Nuclear and subatomic physics and weak interaction

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Abstract

The formalism allowing to calculate a creation cross-section of the neutrino's exoatom and its decay probability as well as to analyze other exotic electroweak processes without use of heuristic assumptions is constructed. The easiest neutrino's exoatom the "neutroneum" considered in ref. [1] can be interpreted as "the quasi-bound state of a quasi-neutron and a quasi-neutrino". Here and below the prefix "quasi" means that the metastable compound system can be considered as exoatom in which neutrino moves near a neutron. The special property of the neutrino is used for this purpose, namely: the boundary condition "zero at infinity" for the neutrino wave function. This condition is not forbidden if the decay of the neutrino exoatom into the residual atom and the electron is allowed by the energy conservation law. But at the same time the exoatom decay with only neutrino emission is forbidden.

1. Introduction

The base of the theory of neutrino exoatom was developed in [1]. The easiest of them named the "neutroneum" can be interpreted as "the quasi-bound state of the quasi-neutron and the quasi-neutrino" [1]. Here and below the prefix "quasi" means that the metastable compound system can be considered as the exoatom in which the neutrino moves near the neutron. The role of different boundary conditions in physics of nuclear β-processes at the low energies was investigated in [1-2]. It was shown that, at least, for the easiest neutrino exoatom the neutroneum the neutrino confinement in compound system the quasi-neutrino plus the quasi-neutron is not caused by the neutron - neutrino interaction but due to the energy and momentum conservation laws.

We develop below the mathematical formalism for description of the induced exotic electron capture by any atomic nuclei resulting in a creation of the neutrino exoatoms. This approach provides the methodical basis for an analysis of the secondary effects in similar β - processes.

2. Main formalism

Let's consider Hamiltonian describing the exotic electroweak resonances (neutrino exoatom) for interaction of leptons with nucleons:

$$\hat{H} = \hat{H}_0 + \hat{V}_c + \hat{V}_w \tag{2.1}$$

Free Hamiltonian $\hat{H}_0$ in the second quantization representation (SQR) is

$$\hat{H}_0 = \sum_{\lambda_e} \varepsilon_{\lambda_e} \hat{e}^+_{\lambda_e} \hat{e}_{\lambda_e} + \sum_{\lambda_\nu} \varepsilon_{\lambda_\nu} \hat{\nu}^+_{\lambda_\nu} \hat{\nu}_{\lambda_\nu} + \sum_{\lambda_p} \varepsilon_{\lambda_p} \hat{p}^+_{\lambda_p} \hat{p}_{\lambda_p} + \sum_{\lambda_n} \varepsilon_{\lambda_n} \hat{n}^+_{\lambda_n} \hat{n}_{\lambda_n} + \sum_{\lambda_{n_\nu}} \varepsilon_{\lambda_{n_\nu}} \hat{\eta}^+_{\lambda_{n_\nu}} \hat{\eta}_{\lambda_{n_\nu}} \tag{2.2}$$

where $\hat{e}_{\lambda_e}, \hat{\nu}_{\lambda_\nu}, \hat{p}_{\lambda_p}, \hat{n}_{\lambda_n}, \hat{\eta}_{\lambda_{n_\nu}}$ - are the electron, neutrino, proton, neutron and the neutroneum annihilation operators, respectively, and Hermitian conjugate terms are the creation operators for the same particles. Index n_ν here and below means that we deal with the neutroneum.

If the wave functions (WF) of all particles belonging to the continuous spectrum are normalized to δ- function, the sums in (2.2) (and further) are integrals over momentum $\vec{p}_x$ and the sums over the spin projections $\underline{m}_x$ of the particles:

$$\sum_{\lambda_x}\hat{R}_x=\sum_{\underline{m}_x}\int d\vec{p}_x\hat{R}(\underline{m}_x) \qquad (2.3)$$

We postulate that the Hamiltonian of interaction is (See (2.1)):

$$\hat{V}=\hat{V}_c+\hat{V}_w \qquad (2.4)$$

and

$$\hat{V}_c=\hat{V}_{ee}+\hat{V}_{pp}+\hat{V}_{ep} \qquad (2.5)$$

where $\hat{V}_{ee}$ - Coulomb potential of electron-electron interaction

$$\hat{V}_{ee}=\frac{1}{2}\sum_{\lambda_e^{(1)},\lambda_e^{(2)},\lambda_e^{(3)},\lambda_e^{(4)}}\left\langle\lambda_e^{(1)}\lambda_e^{(2)}\middle|V_c\middle|\lambda_e^{(3)}\lambda_e^{(4)}\right\rangle\hat{e}^+_{\lambda_e^{(1)}}e^+_{\lambda_e^{(2)}}\hat{e}_{\lambda_e^{(4)}}\hat{e}_{\lambda_e^{(3)}} \qquad (2.6)$$

$\hat{V}_{pp}$ - Coulomb potential of proton - proton interaction

$$\hat{V}_{pp}=\frac{1}{2}\sum_{\lambda_p^{(1)},\lambda_p^{(2)},\lambda_p^{(3)},\lambda_p^{(4)}}\left\langle\lambda_p^{(1)}\lambda_p^{(2)}\middle|V_c\middle|\lambda_p^{(3)}\lambda_p^{(4)}\right\rangle\hat{p}^+_{\lambda_p^{(1)}}\hat{p}^+_{\lambda_p^{(2)}}\hat{p}_{\lambda_p^{(4)}}\hat{p}_{\lambda_p^{(3)}} \qquad (2.7)$$

$\hat{V}_{ep}$ - Coulomb potential of electron - proton interaction:

$$\hat{V}_{ep}=\sum_{\lambda_e,\lambda_p,\lambda'_e,\lambda'_p}\left\langle\lambda_e\lambda_p\middle|V_c\middle|\lambda'_e\lambda'_p\right\rangle\hat{e}^+_{\lambda_e}\hat{e}_{\lambda'_e}\hat{p}^+_{\lambda_p}\hat{p}_{\lambda'_p} \qquad (2.8)$$

To take into account the contribution of the weak interaction we use formulae

$$\hat{V}_w=\sum_{\lambda_e,\lambda_p,\lambda_{n_\nu}}\left\langle ep\middle|V_{n_\nu\to ep}\middle|n_\nu\right\rangle\hat{e}^+_{\lambda_e}\hat{p}^+_{\lambda_p}\hat{\eta}_{\lambda_{n_\nu}}+\sum_{\lambda_\nu,\lambda_n,\lambda_{n_\nu}}\left\langle\nu_e n\middle|V_{n_\nu\to\nu n}\middle|n_\nu\right\rangle\hat{\nu}^+_{\lambda_\nu}\hat{n}^+_{\lambda_n}\hat{\eta}_{\lambda_{n_\nu}}+h.c. \qquad (2.9)$$

It was shown [1-2] that $(\hbar=c=1)$

$$\left\langle ep\middle|V_{n_\nu\to ep}\middle|n_\nu\right\rangle=\frac{G_\beta}{\sqrt{2V^3V_{eff}^{n_\nu}}}(2\pi)^3\delta(\vec{k}_{n_\nu}-\vec{k}_p-\vec{k}_e)\hat{S}(j_{n_\nu},\underline{m}_{n_\nu}|\underline{m}_p,\underline{m}_e)\,. \qquad (2.10)$$

where V is the normalizing volume, and the angular factor in (2.10) is equal

$$\hat{S}(j_{n_\nu},\underline{m}_{n_\nu}|\underline{m}_p,\underline{m}_e)=C^{j_{n_\nu}\underline{m}_{n_\nu}}_{1/2\underline{m}_p\;1/2\underline{m}_e}\phi_{ep}(j_{n_\nu})\,. \qquad (2.11)$$

and

$$\phi_{ep}(j_{n_\nu})=1+6\lambda(-1)^{j_{n_\nu}}\begin{Bmatrix}1 & 1/2 & 1/2\\ j_{n_\nu} & 1/2 & 1/2\end{Bmatrix} \qquad (2.12)$$

The matrix element (2.10) contains wave function in cat-vector $\left|n_\nu\right\rangle$, which by definition is

$$\left|n_\nu\right\rangle=\sum_{\underline{m}_n,\underline{m}_\nu}C^{j_{n_\nu}\underline{m}_{n_\nu}}_{1/2\underline{m}_n\;1/2\underline{m}_\nu}\left|n\right\rangle_{\underline{m}_n}\otimes\left|\nu\right\rangle_{\underline{m}_\nu} \qquad (2.13)$$

and the neutrino WF (2.13) satisfies to the boundary condition

$$\lim_{r\to\infty}\left|\nu\right\rangle_{\underline{m}_\nu}=0 \qquad (2.14)$$

According to [1]

$$\begin{cases}\phi_{ep}(0)=1+3\lambda\approx 4.69\\ \phi_{ep}(1)=1-\lambda\approx -0.23\end{cases} \qquad (2.15)$$

The operators $\hat{e}_{\lambda_e}, \hat{\nu}_{\lambda_\nu}, \hat{p}_{\lambda_p}, \hat{n}_{\lambda_n}, \hat{\eta}_{\lambda_{n_\nu}}$ satisfy to the standard anti-commutation and commutation rules:

$$\{\hat{e}_{\lambda_e}, \hat{e}^{+}_{\lambda'_e}\} = \delta_{\lambda_e\lambda'_e} = \delta(\vec{p}_e - \vec{p}'_e)\delta_{\underline{m}_e\underline{m}'_e} \ ; \qquad \{\hat{e}_{\lambda_e}, \hat{e}_{\lambda'_e}\} = \{\hat{e}^{+}_{\lambda_e}, \hat{e}^{+}_{\lambda'_e}\} = 0 \tag{2.16}$$

$$\{\hat{\nu}_{\lambda_\nu}, \hat{\nu}^{+}_{\lambda'_\nu}\} = \delta_{\lambda_\nu\lambda'_\nu} = \delta(\vec{p}_\nu - \vec{p}'_\nu)\delta_{\underline{m}_\nu\underline{m}'_\nu} \ ; \qquad \{\hat{\nu}_{\lambda_\nu}, \hat{\nu}_{\lambda'_\nu}\} = \{\hat{\nu}^{+}_{\lambda_\nu}, \hat{\nu}^{+}_{\lambda'_\nu}\} = 0 \tag{2.17}$$

$$\{\hat{p}_{\lambda_p}, \hat{p}^{+}_{\lambda'_p}\} = \delta_{\lambda_p\lambda'_p} = \delta(\vec{p}_p - \vec{p}'_p)\delta_{\underline{m}_p\underline{m}'_p} \ ; \qquad \{\hat{p}_{\lambda_p}, \hat{p}_{\lambda'_p}\} = \{\hat{p}^{+}_{\lambda_p}, \hat{p}^{+}_{\lambda'_p}\} = 0 \tag{2.18}$$

$$\{\hat{n}_{\lambda_n}, \hat{n}^{+}_{\lambda'_n}\} = \delta_{\lambda_n\lambda'_n} = \delta(\vec{p}_n - \vec{p}'_n)\delta_{\underline{m}_n\underline{m}'_n} \ ; \qquad \{\hat{n}_{\lambda_n}, \hat{n}_{\lambda'_n}\} = \{\hat{n}^{+}_{\lambda_n}, \hat{n}^{+}_{\lambda'_n}\} = 0 \tag{2.19}$$

$$[\hat{\eta}_{\lambda_{n_\nu}}, \hat{\eta}^{+}_{\lambda'_{n_\nu}}] = \delta_{\lambda_{n_\nu}\lambda'_{n_\nu}} = \delta(\vec{p}_{n_\nu} - \vec{p}'_{n_\nu})\delta_{j_{n_\nu}j'_{n_\nu}}\delta_{\underline{m}_{n_\nu}\underline{m}'_{n_\nu}} \ ; \qquad [\hat{\eta}_{\lambda_{n_\nu}}, \hat{\eta}_{\lambda'_{n_\nu}}] = [\hat{\eta}^{+}_{\lambda_{n_\nu}}, \hat{\eta}^{+}_{\lambda'_{n_\nu}}] = 0 \tag{2.20}$$

$$[\hat{e}_{\lambda_e}\hat{\eta}^{+}_{\lambda_{n_\nu}}, \hat{\eta}^{+}_{\lambda_{n_\nu}}\hat{e}_{\lambda_e}] = [\hat{p}_{\lambda_p}\hat{\eta}^{+}_{\lambda_{n_\nu}}, \hat{\eta}^{+}_{\lambda_{n_\nu}}\hat{p}_{\lambda_p}] = 0 \tag{2.21}$$

$$[\hat{\eta}_{\lambda_{n_\nu}}\hat{e}^{+}_{\lambda_e}, \hat{e}^{+}_{\lambda_e}\hat{\eta}_{\lambda_{n_\nu}}] = [\hat{\eta}_{\lambda_{n_\nu}}\hat{p}^{+}_{\lambda_p}, \hat{p}^{+}_{\lambda_p}\hat{\eta}_{\lambda_{n_\nu}}] = 0 \tag{2.22}$$

As in the theory of finite Fermi-systems (TFFS) [3] we use basic functions for given atom. For example, the wave functions of the electron in the Coulomb field of the proton are usual Coulomb functions of discrete and continuous spectrum, i.e., eigen-functions of the Hamiltonian

$$\hat{H}^{(0)}_{ep} = \sum_{\lambda_e}\varepsilon_{\lambda_e}\hat{e}^{+}_{\lambda_e}\hat{e}_{\lambda_e} + \sum_{\lambda_p}\varepsilon_{\lambda_p}\hat{p}^{+}_{\lambda_p}\hat{p}_{\lambda_p} + \hat{V}_{ee} + \hat{V}_{pp} + \hat{V}_{ep} \, . \tag{2.23}$$

It is obvious that the Hamiltonian (2.23) corresponds to the system of any interacting electrons and protons.

For the free neutrinos we use the plane waves, normalized to unit in volume V, or on δ- function. Transition from one system of normalization to another is carried out by formal substitution of $L \leftrightarrow 2\pi$. All calculations are carried out in $m_l \ll m_N$ approximation. In practice it means that the neutrons, protons and neutroneum considered as infinitely heavy particles in comparison with leptons (the electron and the neutrino).

We use the simplest set of the constants [4]

$$G_\beta \approx 8.76\cdot 10^{-5} \ MeV\cdot fm^3 = 4.439\cdot 10^{-7} \ fm^2, \ \lambda = 1.23\, . \tag{2.24}$$

Let's show that the ratios given above satisfy to the compliance principle and can be used to calculate all the properties of heavy neutrino atoms, and, perhaps, neutroneum stars.

3. Perturbation theory and Green's function in the theory of exotic electro-weak processes (TEEP).

The main properties (mass, spin, life-time, the creation cross-section in eH-collisions) of neutrinos exoatom neutroneum established in [1-2]. An obvious lack of works [1-2] is their heuristic character. The self-consistent theory of the creation cross-section, life-time and “effective sizes” of the neutrino exoatom requires a clear understanding the fact that "switch on" of the weak interaction changes all important properties of the hydrogen atom. The system of the eigenfunctions of the standard Coulomb Hamiltonian added by a new function, orthogonal to all eigenfunctions of the non-perturbed Hamiltonian.

Let's consider a problem of the eigenvalues (EV) and the eigenfunctions (EF) of the Hamiltonian $\hat{H}$. As we use SQR then the number of particles in the exoatom is arbitrary. When we have a small parameter, we can use the perturbation theory. In our case the influence of the weak interaction to the full Hamiltonian is extremely small. According we can use the standard perturbation theory [5].

According to [5] we rewrite our Hamiltonian as:

$$\hat{H} = \hat{H}_0 + \hat{V}(t) \tag{3.1}$$

The Schrödinger equation for the time-dependent WF is:

$$i\hbar \frac{\partial \Psi}{\partial t} = (\hat{H}_0 + \hat{V})\Psi \, . \tag{3.2}$$

Exact solution Ψ we can write as superposition of EF of non-perturbed Hamiltonian:

$$\Psi = \sum_k a_k(t)\Psi_k^{(0)} \tag{3.3}$$

and

$$\hat{H}_0 \Psi_k^{(0)} = E_k^{(0)} \Psi_k^{(0)} \tag{3.4}$$

The transition probability (Fermi's "golden rule") in the framework of zero order perturbation theory is [5]

$$dw_{fi} = \frac{2\pi}{\hbar} \left| \left\langle \psi_f \left| \hat{V} \right| \psi_i \right\rangle \right| \delta(E_i - E_f) dn_f \tag{3.5}$$

where

$$\Psi(x,t) = \psi(x) \cdot \exp(-iEt / \hbar) \tag{3.6}$$

Similar expression for the first order of the perturbation theory is:

$$dw_{fi} = \frac{2\pi}{\hbar} \left| \left\langle \psi_f \left| \hat{V} \right| \psi_i \right\rangle + \int \frac{\left\langle \psi_f \left| \hat{V} \right| \psi_\nu \right\rangle \left\langle \psi_\nu \left| \hat{V} \right| \psi_i \right\rangle}{E_i - E_\nu + i0} d\nu \right|^2 \delta(E_i - E_f) dn_f \tag{3.7}$$

Let's calculate a decay probability of the exoatom neutroneum.

If the neutroneum WF is normalized to unity in the "box" with volume V, then the decay probability to proton and electron is (see [1-2]):

$$w_{n_\nu \to p + e^-} = \frac{2\pi}{\hbar} \int \frac{V d\vec{p}_e}{(2\pi\hbar)^3} \frac{V d\vec{p}_p}{(2\pi\hbar)^3} \delta(\varepsilon_{n_\nu} - \varepsilon_p - \varepsilon_e) \left\langle \left| \left\langle ep \left| \hat{V}_{ep \leftrightarrow n_\nu} \right| n_\nu \right\rangle \right|^2 \right\rangle \, , \tag{3.8}$$

where WF of the initial and the final states in SQR are

$$\begin{cases} \left| n_\nu \right\rangle = \eta^+_{\lambda_{n_\nu}} \left| 0_{l \otimes N} \right\rangle \\ \left| e \right\rangle = \hat{e}^+_{\lambda_e} \left| 0_l \right\rangle \\ \left| p \right\rangle = \hat{p}^+_{\lambda_p} \left| 0_N \right\rangle \end{cases} \tag{3.9}$$

where $\left| 0_l \right\rangle$ - the lepton vacuum, $\left| 0_N \right\rangle$ - the nucleon vacuum, and $\left| 0_{l \otimes N} \right\rangle \equiv \left| 0_l \right\rangle \otimes \left| 0_N \right\rangle \equiv \left| 0 \right\rangle$ - direct production of the lepton and the nucleon vacuum, as well as the external square brackets mean averaging over spin projections in the initial state and summation over spin projections in the final state.

As WF of the initial and the final states are defined by (3.9) so only one term "survives" out of all sum (2.9). Therefore according to (3.8) ($\hbar = c = 1$).

$$w_{n_\nu \to p + e^-} = \frac{\phi^2_{ep}(j_{n_\nu}) G^2_\beta}{8\pi^2 V^{n_\nu}_{eff}} \int d\vec{k}_e \delta(\varepsilon_{n_\nu} - \varepsilon_p - \varepsilon_e) \tag{3.10}$$

is in a full agreement with [1-2].

The problem of a correct calculation of the neutroneum creation cross-section with the perturbation theory is much more difficult. To show it we consider the neutroneum creation reaction in the electron-proton collision.

According to momentum-energy conservation law

$$\left\langle \psi_f \left| \hat{V} \right| \psi_i \right\rangle \sim \delta(\vec{p}_i - \vec{p}_f) \overline{\left\langle \psi_f \left| \hat{V} \right| \psi_i \right\rangle} \tag{3.11}$$

Therefore the neutroneum creation cross-section in ep - collisions (if we take into account the known neutroneum life-time) in the framework of the zero order perturbation theory is [1-2]:

$$\sigma_{ep \to n_\nu} = \frac{2\pi}{\hbar v_e} \left| \overline{\left\langle n_\nu \left| \hat{V}_w \right| ep \right\rangle} \right|^2 \frac{1}{2\pi} \cdot \frac{\Gamma_{n_\nu}}{(\varepsilon_{ep} - \varepsilon_{n_\nu})^2 + \Gamma_{n_\nu}^2 / 4}. \tag{3.12}$$

The cross-section (3.12) depends on the energy of the incoming electron. The width of the resonant peak Γ_{n_ν} in cross-section (3.12) is extremely small value since we deal with the resonance caused by the weak interaction.

4. A role of the third particle in the initial state

The vanishing width Γ_{n_ν} of a quasi-discrete level permits us to replace the Breit - Wigner formula (3.13) by δ - function $\delta(E_e + E_p - E_{n_\nu})$. It means that the radiationless processes of the neutrino exoatom creation in the free particles collisions are actually forbidden by momentum-energy conservation law.

Thus we have to use (3.7) to calculate the neutroneum creation cross-section. We can do it because the electron WF "tail" is mainly placed inside the nucleus-target (for example, in the proton). Therefore “switch on” of the weak interaction is adiabatic.

According this circumstance the ratio (3.7) we rewrite as:

$$d\sigma_{H(e,e')n_\nu} = \frac{2\pi V}{\hbar v_e} \left| \hat{s} \int \frac{V d\vec{p}_e}{(2\pi\hbar)^3} \frac{V d\vec{p}_p}{(2\pi\hbar)^3} \frac{\left\langle n_\nu \left| \hat{V}_w \right| ep \right\rangle \left\langle epe' \left| \hat{V}_{ee} \right| eH \right\rangle}{E_i - E_{ep} + i0} \right|^2 \delta(E_i - E_f) \frac{V d\vec{p}_{e'}}{(2\pi\hbar)^3} \frac{V d\vec{p}_{n_\nu}}{(2\pi\hbar)^3} \tag{4.1}$$

Expression for reaction cross-section $H(e,e')n_\nu$ demands detailed comments.

The neutroneum creation in the gas target was studied in detail [1-2]. From the quantum mechanics point of view formula (4.1) contains the two-particle Green function $\hat{G}_{ep}$:

$$\hat{G}_{ep} \equiv \sum_{e,p} \frac{\left| ep \right\rangle \left\langle ep \right|}{E_i - E_{ep} + i0} = \int \frac{V d\vec{p}_e}{(2\pi\hbar)^3} \frac{V d\vec{p}_p}{(2\pi\hbar)^3} \frac{\left| ep \right\rangle \left\langle ep \right|}{E_i - E_{ep} + i0} \tag{4.2}$$

The sum over e, p includes all eigenfunctions of the Hamiltonian $\hat{H}_c = \hat{H}_0 + \hat{V}_c$:

$$\hat{H}_c \left| ep \right\rangle = E_{ep} \left| ep \right\rangle \tag{4.3}$$

During the derivation (4.2) we have used the fact that the WF of the initial, final and intermediate states are known. For example, WF of the initial state in obvious designations is

$$\left| \psi_i \right\rangle = \left| e \right\rangle \otimes \left| H \right\rangle \equiv \left| eH \right\rangle \tag{4.4}$$

and the hydrogen atom WF is

$$\left|H\right\rangle = \int d\vec{q}\varphi_{nl\underline{m}_l}(\vec{q})\hat{e}^{+}_{\vec{q}\underline{m}_s}\hat{p}^{+}_{\vec{p}_H-\vec{q}\,\underline{m}_p}\left|0\right\rangle \tag{4.5}$$

The main question of the perturbation theory in the framework of TEEP - is the problem of the eigenfunctions of the Hamiltonian

$$\hat{H}_c = \hat{H}_0 + \hat{V}_c \tag{4.6}$$

The set of these WF is incomplete if we deal with the eigenfunctions of the Hamiltonian

$$\hat{H} = \hat{H}_c + \hat{V}_w \tag{4.7}$$

To make the set of two-particle electron-proton eigenfunctions complete, it is necessary to take into account a possibility of the quantum transition $ep \leftrightarrow n_\nu$. As a result the Schrödinger equation is

$$\hat{H}\psi_\lambda = E_\lambda \psi_\lambda \tag{4.8}$$

and the two-particle Green function (4.2) is modified:

$$G_{ep} = \sum_\lambda \frac{\left|\psi_\lambda\right\rangle\left\langle\psi_\lambda\right|}{E - E_\lambda + i\gamma} \tag{4.9}$$

Set $\left|\psi_\lambda\right\rangle$ contains the following basic functions:

1) the functions of a discrete spectrum of the electron in the hydrogen atom;
2) the Coulomb wave functions of a continuous spectrum;
3) the resonant state of neutroneum (state λ_0).

As the matrix element $\left\langle n_\nu\left|\hat{V}_w\right|ep\right\rangle$ is non-equal zero only for the state λ_0, so the contribution into integral (4.1) gives only a pole. This fact was taken into account in [1-2], and in the cross-section the index $\hat{s}$ stresses that the contribution gives only a pole and we have to neglect the contribution of the principal value of the integral into (4.1).

When obtaining the eigenfunctions and the eigenvalues of a Hamiltonian $\hat{H}$ it is necessary to take into account a coupling of the reaction channels: namely, the virtual transition $ep \leftrightarrow \nu n$ and the real transition $ep \leftrightarrow n_\nu$ (in the quarks "language" $eu \leftrightarrow \nu d$ emission of the d - quark at the mass surface).

A small component of the neutroneum WF in the full WF of the ep - pair we can calculate according to [5]

$$\psi_{ep} = \psi^{(0)}_{ep} + \delta\psi_{ep} \tag{4.10}$$

where $\delta\psi_{ep} = c_{n_\nu}\left|n_\nu\right\rangle$.

Above in we use WF normalized in the "box". For the further purposes is more convenient to use δ - function WF normalization. Thus

$$c_{n_\nu} = \int d\vec{k}_{n_\nu} \frac{\left\langle n_\nu\left|\hat{V}_w\right|ep\right\rangle}{E_i - E_f + i0} \tag{4.11}$$

where (see (2.10))

$$\left\langle ep\left|V_{n_\nu \to ep}\right|n_\nu\right\rangle = \frac{G_\beta}{\sqrt{2(2\pi)^3 V^{n_\nu}_{eff}}}(2\pi)^3\delta(\vec{k}_{n_\nu} - \vec{k}_p - \vec{k}_e)\hat{S}(j_{n_\nu}, \underline{m}_{n_\nu}|\underline{m}_p, \underline{m}_e) \tag{4.12}$$

In the plane wave approximation the WF of the particles in the continuous spectrum are:

1) the ep - pair

$$\psi_{ep}^{(0)} = \frac{1}{(2\pi)^3}\exp[i(\vec{k}_e\vec{r}_e + \vec{k}_p\vec{r}_p)]\cdot\chi_{1/2\underline{m}_p}(\vec{s}_p)\chi_{1/2\underline{m}_e}(\vec{s}_e) \qquad (4.13)$$

2) the neutroneum

$$\psi_{n_\nu} = \frac{1}{(2\pi)^{3/2}}\exp(i\vec{k}_{n_\nu}\vec{r}_{n_\nu})\cdot\chi_{j_{n_\nu}\underline{m}_{n_\nu}}(\vec{s}_{n_\nu}). \qquad (4.14)$$

We can easy take into account the Coulomb distortions of the WF [4].

The two-particle Green function decomposition includes the singular term c_{n_ν}

$$c_{n_\nu} = \frac{G_\beta\phi_{ep}(j_{n_\nu})}{E_{ep} - E_{n_\nu} + i\gamma}\frac{(2\pi)^{3/2}}{\sqrt{2V_{eff}^{n_\nu}}}\sum_{\underline{m}_{n_\nu}} C_{1/2\underline{m}_p\ 1/2\underline{m}_e}^{j_{n_\nu}\underline{m}_{n_\nu}} \qquad (4.15)$$

Therefore the Green function (4.9) has the singular component proportional to the product $\left|n_\nu\right\rangle\left\langle n_\nu\right|$. It means that in the framework of TEEP exists the state, the transition probability to which is equal:

$$\left|c_{n_\nu}\right|^2 = \frac{G_\beta^2\tilde{\phi}_{ep}^2(j_{n_\nu})}{(E_{ep} - E_{n_\nu})^2 + \gamma^2}\frac{(2\pi)^3}{2V_{eff}^{n_\nu}} \sim \delta(E_{ep} - E_{n_\nu}) \qquad (4.16)$$

The energy dependence (4.16) has a δ - function form since the Breit - Wigner resonance is caused by weak interaction, and is extremely narrow. At the same time the ratio (4.16) includes the ratio

$$\gamma \sim \frac{G_\beta^2\tilde{\phi}_{ep}^2(j_{n_\nu})}{V_{eff}^{n_\nu}} \qquad (4.17)$$

Therefore the singular component of the ep - pair Green function is not depended on the weak interaction constant G_β and an effective volume $V_{eff}^{n_\nu}$ of the neutroneum. Exactly thanks to this circumstance in formula (4.1) for the neutroneum creation cross-section only the polar term is survived and there are no the additional factors demanding measurements in special experiments.

Due to the energy-momentum conservation law the electroweak reaction $e + u \to \nu + d + G$ has a channel with recoil momentum transfer to gluon. At the quarks level of matter the existence of this channel corresponds to the reaction of the induced exotic electron capture, when the neutrino is stopped. It can lead to many the observable secondary effects [6] which are of interest to future investigations.

5. Conclusion

We can sum the results obtain above as follows.

1. The formalism allowing to investigate the exotic electroweak processes by standard methods of quantum mechanics and nuclear physics was developed.
2. The TEEP-results for the main electroweak processes are reproduced by the standard methods of the perturbation theory.

3. It was shown that the new formalism of the theory of the nuclear β- processes at the low energies satisfies to the compliance principle.
4. The correctness of the pole approximation in the TEEP was proved.
5. The base of the general theory of the neutrino atoms is created.
6. The investigations of the secondary effects which accompany the new type of the β-processes, is of huge interest, and still not described in scientific literature.

References

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