both fragments. The valley presents a good opportunity for scission and separation of all three fragments somewhere at $r_{12} \gtrsim 30$ fm. Indeed, the TKE value $T = Q$ would be achieved if scission occurred at $r_{12} = 25.56$ fm. In practice, part of the released energy is stored in the deformation energy of the fragments, while scission occurs at a larger distance. Baring this in mind, we varied the parameter $D = r_{12}$ in the range up to 40 fm. Experimental results [15] confirm such an expectation. The results of the trajectory simulation are presented in Fig. 2 and Table 1. The calculated kinetic energies of each of the fragments, together with their TKE, are presented in Fig. 2 *versus* the distance between the extreme heavy and light fragments D at scission. All the energies smoothly decrease with increasing D, with TKE changing from $T = Q = 251$ MeV for $D = 25.6$ fm down to $T = 160$ MeV for $D = 40$ fm. As well as in ordinary binary fission, the heavy fragments are produced with lower kinetic energies. We note a characteristic feature of TTF displayed in Fig. 2: the ternary fragments, which are formed between the heavy and light ones, turn out to be very slow, with the kinetic energies of approximately 5 MeV. This is 15 – 20 times as small as the energies of the main fragments. This is in accordance with ref. [29].

The results concerning the angular distribution of the fragments are presented in Table 1. As a consequence of rotation of the fission axis, neither of the fragments continues to move along the z axis anymore, if $L \neq 0$. This is the same phenomenon which causes the ROT effect. For the configuration presented in Fig. 1, where the momentum L is directed towards the reader, the light Ni fragment goes below the z axis. In turn, the heavy Te fragment goes upwards. And the asymptotic rotation angle appears to be close to the value which can be expected, based on the value which is observed and reproduced by numerical simulations for the ROT effect. Thus, it comprises $\sim 2°$ for $L = 20$, that is about 0.1 degree per unit angular moment. The folding angle Θ between the fragments remains 180° in the case of binary fission. In our case of TTF, however, it changes, depending on the L value, diverging by 1 – 2 degrees from 180°.

The values of the angle Θ between the asymptotic directions of the two main fragments are presented in the Table for various L values. They are close to 180°. The ternary and the light fragments always fly in the same direction. The angle of divergence Φ between them is also presented against the value of the total transverse angular momentum L, which was varied in a wide range $0 \leq L \leq 20$. All three fragments remain in the same plane, wherein the projections of the velocities of the two fragments, the light and the ternary one, per axis perpendicular to the direction of emission of the heavy fragment have opposite signs. Flight off of the ternary fragment is equally probable into the upper and lower half-planes. From the presented results it follows that two fragments moving in the same direction diverge within one-two degrees at most, for all the considered L values. Such a divergence can be prettily neglected in the first approximation, at least in the conditions of the conducted experiments on FOBOS and

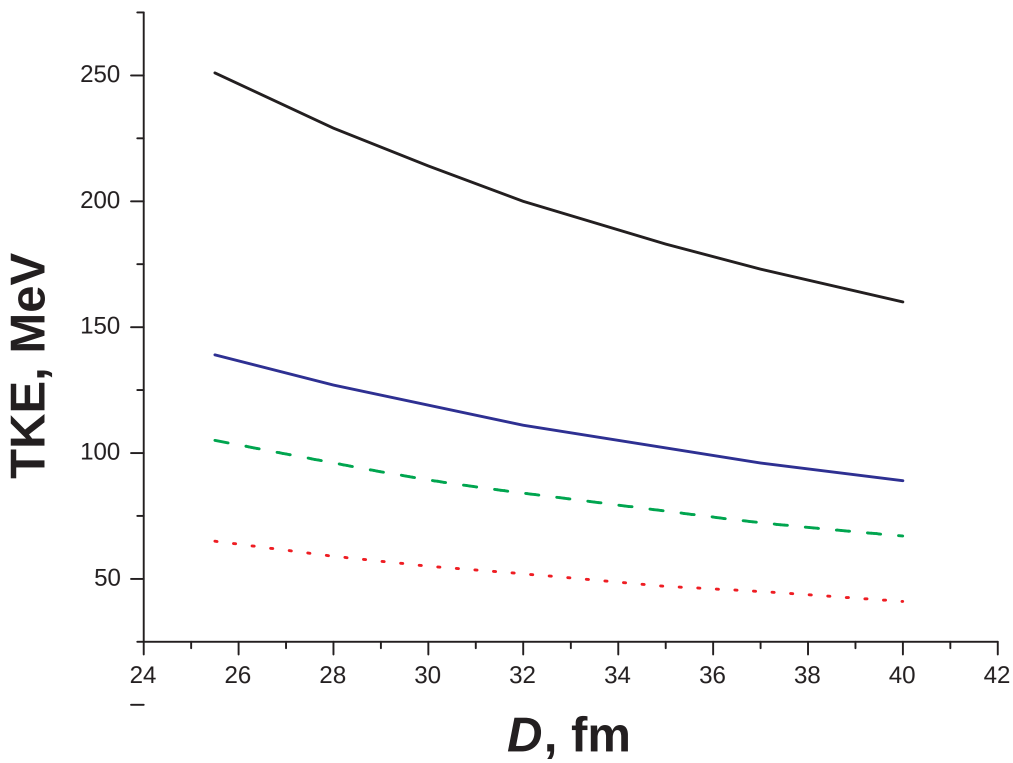

Figure 2: Kinetic energies of the fragments and their TKE values against the scission point D: fragments 1 and 2 — lower full and dashed lines, respectively, dotted line — the energy of the ternary fragment, scaled by a factor of 10, and full upper line — the total kinetic energy.

mini-FOBOS. This quite justifies the observed picture of collinear flight of the fragments in the TTF.

Conclusion

It follows from the considered model of scattering of the fragments of TTF that the approximately collinear picture of spreading of all the three fragments is most probable. In the case of $K = 0$, the reason is axially-symmetric shape of the fissile nucleus on its path towards fission. Because of the axial symmetry, there is a sole way of formation of the fragments, when they remain co-axial till scission. This is a remarkable illustration of the collective Bohr's model. Before, the Bohr's hypothesis marked application of the principles of symmetry in fission, which are laid in the base of the collective model. The hypothesis also works in the case of fission of ^{235}U by thermal neutrons, in which case the compound nucleus is characterized by full chaos over the K values: all the possible K values from 0 to 4 become equally probable due to the Coriolis mixing [30]. Also in this case, most probable channels with a certain K turn out to be those which reply to minimal energy over the fission barrier. The barrier works as the filter [30]. However, the angular distribution becomes more complicated because of the chaos on the stage

Table 1: Calculated angular distributions of the fragments of true ternary fission of ^{252}Cf (6) *versus* the scission point D and the relative angular momentum L in the c. m. system. Θ is the folding angle between the directions of the heavy and light fragments, Φ — the divergence angle between the light ^{72}Ni and ternary ^{48}Ca fragments

D, fm	L	Θ°	Φ°
25.6	5	179.9	0.7
	10	179.8	1.4
	15	179.7	2.1
	20	179.6	2.8
30	5	179.9	0.6
	15	179.7	1.9
35	5	179.9	0.6
	15	179.7	1.8
	20	179.6	2.4

of compound nucleus. In view of the results obtained above, the true ternary fission presents a bright example where the principles of symmetry comprising the foundation of the collective model, and specifically the Bohr's hypothesis concerning fission, manifest themselves in full shine.

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