TWO METHODS OF THE DETERMINATION OF THE PARITIES OF LOW-LYING STATES IN ^{159}Gd FROM ANALYSIS OF THE γ-RAY INTENSITIES FROM REACTION ^{158}Gd(n$_{res}$,γ)^{159}Gd

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ABSTRACT

Energy levels and transitions in ^{159}Gd were studied by means of radiative capture of resonance neutrons at 12 isolated resonances of ^{158}Gd. The time-of-flight technique was used on an enriched target at the IBR-30 reactor at JINR Dubna. A total of 80 primary gamma-transitions were recorded and their absolute intensities were determined resulting in the observation of $1/2^{\pm}$, $3/2^{\pm}$ levels up to 2.4 MeV. Parities of found levels were recalculated using two methods: the first method consists in analyzing of intensities averaging in 12 resonances and in the second method individual intensities are analyzed. The second method is described for the first time.

1. INTRODUCTION

We extensively studied spectroscopic properties of ^{159}Gd by resonance neutrons [1,2], by neutrons from neutron filters [3], by thermal neutrons and from (d,p) and (d,t) reactions. Full information was published in [4].

In [2] together with the other information the determination of parities of low lying levels of ^{159}Gd was obtained. Due to big volume of information in that work description of the method of this determination is very short. In this work we must to give more detailed description of procedure of parity determinations. During preparation of this work we recalculated all data and there are some differences between results of previous work ([2], Table 3) and new results. Analysis of these differences will be done later.

2. EXPERIMENT

Energy levels and transitions in ^{159}Gd were studied by means of radiative capture of resonance neutrons at 12 isolated resonances of ^{158}Gd. The time-of-flight technique was used on an enriched target at the IBR-30 reactor at JINR Dubna. A total of 80 primary gamma-transitions were recorded and their absolute intensities were determined. Absolute intensities of 80 transitions in 12 resonances of ^{158}Gd are presented in [2] (Table 1).

As ^{158}Gd is even-even nucleus all resonances in measured range of neutron energy have $J^{\pi} = 1/2^{+}$. Primary γ-rays usually are E1 (transitions on the levels with the

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opposite parity) and M1 (transitions on the levels with the same parities). Levels on which transitions pass have J=1/2, 3/2 with parity + if transition is M1 or - if transition is E1. M1 transitions have energy dependence E_γ^3 and dependence of E1 transitions is more sharply namely about E_γ^5. It is known that intensities of E1 transitions in measured range of energies are in 5-7 times more than of M1 transitions. From spectroscopic data 8 M1 transitions and 9 E1 transitions are known.

3. TASK OF PARITY DETERMINATION

Using the difference in intensities of E1 and M1 transitions we formulated and decided quantitative conditions of parity determination of low lying levels in ^{159}Gd. Two methods of the parity determination were used by us: the first method consists in analyzing of intensities averaging in 12 resonances and in the second method individual intensities are analyzed. The second method is described for the first time.

3a. The first method

80 intensities averaged in 12 resonances were obtained. From these data dependence E_γ^3 was excluded and reduced data were normalized on mean value of reduced intensities of 8 known M1 transitions. Here formula of reducing is shown:

$$\xi_{\gamma f} = \{< I_{\gamma\lambda f} >_\lambda / E_\gamma^3\} / \{<< I(M1)_{\gamma\lambda f} >_\lambda / E_\gamma^3 >_{M1}\}, \qquad (1)$$

where λ is sign of resonance and f is sign of final state.

Values $\xi_{\gamma f}$ of 80 transitions are presented in Fig. 1. There are two curves and straight line in this Figure: curves are dependences of E1 transitions (E_γ^5) which normalization was from 9 known E1 transitions - upper curve and from 8 known E1 transition - lower curve. Straight line is mean of 8 known M1 transitions, its y = 1.

As point about energy 5400 keV (transition 5384.70 keV) is very high two curves for E1 normalizing are plotted: upper curve with use this point and lower curve is without this point. Later meaning of these two curves will be discussed.

It is clearly seen that intensities of E1 transitions are good separated from intensities of M1 in range 4700-6000 keV. This fact gives opportunity to use intensities for separation all intensities in two groups. For this we use statistical properties of intensities. It is known that intensities of transitions from resonances of one type to the levels with certain J^π follow Porter-Thomas (PT) law of distribution [5]. This distribution is from class of χ^2 distributions, namely χ^2 distribution with ν=1. Mean value of intensities from 12 resonances will obey χ^2 distribution with ν=12. To determine that some transition is E1 we must show that it can not be from distribution which describe M1 transitions.

For deciding if point of examined transition obeys or not obeys analyzed distribution the confidence level must be chosen. In our works we use two confidence levels: 1% and 0.1%. For these two confidence levels critical values must be found. These are values of x integrals to which from the left or from the right are equal to confidence level. When event falls in the piece p1 with confidence level 0.1% this hypothesis is abandoned hardly and result of other hypothesis is set without parentheses and when event falls in the piece p2 with confidence level 1% other hypothesis is set in parentheses. We shell use term reability which determines integral of the distribution from the left to the tested point. This value will be close to 0 if point is to the left from maximum of the distribution or close to 1 if point is to the right from maximum.

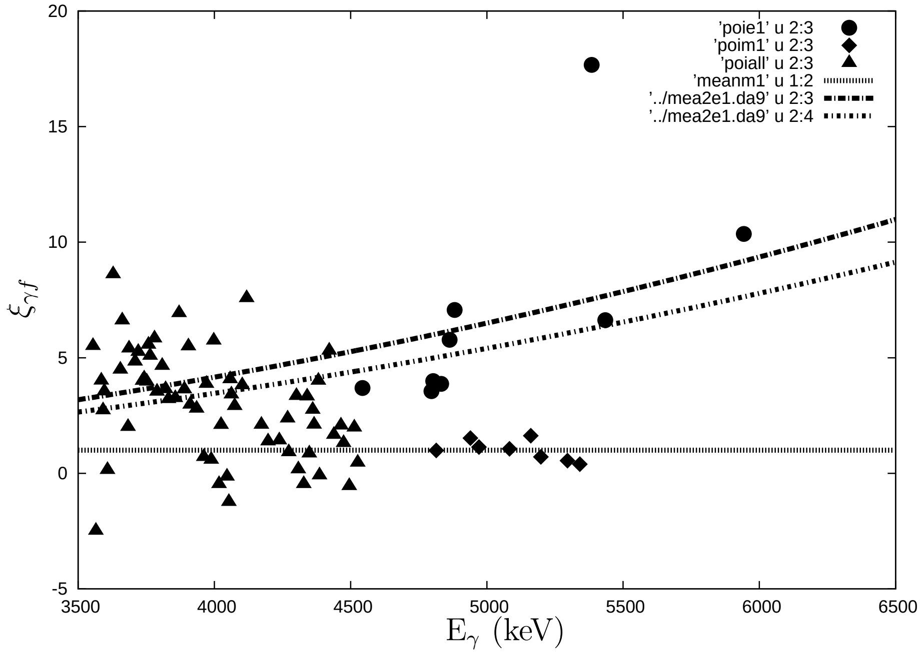

Fig. 1. Values $\xi_{\gamma f}$ of 80 γ-transitions (points), 2 curves – dependences of E1 and stright line – dependence of M1; ● –known E1, ♦ – known M1, ▲ – unknown parities.

We must create statistical distribution which picture this hypothesis and see how point of examined transition is far from the center of distribution. As distribution is normalized on 1 probability to appear accidentally in any piece of this distribution is equal to area of distribution within this piece. For events far to the left and far to the right from the center of distribution probabilities are integrals from the beginning of distribution to this event for events far to the left and from this event to the end of distribution for events far to the right. An example of arbitrary distribution is shown in Fig.2.

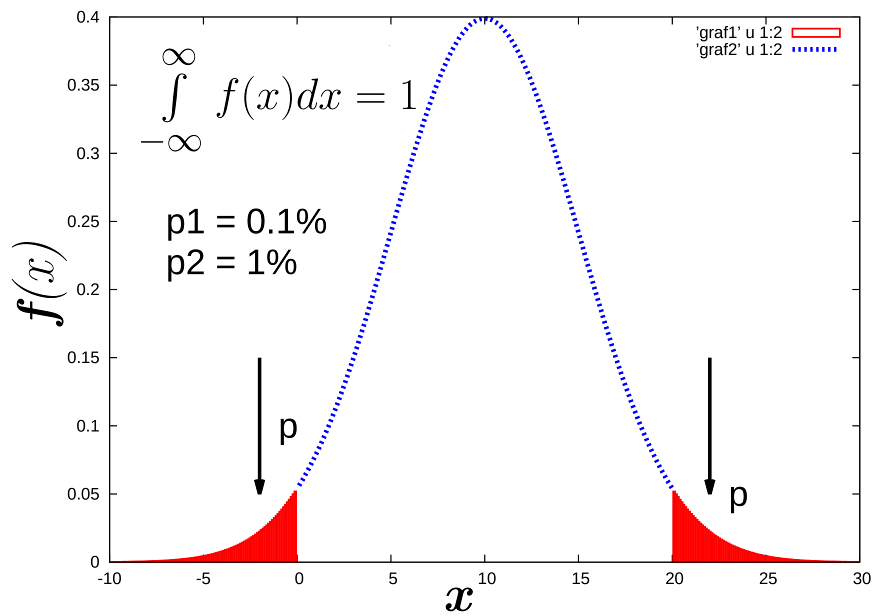

Fig. 2. Arbitrary function $f(x)$ normalyzed on 1

It was said that mean value of intensities from 12 resonances will obey χ^2 distribution with ν=12. But apart of this distribution each intensity of transition has its statistical error which obeys Gaussian distribution. Hence χ^2 distribution with ν=12 must be convoluted with Gaussian with individual statistical error of each intensity.

To find E1 transitions it is need:

- to normalize all experimental intensities on mean M1 value;
- to obtain χ^2 distribution with ν=12;
- to make convolution of this distribution with Gaussian which is individual for each point;
- to find critical values for two confidence levels;
- to find value reability which will be integral from the left, this value will be close to 1 as E1 transitions are greater than M1;
- to compare this value with two critical values.

To find M1 transitions we must compare intensities which are normalized on mean of E1 transitions after excluding of energy dependance of these transitions with χ^2 distribution. For this dependence of E1 transitions must be eliminated. In [2] two dependences of E1 transitions were discussed: Lorencian and Markushev-Furman model. But in this work we use simple dependence E_γ^5 because this dependence is more sharply than others and this gives more reliable results in energy range from 3.5 to 4.5 MeV as curve E_γ^5 is more near to M1 points.

Now we must discuss about two curves in Fig.1. Upper curve is obtained with use point 5384.70 keV. During analysis of points with normalizing on E1 with lower curve this point was very big and it was determined as doublet. At next doublets will be marked as D or (D). In view of this upper curve did not used and all calculations were made with lover curve. Using method described upper points which are lover than critical values can be determined as M1 or (M1) and points which are higher than critical values with integrals near 1 can be determined as D or (D).

3b. The second method

This method consists in analysis of absolute intensities of each transition from resonances to each level. As number of resonances is 12 we have twelve distributions for each level. Statistical distribution of these values is PT distribution. This distribution must be convoluted with Gaussian from the statistical error of transition. By such way twelve values of reability can be obtain. These values must be analyzed. They represent 12 points on [0,1] cut. Now we try to obtain quantitative conditions to refuse hypothesis that twelve transitions on one level belong to obtained distributions.

Divide [0:1] cut by point on two pieces p and q. We know that p is small part of cut [0:1].

$$p + q = 1$$

From 12 resonances we have 12 independent probabilities and whole probability is:

$$(p + q)^{12} = 1$$

It is Newton's binomial. Reveal parenthesis:

$$p^{12} + 12p^{11}q + 66p^{10}q^2 + 220p^9q^3 + 495p^8q^4 + 792p^7q^5 + 924p^6q^6 + 792p^5q^7 + 495p^4q^8 + 220p^3q^9 + 66p^2q^{10} + 12pq^{11} + q^{12} = 1$$

Now analyze some member of this expression, for instance, the forth: $220p^9q^3$. This member is the probability that 9 points fall to part p. If we solve the equation $220p^9q^3 = 0.001$ or $220p^9q^3 = 0.01$ we can obtain critical values which say that probability to have 9 point in piece p is 0.1% or 1%.

By such way we can obtain all 12 pairs of critical values for all members with p of this expression. Now for all 80 transitions it is need to do convolutions of one function of PT with the individual Gaussian from experimental errors and to find set of 12 reability values. These data can be compared with critical values.

4. CALCULATIONS

We must:

- to obtain Gaussian, χ^2 distribution with ν=1, χ^2 distribution with ν=12;
- to make convolution of χ^2 distributions with Gaussians which are individual for each intensity;
- to find critical values for two confidence levels for χ^2 distribution with ν=12;
- to find reability values for ν=12;
- to calculate critical values for 12 pairs of confidence levels for ν=1;
- to analyze all data and obtain parities by two methods.

It is seen in Fig. 2 that bounds of p pieces are set far from maxima of distributions and integrating must be to the area with small values of functions.

For better precision all calculations were made in Fortran codes with variables REAL*16. Calculations with such variables give precision of results with about 33 right signs.

For making of convolution all curves were replaced by piecewise continuous polynomials of the third power throw each 4 points of x coordinates with different steps which were determined as gave high precision of replacing. Precision of replacing was tested by integrating of curves and comparing of integrals with 1. In Table 1 parameters of fit of some functions by polynomials are shown: N – numbers of points of x coordinates, Area – integrals and NR – number of right signs in integrals obtained. It is seen that precision depends on number of points x.

Table 1. Parameters of polynomials for 4 functions: N – number of points x, Area – integral, NR – number of right signs in integrals

Function	N	Area	NR
Gaussian	2031	0.999999999897369E+00	9
χ^2 with $\nu = 1$	1463	0.999999991390689E+00	8
χ^2 with $\nu = 2$	5594	0.999999999999126E+00	12
χ^2 with $\nu = 12$	1950	0.999999998204125E+00	8

Function χ^2 with $\nu = 2$ is very simple function: $exp(-x)$. It was not used and provides as example. The most computer time is spent for convolutions. In [2] these calculations were provided in old system UNIX or SANOS and time of calculations was limited. Because of this calculations were made with smaller precision of convolutions: namely, 5 or 6 signs.

Now calculations were made on modern desktop computer and 8 signs of precision were achieved. This is source of differences between old and new results which will be discussed later. The most time was spent in method 2 as there must be 12 convolutions for each of 80 levels.

Borders of reability for 4 confidence levels were calculated and are shown in Table 2. These borders were tested by Monte-Carlo simulation. In Monte-Carlo 12 random numbers were generated and events which fall to pieces with borders from Table 2 were added in four counters. Results are shown in Table 3.

Table 2. Borders of reability for 4 confidence levels (Led) at the end of the cut [0,1].

Led n	20%	10%	1%	0.1%
1	0.97893530	0.99077201	0.99915892	0.99991661
2	0.91343027	0.94958514	0.98684847	0.99602932
3	0.83148438	0.89147431	0.95961344	0.98253584
4	0.74358207	0.82471770	0.92095554	0.95900744
5	0.65528965	0.75295264	0.87335509	0.92642683
6	0.57046992	0.67804360	0.81823516	0.88558257
7	0.49108195	0.60097700	0.75631922	0.83685452
8	0.41670281	0.52218020	0.68779033	0.78017426
9	0.34546533	0.44160458	0.61227202	0.71491122
10	0.27519742	0.35861242	0.52855957	0.63952595
11	0.20343767	0.27143469	0.43368423	0.55049646
12	0.12551473	0.17459582	0.31870794	0.43765867

Table 3. Monte-Carlo test of 4 borders of confidence levels (Led). 1 billion of 12 random numbers was generated.

Led n	20%	10%	1%	0.1%
1	199988356	99997723	9999206	1000541
2	200003147	99993970	10001449	1001716
3	199990133	100008878	9997067	1000800
4	199982567	100009826	9999704	999397
5	199995607	100006117	9996229	1001007
6	199991386	100020255	10002878	1001937
7	200022638	100009744	9999832	1002330
8	200021065	100013733	9995901	1001405
9	200017754	100004531	10001494	1000768
10	200010661	99996432	10001575	1000500
11	200009661	100005569	9998226	1000163
12	200001707	100005309	10003698	1000244

5. RESULTS AND DISCUSSION

Results of the determination of parities are lead in Table 4. This Table is similar to Table 3 in [2]. Only when in the second column of Table 3 in [2] f_γ are listed here in column 5 values $\xi_{\gamma f}$ from the expression (1) are lead and when in that Table there was column Resonance here number of points on [0,1] cut for which appropriate confidence level was performed are listed. The more of these groups results are reliable.

Now results which are worse than in [2] will be listed. As for transition 4474.4 keV in [2] was parity (+) now parity is not determined. For transitions 4348.4, 4308.0 and 4053.5 keV results from + became (+).

For some transitions now results are better: for 4381.7 and 3720.6 keV results are changed from (-) to - without parenthesis. In Table 3 in [2] there are 3 columns with the determination of XL. For 5384.7 keV in these columns there are E1 determinations. Now all 3 values of XL of transition 5384.7 keV are changed from E1 to D. This indicates probability of non-statistical big intensity of this transition. For transitions 3780.4, 3720.6, 3662.1 and 3655.0 keV values of XL became (D).

These results demonstrate importance of precision of calculations for more reliable determination of parities.

Now analysis of differences in parity determination by two methods will be obtained. Schematic comparison of results is shown in Fig. 3.

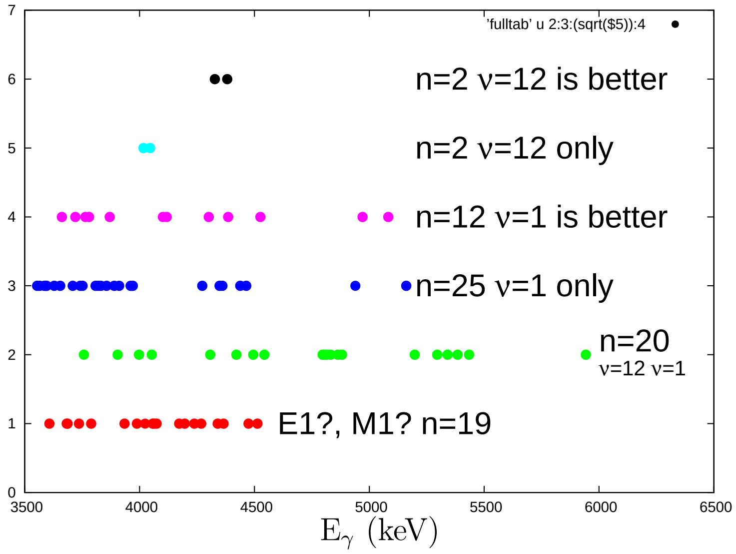

Fig. 3. Scematical show of the differences of results of the determination of parities by two methods (method 1 and method 2).

In this scheme is seen that for 19 transitions parities were not determinated. For 20 transitions determinations by two methods coincide. For 25 transitions determinations are obtained only by the second method and for two transitions determination by the second method is more precise. For two transitions determination is obtain only by the first method and for two transitions this method gave more precise result. This demonstrate that method 2 is more hard criterion then method 1.

There are many D and (D) determinations for transitions in the region 3500 - 4500 keV. This can demonstrate that E_γ^5 is very sharp and Lorencian or Markushev-Furman model may be more real.

Table 4. Final assignments of transition multipolarity XL are based on the analysis of analytically averaged (a) and of individual (b) γ-ray intensities. All observed levels have either spin 1/2 or 3/2 with parity as indicated. Assignments are made within 0.1% significance level (assignments within 1% are indicated in parenthesis)

E_γ [keV]	ΔE_γ	$\xi_{\gamma f}$	$\Delta\xi_{\gamma f}$	Groups[a]	Multipolarity[b]			E_f [keV]	j^π $\frac{1}{2}^\pi, \frac{3}{2}^\pi$
					$XL(a)$	$XL(b)$	XL		
5943.0	0.2	10.4	0.3	1 2 3 4 5 6 7 8 9 10 11	E1	E1	E1	0.0	-
5434.7	0.2	6.6	0.3	1 2 3 4 9 10 11 12	E1	E1	E1	508.3	-
5384.7	0.2	17.7	0.6	2	D	D	D	558.3	-
5341.1	0.3	0.4	0.3	9 10 11 12	M1	M1	M1	601.9	+
5295.9	0.5	0.6	0.3	11 12	M1	M1	M1	647.1	+
5198.1	0.2	0.7	0.3	10 11 12	M1	M1	M1	744.9	+
5161.0	0.5	1.6	0.3	11		M1	M1	782.0	+
5083.0	0.6	1.1	0.3	11	(M1)	M1	M1	860.0	+
4971.0	1.3	1.1	0.5	11	(M1)	M1	M1	972.0	+
4939.7	1.0	1.5	0.4	12		(M1)	(M1)	1003.3	(+)
4881.7	0.3	7.1	0.5	1 2 3 4 5 6 10	E1	E1	E1	1061.3	-
4863.5	0.2	5.8	0.4	1 2 3 4 5 6 7	E1	E1	E1	1079.5	-
4832.2	0.3	3.9	0.5	3	E1	E1	E1	1110.8	-
4814.4	0.3	1.0	0.4	2 10	(M1)	(M1)	(M1)	1128.6	(+)
4803.3	0.2	4.0	0.5	3 4 5 6	E1	E1	E1	1139.7	-
4796.9	0.2	3.5	0.5	2 3 4 5	E1	E1	E1	1146.1	-
4543.7	1.1	3.7	0.6	7 8 9	E1	E1	E1	1399.3	-
4526.1	1.3	0.5	0.6	11	(M1)	M1	M1	1416.9	+
4513.4	1.2	2.0	0.6					1429.6	
4495.2	1.3	-0.6	0.6	3 5 12	M1	M1	M1	1447.8	+
4474.4	1.2	1.3	0.6					1468.6	
4464.7	0.3	2.1	0.6	5		(E1)	(E1)	1478.3	(-)
4438.6	0.8	1.7	0.6	10		(M1)	(M1)	1504.4	(+)
4421.7	0.5	5.3	0.7	1 4 5 6 7 8	E1	E1	E1	1521.3	-
4385.9	1.0	-0.1	0.8	10	(M1)	M1	M1	1557.1	+
4381.7	0.5	4.0	0.8	1 6 7 8 9 11	E1	(E1)	E1	1561.3	-
4366.0	1.4	2.1	0.8					1577.0	
4360.7	0.9	2.8	0.8	5 9 10 11		(E1)	(E1)	1582.3	(-)
4348.4	1.5	0.9	0.8	10 11 12		(M1)	(M1)	1594.6	(+)
4340.4	1.5	3.3	1.4					1602.6	
4327.9	0.6	-0.5	0.8	6 7 9 10 11 12	M1	(M1)	M1	1615.1	+
4308.0	1.0	0.2	0.8	9 10 11	(M1)	(M1)	(M1)	1635.0	(+)
4301.4	0.8	3.3	0.8	9	(E1)	E1	E1	1641.6	-
4273.2	1.5	0.9	0.9	10		(M1)	(M1)	1669.8	(+)
4268.8	0.8	2.4	0.9					1674.2	
4238.4	1.0	1.4	0.8					1704.6	
4197.0	1.0	1.4	0.9					1746.0	
4172.7	0.9	2.1	0.9					1770.3	
4118.5	0.4	7.6	1.0	10	E1	D	D	1824.5	-
4102.1	1.5	3.8	1.0	10	(E1)	E1	E1	1840.9	-
4074.5	0.3	2.9	1.0					1868.5	
4062.3	1.0	3.4	1.2					1880.7	

(cont.)

Table 4 (continued)

E_γ [keV]	ΔE_γ	$\xi_{\gamma f}$	$\Delta\xi_{\gamma f}$	Groups[a]	Multiplicity[b] $XL(a)$	$XL(b)$	XL	E_f [keV]	j^π $\frac{1}{2}^\pi, \frac{3}{2}^\pi$
4057.7	1.5	4.1	1.3					1885.3	
4053.5	2.0	-1.2	1.2	5 9 10 11 12	(M1)	(M1)	(M1)	1889.5	(+)
4046.2	3.0	-0.2	1.0		(M1)		(M1)	1896.8	(+)
4024.5	1.0	2.1	1.1					1918.5	
4017.0	0.9	-0.5	1.1		(M1)		(M1)	1926.0	(+)
3997.6	0.8	5.7	1.2	2 3 4 5 6 7 8 9	E1	E1	E1	1945.4	-
3988.6	0.5	0.6	1.2					1954.4	
3970.9	0.3	3.9	1.3	1 2		(E1)	(E1)	1972.1	(-)
3961.4	1.0	0.7	1.3	12		(M1)	(M1)	1981.6	(+)
3934.9	2.5	2.8	1.4					2008.1	
3911.3	1.7	3.0	1.3	2 6		(E1)	(E1)	2031.7	(-)
3904.7	1.7	5.5	1.3	3 4 5 6 8	E1	E1	E1	2038.3	-
3890.0	2.5	3.6	1.3	4 5 6		E1	E1	2053.0	-
3870.4	0.4	6.9	1.4	12	E1	(D)	(D)	2072.6	-
3856.8	1.7	3.3	1.4	1		(E1)	(E1)	2086.2	(-)
3831.6	0.7	3.2	1.5	6		(E1)	(E1)	2111.4	(-)
3821.1	1.7	3.6	1.5	9 10		(E1)	(E1)	2121.9	(-)
3808.7	1.7	4.7	1.5	5 6		(E1)	(E1)	2134.3	(-)
3789.6	1.0	3.5	1.5					2153.4	
3780.4	1.0	5.8	1.6	12	(E1)	(D)	(D)	2162.6	-
3764.3	1.0	5.1	1.5	2 3 4 5	(E1)	E1	E1	2178.7	-
3758.0	1.0	5.6	1.6	2 7 9 10 11 12	(E1)	(E1)	(E1)	2185.0	(-)
3752.2	1.4	4.0	1.6	3 7		(E1)	(E1)	2190.8	(-)
3742.2	1.0	4.1	1.6	8		(E1)	(E1)	2200.8	(-)
3736.4	1.6	4.0	1.7					2206.6	
3720.6	0.8	5.3	1.7	11	(E1)	(D)	(D)	2222.4	-
3709.0	1.0	4.8	1.7	11		(E1)	(E1)	2234.0	(-)
3686.9	1.0	5.4	2.0					2256.1	
3683.2	0.8	2.0	2.0					2259.8	
3662.1	0.7	6.6	2.0	8 9	(E1)	(D)	(D)	2280.9	-
3655.0	1.0	4.5	1.9	7		(D)	(D)	2288.0	-
3628.7	0.7	8.6	3.7	12		(D)	(D)	2314.3	-
3607.6	0.8	0.2	2.1					2335.4	
3596.0	0.5	3.5	2.2	1		(E1)	(E1)	2347.0	(-)
3591.7	1.7	2.7	2.1	1		E1	E1	2351.3	-
3585.2	0.5	4.0	2.2	1		(E1)	(E1)	2357.8	(-)
3565.4	0.8	-2.5	2.1	7		(M1)	(M1)	2377.6	(+)
3554.4	0.8	5.5	2.2	1 2 3 4		(E1)	(E1)	2388.6	(-)

[a] Numbers of points on [0,1] cut for which appropriate confidence level was performed
[b] Transitions with unusually strong intensities are labelled by D to indicate the possibility of a doublet or of a non-statistically large fluctuation.

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SUPPLEMENT

As we have software toolkit to calculate integrals of some functions we prepared some tables. We hope that these data will have more wide set of arguments and more precise values than that which usually can be received from the Internet.

S1 Table of values of $\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} exp(-\frac{z^2}{2})dz$ for integer values of argument.

x	$\Phi(x)$
1.00	1.5865525E-01
2.00	2.2750132E-02
3.00	1.3498980E-03
4.00	3.1671242E-05
5.00	2.8665157E-07
6.00	9.8658764E-10
7.00	1.2798125E-12
8.00	6.2209606E-16
9.00	1.1285884E-19
10.00	7.6198530E-24
11.00	1.9106596E-28
12.00	1.7764821E-33
13.00	6.1171566E-39

S2 Table of reverse values for $\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} exp(-\frac{z^2}{2})dz$ for values of 10^{-n} of $\Phi(x)$

$\Phi(x)$	x
1.0E-01	1.2815516E+00
1.0E-02	2.3263479E+00
1.0E-03	3.0902323E+00
1.0E-04	3.7190165E+00
1.0E-05	4.2648908E+00
1.0E-06	4.7534243E+00
1.0E-07	5.1993376E+00
1.0E-08	5.6120012E+00
1.0E-09	5.9978070E+00
1.0E-10	6.3613409E+00
1.0E-11	6.7060232E+00
1.0E-12	7.0344838E+00
1.0E-13	7.3487961E+00
1.0E-14	7.6506281E+00
1.0E-15	7.9413453E+00
1.0E-16	8.2220822E+00
1.0E-17	8.4937932E+00
1.0E-18	8.7572903E+00
1.0E-19	9.0132712E+00
1.0E-20	9.2623401E+00
1.0E-21	9.5050250E+00
1.0E-22	9.7417899E+00
1.0E-23	9.9730456E+00
1.0E-24	1.0199157E+01
1.0E-25	1.0420452E+01
1.0E-26	1.0637224E+01