A RELIABILITY OF THE RESULTS OF A STUDY OF THE NUCLEAR SUPERFLUIDITY AND HIDDEN PARAMETERS OF THE GAMMA DECAY OF THE COMPOUND STATE

Vu D.C.[1,4], Sukhovoj A.M.[1], Mitsyna L.V.[1], Nguyen X.N.[2], Pham D.K.[3], Nguyen N.A.[2]

[1]*Joint Institute for Nuclear Research, Dubna, 141980, Russia*
[2]*Dalat Nuclear Research Institute, Vietnam Atomic Energy Institute, Hanoi, Vietnam*
[3]*Hanoi University of Science and Technology, 1 Dai Co Viet, Hanoi city, Vietnam*
[4]*Vietnam Academy of Science and Technology Institute of Physics, Hanoi, Vietnam*

An analysis of the intensities of the two-step cascades (TSC) of reaction products emitted sequentially is now the only possibility of a study of an interaction between Fermi- and Bose-states in an excited nucleus, at least, up to the binding energy of the last nucleon. To obtain reliable information about this interaction, it is necessary to recognize sources of the process distortion. In the present work we attempted a difficult task to evaluate the hidden parameters influence on the required parameters.

1. INTRODUCTION

At small excitation energy, in a nucleus the levels of different types can be excited. In even-even deformed nucleus, for example, quasi-particle and vibrational levels as well as rotational bands have been observed. When the excitation energy grows, the nuclear levels appear, the wave functions of which contain components of all types [1,2]. It is caused by a fragmentation of nuclear states in the gamma-decay process. So at the nuclear excitation energies E_{ex} above some MeV, in the nucleus there are no, in pure form, quasi-particle, vibrational and rotational nuclear levels.

A common defect of a majority of modern nuclear models is an incorrect representation of the excited nucleus as a purely fermion system. The only realistic model of the level density, which takes into account the boson excitations [3] in the nucleus, postulates an existence a phase transition at $E_{\text{ex}} \geq B_{\text{n}}$ (B_{n} is the neutron binding energy). But this model does not explain a real process of nuclear transition to the superfluid state. So, for a practical investigation of the interaction between Fermi- and Boson-systems, the models for the excited nucleus and its cascade decay (as far as possible, including nucleon products of nuclear reaction) are needed.

2. A POSSIBILITY OF EXPERIMENTAL INVESTIGATION OF THE NUCLEAR SUPERFLUIDITY

To detect the effects connected with the nuclear superfluidity, it is necessary to determine [4] the level density ρ and the partial widths Γ of emission of the products at the decay of high-excited nuclear levels (above 5–10 MeV). At that, the ρ and Γ values must be defined in the whole energy region (from the nuclear compound-state up to the ground state). As there are no modern spectrometers to determine the parameters of all nuclear levels, so the experiment must provide the accuracy, at least, of a few tens of percent. Such accuracy is just enough to determine the ρ and Γ values in the required energy region as it corresponds to real large coefficients of transfer of errors of the experimental spectra and cross sections to errors of desired gamma-decay parameters.

As shown in [5 - 7], in order to get reliable information about the superfluid phase of the nuclear matter it is necessary to obtain the level density up to E_{ex} not less than $4\Delta_0$ (Δ_0 is the pairing energy of the last nucleon in the nucleus) with an acceptable accuracy. The level density g of quasi-particle levels near Fermi-surface is about 10 MeV^{-1}, whereas the excitation energy of known phonon bands is of order 1 MeV^{-1}. Thus, the only possibility of experimental identification of the boson levels is a feature of the energy dependence of their density, which decreases at a growth of the partial widths of emission products of the nuclear reaction.

It should be noted that the level density and the partial widths of emission are included in the measured spectra and cross sections of reactions as $\rho\times\Gamma$ product. It means that any experiment, in which the coincidence method is not used, may be mathematically presented by only one equation with two (or even more) unknown values. In other words, whatever ρ and Γ values (from zero up to infinity) can ensure an accurate description of the cross sections and spectra.

For experiments, which use the coincidence method for recording of the set of the parameters (intensities of gamma transitions and energies of all excited levels), a situation is more favorable. The energies of both of initial level of TSC and of its final level are determined by the experimental conditions, as well as the intensities of two cascade quanta. The uncertainty of the extracted ρ and Γ parameters was essentially decreased, if to separate carefully the intensity $I_{\gamma\gamma}(E_1)$ of primary transitions of the TSC from its total intensity $I_{\gamma\gamma}(E_1,E_2)$ (E_1 is energy of primary cascade quantum, E_2 is energy of secondary quantum).

Each of experimental spectra of two coincided gamma-quanta is a superposition of two mirror-symmetrical distributions. If the set of resolved pairs of peaks of the cascades are big enough, using information about their energies, we can determine the intensity of the cascades as a function of only primary transitions with a good accuracy. An inevitable distortion of a shape of the $I_{\gamma\gamma}(E_1)$ distribution due to inaccurate accounting [5] of a contribution of unresolved transitions can be minimized with statistics increase. Our analysis of the experimental TSC intensities, for 44 nuclei of the mass region $28 \leq A \leq 200$ [6 – 10], unambiguously showed that at TSC recording with a statistics of ~ 4000 events (and more) in a summary peak, this distortion has no effect on the errors of extracted parameters and can be neglected. In all our experiments, three of quarters of all investigated nuclei had a minimal required statistics, and it was some times more for the rest of nuclei (in the case of ^{172}Yb, for example, the percentage of resolved TSC to the ground state was 70%).

The method [11, 12] of decomposition of experimental spectrum into two required distributions $I_{\gamma\gamma}(E_1)$ and $I_{\gamma\gamma}(E_2)$ was firstly realized in Dubna.

3. THE METHOD FOR A STUDY OF THE GAMMA DECAY

For the first time, the method for simultaneous determination ρ and Γ values from measured intensities $I_{\gamma\gamma}(E_1)$ of the two-step cascades at radiative capture of the thermal neutrons by stable target nuclei was proposed in Dubna [13, 14]. In the experiments the intensities $I_{\gamma\gamma}$ to a group of low-lying levels of a target nucleus were measured, when the high-lying compound state decays and the TSC quanta were recorded by two Ge-detectors, as coincidences. At that, the interval of spins, parities and the partial widths of excited levels, density of which must be determined for evaluation of the nuclear entropy [3], are limited by selection rules on multipolarity.

Our method allows us to get unique information about gamma decay of any nuclei [5, 6]. The intensity distributions $I_{\gamma\gamma}(E_1)$, derived using the procedure [13] were approximated by fitted parameters $p_1,p_2..$ and $q_1,q_2..$ of suitable functions $\rho(E_{ex}) = \varphi(p_1,p_2,...)$ and $\Gamma(E_1) = \psi(q_1,q_2..)$. A choice of the models, which define the hypothetical $\rho(E_{ex})$ and $\Gamma(E_1)$ functions with an optimal number of fitted parameters, is very important, as it is just a source of the main systematics error of the analysis.

A cascade decay of a neutron resonance (or any compound-state) λ occurs through the intermediate levels i to the final levels f. The part of primary transitions $I_{\gamma\gamma}(E_1)$ for any small energy interval ΔE_j of cascades is expressed by an equation:

$$I_{\gamma\gamma}(E_1) = \sum_{\lambda,f}\sum_{i}\frac{\Gamma_{\lambda i}}{\Gamma_{\lambda}}\frac{\Gamma_{if}}{\Gamma_{i}} = \sum_{\lambda,f}\sum_{j}\frac{\Gamma_{\lambda j}}{<\Gamma_{\lambda j}>M_{\lambda j}}n_j\frac{\Gamma_{jf}}{<\Gamma_{jf}>m_{jf}}. \quad (1)$$

A sum of the partial widths $\Sigma_j\Gamma_{\lambda j}$ of primary transitions to $M_{\lambda j}$ intermediate levels is $<\Gamma_{\lambda j}>M_{\lambda j}$, and a sum $\Sigma_j\Gamma_{jf}$ for secondary transitions to m_{jf} final levels is $<\Gamma_{jf}>m_{jf}$, inasmuch as $<\Gamma_{\lambda j}> = \Sigma_j\Gamma_{\lambda j}/M_{\lambda j}$ and $<\Gamma_{jf}> = \Sigma_j\Gamma_{jf}/m_{jf}$. In a small energy interval ΔE_j a sum of intermediate levels is $n_j = \rho\Delta E_j$.

One can see that in the equation (1) for any i and f levels of TSC there are two independent parts:

1) $I_1=\rho\times\Gamma/\sum(\rho\times\Gamma)$ distribution of the spectrum of primary gamma-transitions and
2) $I_2=\Gamma/\sum(\rho\times\Gamma)$ distribution of branching coefficients.

At that, anti-correlation between ρ and Γ values in I_1 distribution is absolute (100%), but correlation between I_1 and I_2 distributions is practically absent (of course, by a condition that a probability of emission of secondary gamma quantum to f levels does not depend on a probability of gamma transition between the levels λ and i). As in $I_{\gamma\gamma}(E_1)$ distribution the correlation between the ρ and Γ values becomes weaker, it limits an area and increases the precision of possible solutions of equation (1).

Correlation coefficients are different for products of type ρ×Γ as well as ρ×ρ and Γ×Γ (for various energies of gamma-transitions and excitation energies). But as correlation coefficients between the spectrum I_1 of primary transitions and branching coefficients I_2 are almost zero, this fact provides a possibility to avoid the fatal errors at analysis. A high statistics allowed us to obtain $I_{\gamma\gamma}(E_1)$ distributions carefully enough (with a required accuracy of several tens of percent in any energy bins).So we can evaluate the errors Δρ and ΔΓ for a majority of obtained values [6, 7, 10] by several tens of percent at $\Delta I_{\gamma\gamma}$ error of about 1%. How $\Delta I_{\gamma\gamma}$ connects with Δρ and ΔΓ at any step of iteration process of solving the system (1) by Monte-Carlo method, one can see in detail in [5–10].

In order to get a reliable data by the method of analysis of TSC intensities, the following representations about the required $\rho(E_{ex})$ and $\Gamma(E_1)$ functions are absolutely necessary:

1) the number of quasi-particles begins to grow from zero (for even-even and odd-odd nuclei) or from unity (for even-odd and odd-even nuclei) at an increase of E_{ex} (as in the model [2,15], for example);
2) the levels of collective type must appear at the breaking of each Cooper pair (as in the model [4], for example);
3) the dependences of the radiative widths of the dipole transitions on the excitation energy can be nonmonotonical functions [16].

4. EVALUATION OF AN INFLUENCE OF HIDDEN PARAMETERS ON THE ANALYSIS RESULTS

As was already said, a necessity of simultaneous fitting of the parameters included in the intensity $I_{\gamma\gamma}$ distribution is evident due to the strong correlation between the $\rho(E_{ex})$ and $\Gamma(E_1)$ functions. The first step to solve this difficult problem was done in Dubna. There are no yet both the experiments on a study of an interaction between nuclear Fermi- and Bose-states, which can really compete with our data, and the modern models to describe this process with some degree of certainty. For now, our purpose is to determine and evaluate factors, which exert strong influence on the investigated process, when comparing different variants of our empirical model.

At approximation of $I_{\gamma\gamma}(E_1)$ intensities, the fitting of the parameters of the $\rho(E_{ex})$ and $\Gamma(E_1)$ functions is practically unambiguous in a wide region of required values. The best fits have usually a small scatter for the iteration processes with different vectors of initial values. Nevertheless, a correctness of $\rho(E_{ex})$ and $\Gamma(E_1)$ energy dependences in used models (and phenomenological representations) cannot be determined in the framework of the experiment. The most probable $\rho(E_{ex})$ and $\Gamma(E_1)$ functions can be chosen only analyzing different alternative models.

As the required $\rho(E_{ex})$ and $\Gamma(E_1)$ functions can be obtained only from indirect experiment, hidden parameters of the decay process (the breaking thresholds of the Cooper pairs of nucleons, local peaks in the energy dependence of the radiative widths, etc.) represent as serious problem. So it is very important to describe reliably the nuclear properties in the points of breaking the Cooper pairs.

In a base of our experimental data, the fact is established that in these points there are breaks in the energy dependence of the nuclear level density as well as distinctive local changes in the dependences of the radiative widths on the excitation energy. At that, our results won't be in a contradiction with smoothness of the evaporating cross sections, if there are increases in the partial widths (or radiative strength functions) in the points of the Cooper pairs' breaking. This effect can be rather confirmed by pygmy-resonance, which was discovered in many nuclei [17]. It should be noted that pygmy-resonyance take place not in all nuclei. But if to take into account that pygmy-resonance strength varies for different nuclei, whereas its center coincides well with the breaking point of the second Cooper pair, and the level densities of different nuclei noticeably vary in the points of Cooper pairs' breaking [6, 7, 8], then our experiments could explain the pygmi-resonance specific.

For a local intensification of the partial widths, an additional fitted coefficient M was introduced to their expressions [9]. The maximal value of the intensification was at $M = \rho_{fg}/\rho_{exp}$, where ρ_{fg} is the highest possible density of quasi-particle levels and ρ_{exp} is the level density, which provides an accurate description of experimental intensity distribution. In order to evaluate the probable increasing of the radiative strength functions, the parameter M varied at solving of the system of equations (1).

In order to investigate an interconnection between stepwise changes in the energy dependence of the nuclear level density and changes of the intensities of gamma transitions, four nuclei with different nucleon parities (^{172}Yb, ^{176,177}Lu and ^{193}Os) [6, 7] were analyzed. At that, the TSC intensities for these nuclei were fitted with an additional parameter M. The results of the analysis are presented in Figs.1–4.

An inability to describe $I_{\gamma\gamma}(E_1)$ distribution with help of $\rho(E_{ex})$ function calculated using Fermi-gas model is evident in Fig. 1. One can see also in Fig. 1 that the changes in the radiative strength functions (at various M) not lead to a noticeable modification of the approximated TSC intensity, whereas a mutual scatter of ρ and Γ became significantly bigger in the presence of M in the fittings (see Figs. 2, 3).

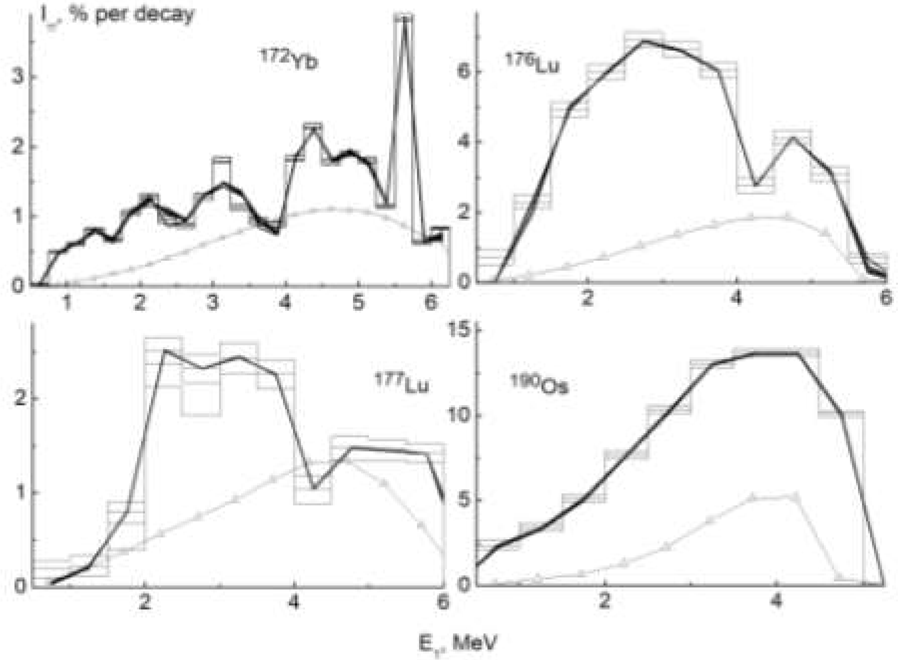

Fig. 1. The intensity distribution $I_{\gamma\gamma}(E_1)$ of primary transitions of TSC for ^{172}Yb, ^{176}Lu, ^{177}Lu and ^{193}Os nuclei. The broken solid line is the best fit; histogram is a total intensity of TSC in energy intervals with width of 0.5 MeV (0.25 MeV for ^{172}Yb); triangles is calculation using the models [18,19] with a constant value for $M1$-strength functions.

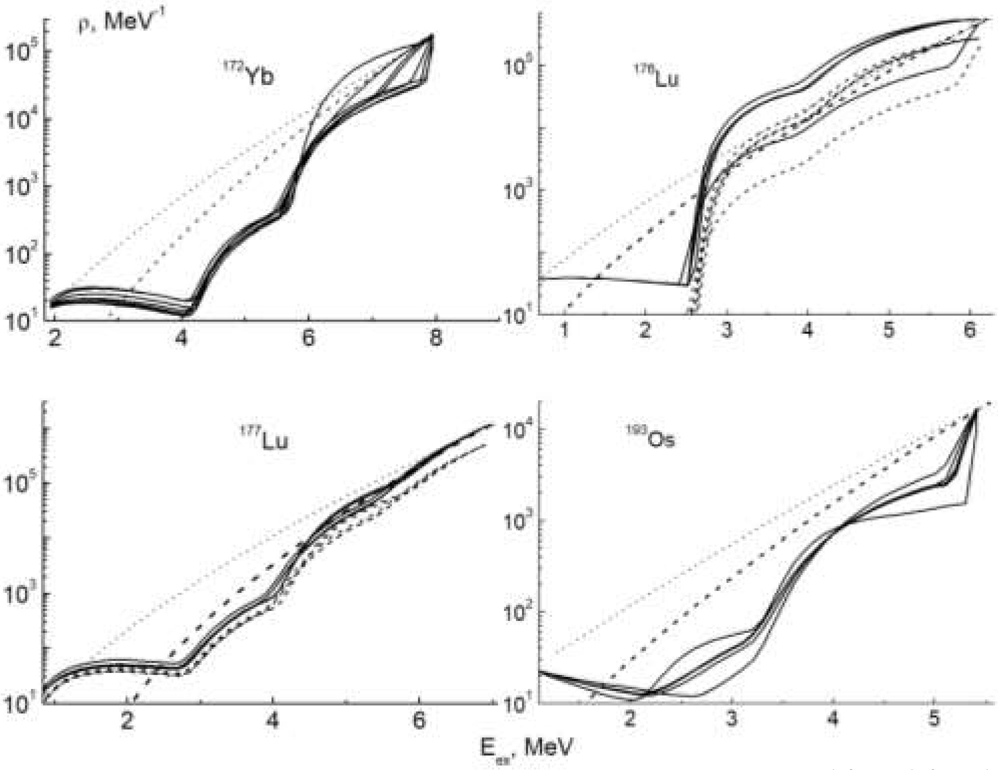

Fig. 2. The level density dependences on the excitation energy for TSC of ^{172}Yb, ^{176}Lu, ^{177}Lu and ^{193}Os nuclei. Solid lines are the best fits; dashed and dotted are level densities calculated using the Fermi-gas model with and without taking into account the shell inhomogenities of single-partial spectrum, correspondingly.

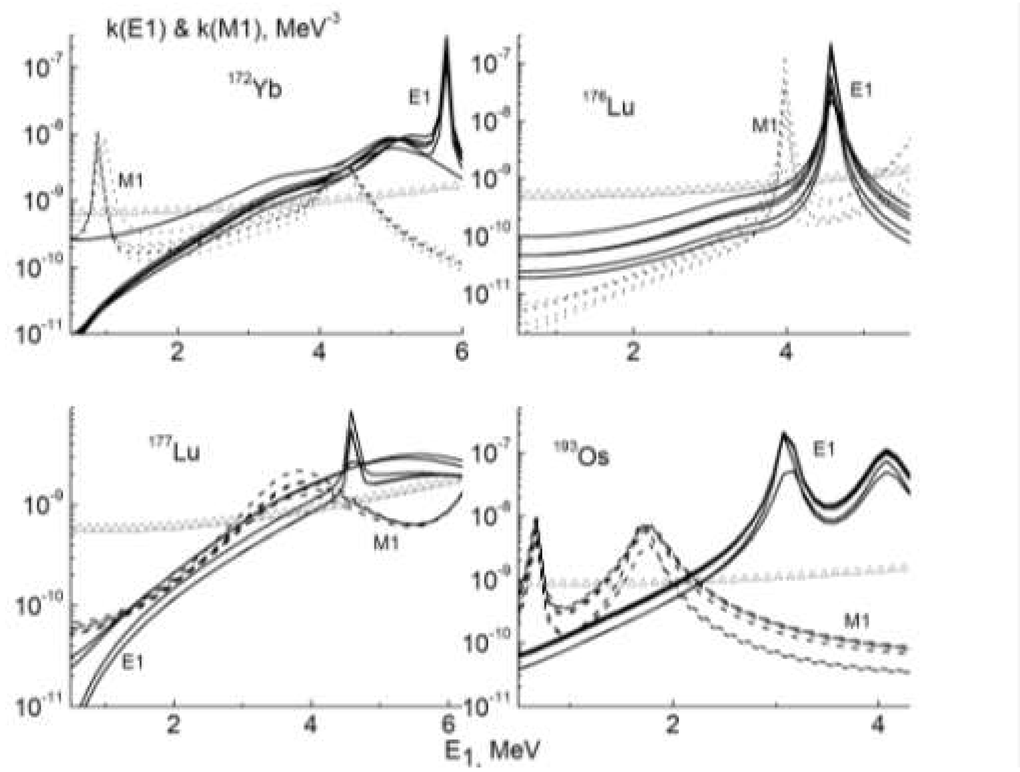

Fig. 3. The dependences of the radiative strength functions on energy of primary transitions of TSC ^{172}Yb, ^{176}Lu, ^{177}Lu and ^{193}Os nuclei. Solid lines – E1-transitions; dashed lines – M1-transitions.

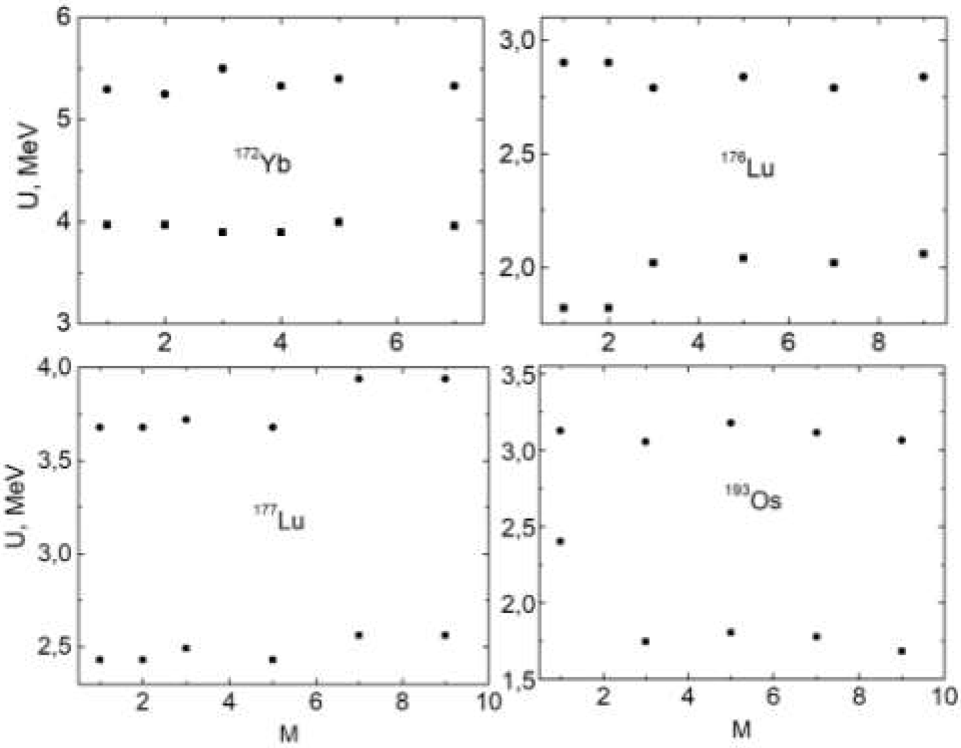

Fig.4. The breaking thresholds of the Cooper pairs of nucleons in ^{172}Yb, ^{176}Lu, ^{177}Lu and ^{193}Os nuclei at various M values. Points are the breaking thresholds U_2 for the second Cooper pairs; squares are the thresholds U_3 for the third pairs.

For ^{172}Yb and ^{193}Os the intense increase in the strength functions is observed at E_{ex} ~B_n. At that, intensity description becomes more precise at the energies lower than the breaking threshold for the second Cooper pair (at $E_1 \leq$ 1–2 MeV). Incidentally, in these nuclei the point of the fourth Cooper pair almost coincides with B_n (and in some nuclei just at this energy the phase transition to the Fermi-system is predicted [3]).

The most probable values of the breaking thresholds of the second and the third Cooper pairs at different parameters M are shown in Fig. 4. From their small scatter follows that a change of the intensity of emitted gamma-quanta weakly influences the positions of points of the Cooper pairs' breaking, but the observed intensity of gamma-transitions from the breaking point, at that, essentially changes for different nuclei and at various energies.

5. CONCLUSION

For the cascades measured with a small background and coincidence statistics high enough, the shape of $I_{\gamma\gamma}(E_1)$ distribution can be determined [11] with an sufficient uncertainty (of about several percent) to unambiguously determine nuclear parameters.

One can accept that experimental errors of the total sum $I_{\gamma\gamma}(E_1, E_1)$ are the same as the errors of the intense primary transition (5–10% per decay), which are used for normalization of the absolute values of the cascades. Such accuracy guarantees a reliable determination of the parameters of the cascade gamma-decay (including the breaking thresholds for Cooper pairs) even at improper representation of $\Gamma(E_1)$function.

Errors of the obtained $\rho(E_{ex})$ and $\Gamma(E_1)$ functions, which are describe the $I_{\gamma\gamma}(E_1)$ distribution with such accuracy, are mainly conditioned by an inaccuracy of their representations used by the empirical model [5].

When analyzing in the framework of the Dubna method, the $I_{\gamma\gamma}(E_1)$ intensities were described for 44 nuclei of the mass region of 28 ≤ A ≤ 200, and a realistic picture of interaction between Fermi- and Bose-states in the nucleus, below the neutron binding energy, was firstly obtained.

The experiments were carried out in Dubna, Riga, Rzhezh and Dalat. Now a group from Belgrad began the experiment at the reactors in Munich and Budapest.

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