Group Delay Time and Neutron Optics

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The concept of group delay time (GDT) was introduced into the theory by D. Bohm and E.P. Wigner as a measure of time shift of a wave packet interacting with a stationary potential structure. It was widely used in the analysis of a number of quantum problems, at the same time playing the role of some sort of "theoretical clock". Later on, A.I. Baz proposed to measure the time of scattering by the potential of a particle with a magnetic moment, calculating the precession angle in a magnetic field presets in the potential region. Subsequently, it became clear that neutron experiments enabled implementation of the Larmor clock based on the constant precession frequency, and the Larmor time measured by it was closely related to the GDT. In the case of neutron reflection from resonant multilayer structures, the GDT can be much longer than in the case of total reflection. Moreover, under certain circumstances, it can be negative. The latter can also be measured by the Larmor clock method. In addition, the GDT concept has proved useful for clarifying new aspects of neutron wave propagation in a refractive matter.

Introduction

The problem of interaction time in quantum mechanics has been the subject of intense discussion for many decades. Apparently, for the first time, the question of how much time a particle spends in the potential region was formulated in [1]. In the 50s, the concept 'interaction time' acquired its mathematical formulation in the works by L. Eisenbud [2], D. Bohm [3], and E.P. Wigner [4]. To date, the number of publications devoted to this problem has reached several dozens. In this paper, let us refer only to reviews [5, 6]. The diversity of opinions expressed in this discussion is largely due to differences in the definition of the physical clock or time measuring procedure.

A very common definition of interaction time going back to works [1–4] is the so-called group delay time (GDT), formerly often referred to as 'phase delay time'

$$\Delta t = \hbar \qquad , \tag{1}$$

where φ is the phase shift of a plane wave passing between points *x1* and *x2* that enclose the area of interaction (potential). Formula (1) corresponds to the total time spent by the particle when passing between *x1* and *x2*, including delay τ associated with the interaction itself. Obviously, to determine the actual time of interaction, it is necessary to subtract the time of free propagation in the absence of potential.

An important step in the study of the issue was made in 1966 by A.I. Baz [7]. Referring to the problem of particle scattering on a spherical potential with effective range r_0, he defined the quantum clock as follows: "Let us suppose that inside a sphere $R > r_0$ there is an infinitely small homogeneous field B directed along the Z-axis, and for $r > R$ field B equals zero. Moreover, let us assume that the scattered particles have a spin s=1/2 and a magnetic moment $\boldsymbol{\mu}$=2sμ. Let the spin (and the magnetic moment) of the particles incident on the potential be polarized along the X-axis. If the particle enters sphere $r=R$, where field B is in effect, the magnetic moment begins to precess with the Larmor frequency

$$\omega_L = 2\mu B/\hbar . \tag{2}$$

Therefore, the spin of the particles scattered and gone beyond sphere $r=R$ will be rotated through a certain angle θ relative to its initial direction. One can calculate this angle and consequently find the average time spent inside the sphere $r=R$: $t(E) = \theta/\omega_L$».

The 'Larmor time' measured by such a clock has a close connection with the GDT. Indeed, the angle of the Larmor precession θ can be identified as phase difference Δφ of the two wave function components corresponding to the two values of the spin projection on the Z-axis, which differ in values. According to A.I. Baz, having determined the time delay associated with the interaction as $\Delta t_L = \Delta\varphi/\omega_L$, and taking into account that in (2) $2\mu B=\Delta E$, we obtain $\Delta t_L = \hbar(\Delta\varphi/\Delta E)$ that coincides with (1) in the limit $B \to 0$.

V.F. Rybachenko used A.I. Baz's method to calculate the time during which the particle tunnels through the potential barrier [8]. Owing to works [7, 8], the term 'Larmor clock' has become deeply embedded in scientific literature.

Larmor clock and neutron spin echo

With a progress of a technique of neutron experiment, it has become possible to put the Larmor clock into practice. To measure the time of neutron interaction with an object by means of this method, the object should be placed in an area with a magnetic field, where the neutron spin should precess. We assume that the magnetic field is normal to the plane of the spin. Thus, the task is to measure the precession angle $\theta = \omega_L \cdot \tau$, where τ is the measured interaction time. In this case, the complete precession angle is determined by the time spent in the precession region $\Phi = \omega_L(t+\tau)$, where the time of flight $t = L/V$, V is the velocity, and L is the length of the region with the magnetic field. With a reasonable length L, time τ is several orders of magnitude shorter than time of flight t. The relative smallness of τ together with the requirement of practical measurability of phase θ imposes a lower limit on the Larmor frequency ω_L. Therefore, the $\omega_L t$ factor becomes sufficiently high. This means that in such a measurement it is necessary to ensure a very high degree of beam monochromatization, so that the dispersion of the Larmor phase was not too high: $\Delta\Phi_L = \omega_L t(\Delta v/v) << 1$. Otherwise, the beam will be depolarized and the measurements will become impossible. In practice, this requirement is difficult to meet, since the necessary degree of monochromatization leads to unacceptably great intensity loss.

The problem of monochromatization can be avoided by using the neutron spin echo method [9]. In this case, neutrons pass not one, but two precessing paths of the opposite precession direction successively. If the neutron velocity is constant all the way, the total Larmor phase becomes zero for all neutron velocities, provided

$$\int_{L_1} Bd\ell \qquad \int_{L_2} \qquad \ell \tag{3}.$$

The degree of monochromatization is limited only by the extent to which condition (3) can be fulfilled. Placing the sample in one of the precession path results in a phase change by $\omega_L\tau$. Several experiments based on this idea were carried out using the spin-echo spectrometer of the Laue-Langevin Institute [10-12]. The Larmor clock method was applied applied to measure the time delay caused by the difference between the classic neutron velocity in vacuum and in a refractive medium. In this experiment, the time measurement error was 3.7×10^{-10} seconds, despite the fact that the total time of neutron flight through two regions with a field was 0.017 seconds, i.e. eight orders of magnitude longer. In addition, the time of Bragg reflection from a multilayer periodic structure and the time of tunneling at the resonance of a quasi-bound state in a three-layer structure analogous to a Fabry-Perot interferometer were measured. In these cases, the delay measured by the Larmor clock was about 10^{-7} seconds.

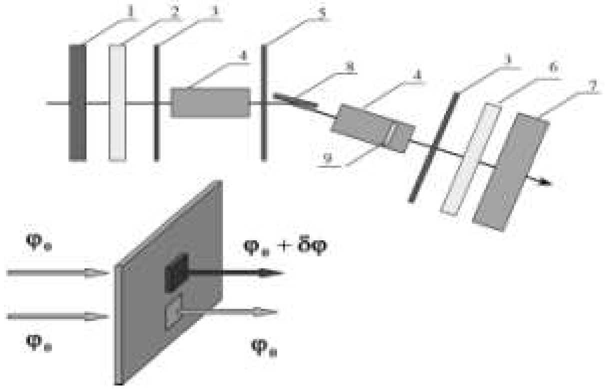

Fig.1. Design of the experiment on the measurement of the delay time of the sample passage, using the Larmor clock method. 1 – velocity selector, 2 - polarizer, 3 - π/2-flippers, 4 – precession solenoids, 5-π-flipper, 6 – analyzer of polarization, 7 – position-sensitive detector, 8 – multilayer mirror monochromator, 9 – sample location. Below is the position of the sample in one of the two beams shaped by the aperture.

Group delay time at reflection from resonant structures

The characteristic delay time of total neutron reflection is about several nanoseconds. To significantly increase the GDT in [13] it was proposed to reflect neutrons from multilayer structures. As a result of multiple interference and formation of standing waves in such structures, one can also speak of emergence of resonant modes.

Fig. 2 shows the calculation results for the neutron reflection from two three-layer Ni/Ti/Ni structures with the parameters indicated at the top. Structure 2 differs from structure 1 in the first and the third layers being swapped. In both cases, a sharp and sufficiently deep dip associated with excitation of the waveguide mode is observed on the reflection curve in the vicinity of 144 neV. For structure 1, the derivative of the phase has its maximum in this point, which corresponds to the GDT τ_{max} = 370 ns. For structure 2, the reflection phase has a pronounced *S*-shaped form with a negative derivative in this region, which leads to a negative delay time with $\tau_{min} = -85$ ns. However, the negative value of the GDT does not contradict the principle of causality [14,15]. The two lower plots of Fig. 2 show the calculation results for the short-time pulse reflection — the Gaussian wave packet – from the same structures. It can be seen that in the case of the negative GDT (the right lower plot), the maximum of the reflected packet does slightly outpace the maximum of the incident flow. However, it occurs due to a change in the pulse shape. The difference in the areas under the curves is obviously due to the neutron passage through the structure

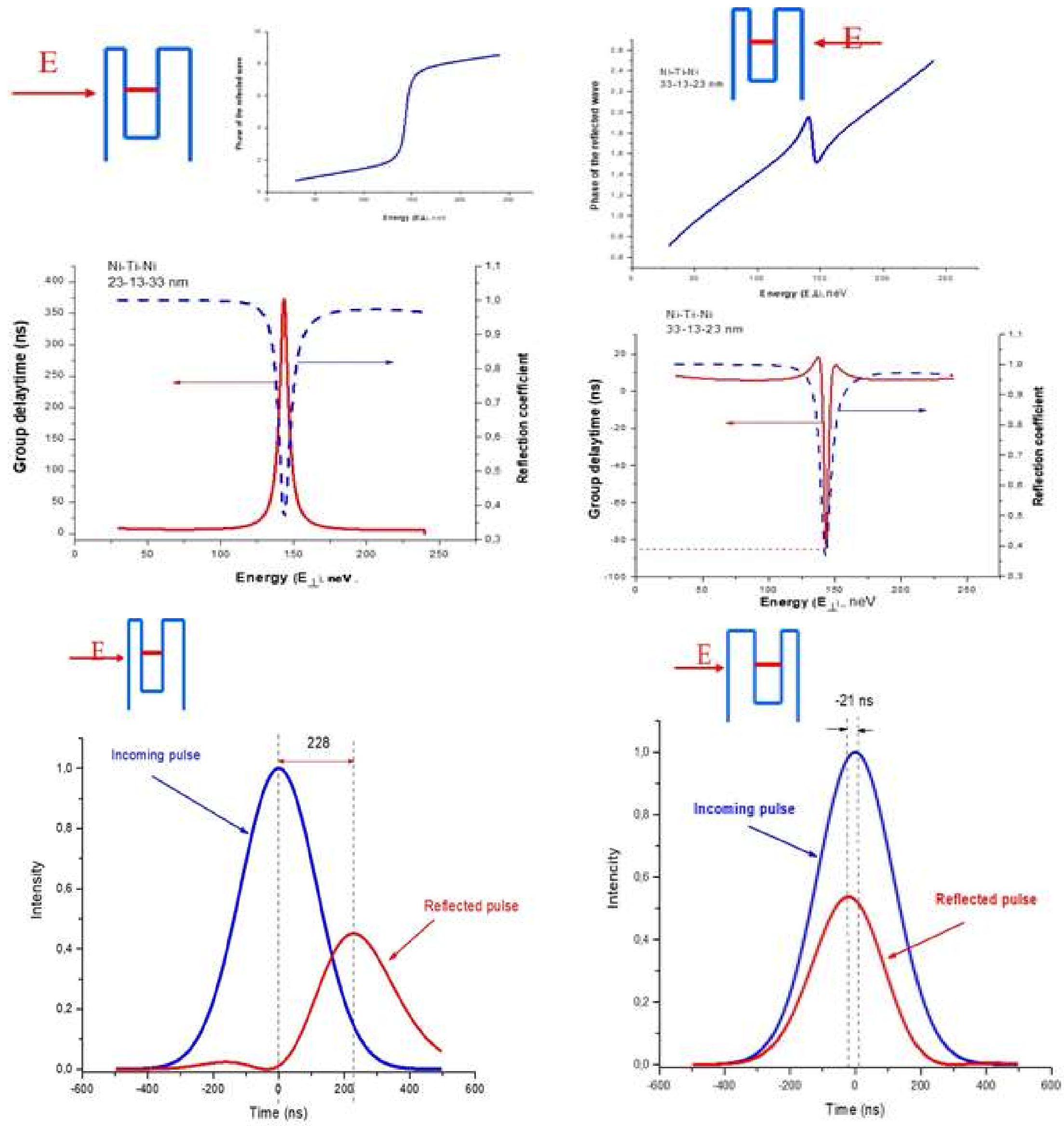

Fig.2. Above: reflection coefficient and the GDT for three-layer structures differing in the order of the layers. Below: relative magnitude and time relations between short wave packets incident and reflected from the same structures.

Group delay time and neutron velocity in a refractive substance

Above, we mentioned the measurement of the neutron GDT conditioned upon refraction in the sample. Let us consider this question in more detail. Suppose that a neutron passes through a refractive sample of length L with a refractive index $n(k_0)$. Let us define the medium dispersion law as $k = F(k_0^2)$ and calculate the neutron velocity in the medium $v = L/\tau$, where τ is the time of passage through the sample. Using group time (1) as the latter and taking into account that the phase shift $\Delta\Phi$ can be obviously determined as $\Delta\Phi = kL$, for the velocity in the medium we obtain

$$\mathrm{v} = \frac{\hbar}{2\mathrm{m}}(\mathrm{F} \;) \quad , \tag{4}$$

where m is the neutron mass, and $F' = dF/d(k_o^2)$. Thus, the velocity in the medium depends not only on the refractive index n, but also on the medium dispersion law. From (4) it follows that by demanding fulfillment of $v = nv_0$, we immediately face the requirement $k^2 = k_0^2 + \chi^2$, where χ^2 is an arbitrary constant. Therefore, the relation that was considered as obvious turns out to be valid only for the potential dispersion law. For very slow neutrons, the following dispersion law form is valid:

$$\mathrm{k}^2 = \mathrm{k}_0^2 - 4\pi\rho\mathrm{b}, \tag{5}$$

where ρ is the number of nuclei per unit volume, and b is the scattering length. Nevertheless, in the case of thermal neutrons, the form of the dispersion law may differ significantly [16].

The result is easily comprehended. Keeping in mind that the nature of the refractive index for any type of waves is associated with interference of primary and secondary waves generated by the scattering by nuclei inside the medium, it is natural to assume that the neutron in the medium is not a true particle, but a quasi-particle with an effective mass m^*. Supposing that the neutron velocity is $v = \hbar$, we get the following relation from (4)

$$\mathrm{m}^* = 2\mathrm{mkF}', \tag{6}$$

and for the potential dispersion law, $m^* = m$ is valid. It also should be noted that from the proportionality of the effective mass of the dispersion function derivative, it follows that the mass itself can be negative. In a refractive medium, a negative effective mass can appear in the case of resonant behavior of the scattering amplitude.

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