Problems and Possibilities of a Study of the Cascade Gamma-Decay of a Nucleus Excited below the Neutron Binding Energy

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1. Introduction

In modern experiment, in which technologically advanced equipment is used for investigation a cascade gamma-decay of excited nucleus, a determination of the parameters of excited nuclear levels (spin, parity, lifetime, excitation energy) is of less importance than an obtaining of fundamental data about intranuclear processes (interaction of nucleons).

Precisely for this reason, it is necessary to determine, first of all, a sequence of gamma-quanta in the cascades of different multiplicities M. There are $M!$ variants of arrangement for any primary cascade-transition in the gamma-ray spectrum (in the decay scheme). If the two-step gamma-cascade (TSC) can be placed in the decay scheme by only two ways (one of them is false), then primary transition of the cascade with M=5, for example, have 120 variants of probable arrangement.

A task of determination of primary-transition spectra is solving in various indirect experiments differently:

- using a difference of total gamma-spectra for different excitation energies of the same nuclear reaction [Oslo];
- by calorimeter of total energy of the gamma-spectrum [Los-Alamos];
- by decomposition of the two-step gamma-cascade to two spectra of only primary and only secondary transitions with the use of both spectrometric information and shape difference of spectra of energy-resolved gamma-transitions and of continuum of unresolved transitions [Dubna].

Each of these ways without doubt has problems of analysis of indirect experiment which lead to inevitability of unknown sizeable systematical errors. Minimization of these errors is possible only by using potentials of these three methods.

An experimental obtaining of the reliable parameters of the cascade gamma-decay of any compound-states is exclusively important to understand processes which take place in an excited nucleus. The ground state of even-even nucleus, in terms of theorists, is “quasi-particle vacuum”, in which free fermions appear at an excitation energy E_{ex} (not just at $E_{ex}>B_n$, but also at $E_{ex}<B_n$, where B_n is the neutron binding energy in the nucleus). While the representation remained in being that the nucleus is a system of non-interactive Fermi-particles, a possibility for nucleons to form Cooper pairs could not be ruled out, and such pairs can break at any excitation energy. A process of breaking of Cooper pairs has not been experimentally investigated until now, as there are no high-aperture spectrometers of gammas with an electron-volt resolution.

When the B_n values compared with the nucleons' pairing energy Δ [1], it is reasonable to expect that 3–4 breaks of Cooper pairs of nucleons below the neutron binding energy must occur, at least, in investigated by us nuclei from mass region of $28 \le A \le 200$. To form a clear picture of intranuclear processes, it is necessary to determine in experiment the parameters of the cascade gamma-decay (the partial radiative widths Γ or the strength functions $k = \Gamma/(A^{2/3} \cdot E_\gamma^3 \cdot D_\lambda)$, where A is the nuclear mass number, E_γ is the energy of γ-quantum, D_λ is an average space between nuclear compound states, and the nuclear level density ρ) simultaneously

For the first time a technique of simultaneous determination of the nuclear level density ρ and partial widths Γ of emission of primary transitions from a total TSC spectrum was proposed and realized in Dubna, at LNP JINR in 1984 [2–4]. The first experiments with recording of the cascades of two quanta emitted sequentially, which have summarized energy of 5–10 MeV, were carried out with two Ge(Li)-detectors and statistics was amounted to several thousands of events of full absorption of cascade energy. From 2000 we began to use in our experiments HPGe-detectors, efficiency of which is essentially greater. By now, using this developed technique, the parameters of gamma decay for 44 nuclei [5, 6] have been determined from measured TSC intensity spectra.

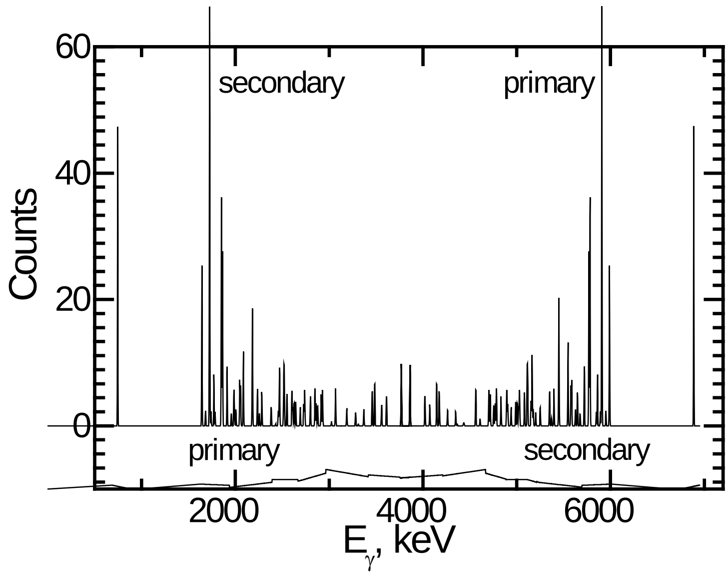

Fig.1. Quanta-energy dependence of the intensity of TSC to the first excited level with the energy E_f = 73 keV of ^{164}Dy nucleus. On top: the intensity of primary and secondary energy-resolved gamma-transitions; below: continuous distribution of unresolved transitions and small background.

Detectors with even tempered efficiency (~10%) allow from a lot of coincidences to extract events of simultaneous recording of two (and more) quanta, if their total energy is equal to total energy of decaying compound state or ~0.5–1 MeV less than this energy. Experimental spectra are composed of isolated energy-resolved (intense) gamma-transitions and continuum of unresolved ones. Latest transitions represent a continuous distribution with zero average and small amplitude [7]. A center of the distribution is at the energy of $0.5(E_1+E_2)$ in the quanta-energy scale, where E_1 and E_2 are energies of primary and secondary quanta of the cascade, correspondingly. A typical quanta-energy dependence of intensity of

TSC is shown in Fig.1 with an example of the intensity distribution of the cascade with the total quanta energy E_1+E_2= 7585 keV to the first excited level of ^{164}Dy nucleus.

Let's compare two experiments, in which the cascade gamma-decay at radiative capture of neutrons by ^{163}Dy nucleus was investigated. Both experiments with different approaches to the data analysis have problems of the alternative ways of obtaining of the nuclear parameters.

1) In Dubna experiment reaction of the radiative capture of thermal neutrons by ^{163}Dy nucleus has been investigated using two Ge(Li)-detectors placed in close geometry (opposite to each other and perpendicularly to the beam line). By the likelihood method (using some of appropriate models of the nuclear level density and partial widths) from the intensity of measured TSC the most probable nuclear level density and radiative strength functions were obtained in the energy range below B_n.
2) In Los-Alamos the same reaction for idem nucleus has been studied in the experiment with 4π-calorimeter of the cascade gammas [8]. Analysis of measured total gamma-spectra of the decay of neutron resonances of ^{164}Dy has been done using developed in Praha algorithm DICEBOX [9] for a simulation of the cascades of gamma-transitions of all possible multiplicities [10].

2. Problems of extracting of the nuclear parameters from gamma-spectrum

When analyzing the total gamma-spectrum, information about the nuclear parameters (Γ (or *k*) and ρ values) is always extracted from the data of *indirect* experiment. In order to obtain the parameters of the investigated nucleus approximating the experimental gamma-spectrum it is always required to use model representations about $\Gamma(E_\gamma)$ and $\rho(E_{ex})$ functions. And a transfer of errors of the experimental spectra to the errors of the determined parameters has to be carefully examined. At a difference between relative errors of experimental spectra $\delta S/S$ and of the required nuclear parameters, $\delta\rho/\rho$ and $\delta\Gamma/\Gamma$, in ~1.5–2 times, the absolute errors $\delta\rho$ and $\delta\Gamma$ can exceed δS in 5–10 times and more. We established that, in the detectors used by us, a difference between areas of the experimental and approximating spectra is usually not more than ~1% when spectra of gamma-cascades are averaged over 200–250 keV.

In spite of the fact that the different types of spectrometers are used in various experiments to measure the intensities of multiple gamma-cascades, for the same investigated nucleus a certain part of TSC intensity $I_{\gamma\gamma}$ at any measured spectrum is changeless. Therefore, a qualitative normalization of measured gamma-spectra per a decay of the nuclear compound state (neutron resonance) is necessary both in analysis with the use of simulation and in analysis by likelihood method.

2.1. Normalization of the cascade intensity

Collected to now and presented in files [11] and [12] data about spectra of the radiative capture of thermal neutrons allow determination of absolute intensity of the TSC $i_{\gamma\gamma}$ (in percent per one decay) with a good accuracy.

For all nuclei investigated in Dubna [5, 6] the normalization of the two-step cascades was carried out using absolute intensity of strong primary gamma-transitions from [11, 12]. For this to be done, from the experimental results the branching coefficients b_r for excited intermediate levels of nuclei were accurately determined offline as well as the intensities of primary transitions of the compound-states decay. Intensity of the decay of individual cascades, $i_{\gamma\gamma}=i_\lambda\cdot b_r$, where i_λ is compound-state decay intensity per one decay, allows to obtain a value of sum of all possible (both intense resolved and continuum of weak unresolved) cascades between initial level E_λ, intermediate levels E_i and final ones E_f. Result sum ($I_{\gamma\gamma}=\sum i_{\gamma\gamma}$) is enough for a determination of the nuclear level density and partial widths of primary cascade transitions in iteration process of solving the system of equations (1), which connect experimental intensities of the TSC gamma-transitions $I_{\gamma\gamma}$ with used parametrical functions $\rho(E_{ex})$ and $\Gamma(E_\gamma)$. The shape and area of measured spectra $I_{\gamma\gamma}$ are determined by a convolution of the required functions $\rho(E_{ex})$ and $\Gamma(E_\gamma)$.

Experimental distribution $I_{\gamma\gamma}(E_1,E_2)$ of the total intensity of the TSC can be fairly accurately described by infinite number of essentially different functions $\rho(E_{ex})$ and $\Gamma(E_\gamma)$ because of nonlinearity of the system of equations (1), which connect these strong correlated functions with $I_{\gamma\gamma}(E_1,E_2)$. And determination of the intensities $I_{\gamma\gamma}(E_1)$ of only primary transitions of the cascades limits a set of solutions of foresaid system of equations, so established sequence of cascade gamma-quanta is a necessity of simultaneous determination of reliable nuclear parameters using likelihood method.

In order to obtain the $I_{\gamma\gamma}(E_1)$ spectrum, which is wanted for further approximation, from the total spectrum $I_{\gamma\gamma}(E_1,E_2)$, it is necessary to establish the sequence of the gamma-quanta in the cascades. It is possible only when there is additional experimental information. In the Dubna technique, by means of the procedure described in [7], spectroscopic information was used thereto. Any TSC (after the procedure of numerical improvement of resolution [13]) is a mirror-symmetrical energy distribution of the intensities of primary and secondary gamma-transitions. Taking into account zero average and dispersion smallness of a noise line of a continuous distribution (see Fig.1) its subtraction from isolated peaks isn't a problem. Procedure [7] allows to determine a part of primary transitions of the cascades, $I_{\gamma\gamma}(E_1)$, with relative accuracy not worse than 10–20% in any energy interval, practically without a distortion of normalization of TSC intensity. For ^{164}Dy we obtained $I_{\gamma\gamma}(E_1)$ = 45.9% per one decay [14].

In analysis of the experiment with scintillation detectors [8] there is no possibility to use the ^{164}Dy decay scheme from [14] and the files [11, 12] for spectra normalization, as insufficient resolution of these detectors does not allow identification of individual intense transitions. In the paper [8] a normalization of the intensities of the two-step gamma-cascades per a decay is absent. It must be note that experimental intensity of the cascades with multiplicity of M =2 in experiment with 4π-calorimeter (4π-C experiment) was, most likely, underrate because of existent irremovable transfer of annihilation gamma-quanta between detectors' crystals at bad resolution of the spectrometer at low energies (at 1 and 6 MeV the resolution is about 16% and 7%, respectively). In energy region near B_n a cross-section of pair production is, by order of magnitude, greater than it is at the energy of gamma-transition of 1–2 MeV. Such "inter-scattering" increases areas of spectrum components with multiplicities of $M \geq 3$ and distorts (decreases) an area of TSC-component.

2.2. Possibility of determination of quanta sequence in the cascades

The cascades' quanta at a neutron capture are emitted sequentially but there is no possibility to determine experimentally their quanta sequence. If the cascade of two emitted quanta has only two variants of quanta placing in the decay scheme, then for three quanta there are 3!=6 variants of replacement, i.e. it is impossible to obtain reliable information about gamma-decay from spectra with multiplicity of $M \geq 3$.

When the data of 4π-C experiment analyzing, an objective information about Γ and ρ values can be extracted only from a spectrum with multiplicity of M = 2. But one should take into account that one of possible variants of quanta sequence for the decay scheme is *false* and must be excluded from analysis.

On a base of investigated TSC of more than four tens of nuclei [5, 6], it was experimentally established that at a statistics of ~5000 (or more) of recorded events of total cascade-energy absorption

- ✓ about 2/3 of the intensity of all primary transitions accounts for energy-resolved ones (Fig.1, on top), and gamma-quanta of resolved primary transitions, as a rule, have energy $E_1 > 0.5B_n$, i.e. the levels excited by them are in the "lower" half of the decay scheme with $E_{ex} < 0.5B_n$;
- ✓ about 1/3 of the intensity of all primary transitions accounts for a continuum of unresolved transitions with the energies $E_1 < 0.5B_n$ and excite levels at $E_{ex} \geq 0.5B_n$ (Fig.1, below), where tenth part of all intensity are transitions into energy region near $E_i \approx 0.5B_n$, in which weak primary and weak secondary transitions are mixed.

To obtain the best values of ρ and Γ, iteration process of approximation in our analysis is repeated many times with different initial parameters of required functions $\Gamma(E_\gamma)$ and $\rho(E_{ex})$ variating a correction-vector value. In DICEBOX simulation [9] used by the authors of [8], for three functions of the level density the most suitable radiative strengths are chosen from few variants from [15].

It must be noted that the level-density model of Strutinsky [16], which is used by us and not taken into account by the authors of [8], can be included to the analysis in any experiment on a study of the cascade gamma-decay. This model is successfully used in practice for a description of pre-equilibrium reactions [15], and an existence in a nucleus of the collective levels (vibrational and rotation ones) [17] is a basis of modern representations about gamma-decay.

3. The possibilities of indirect experiment

In Dubna technique, for determination of ρ and Γ values the standard likelihood method is used. When solving the system of equations (1), which connect in small intervals of primary transitions with unknown numbers of intermediate cascade levels n_j, where $M_{\lambda j}$ is the number of γ-transitions from level λ to intermediate levels n_j and m_{jf} is the number of secondary transitions to final level f of the cascade, corresponded unknown partial widths Γ and the experimental cascade intensities

$$I_{\gamma\gamma}(E_1) = \sum_{\lambda,f}\sum_{i}\frac{\Gamma_{\lambda i}}{\Gamma_\lambda}\frac{\Gamma_{if}}{\Gamma_i} = \sum_{\lambda,f}\sum_{j}\frac{\Gamma_{\lambda j}}{<\Gamma_{\lambda j}> M_{\lambda j}} n_j \frac{\Gamma_{jf}}{<\Gamma_{jf}> m_{jf}}, \quad (1)$$

the results of each successful iteration (trajectories of changing of the best ρ and Γ values and approximated $I_{\gamma\gamma}$ intensities) are presented graphically by approximation program. It allows monitoring the search process of the absolute minimum

$$\chi^2 = \sum_{n_j}\frac{(I_{\gamma\gamma}^{cal}(E_1) - I_{\gamma\gamma}^{\exp}(E_1))^2}{\sigma^2}, \quad (2)$$

where $I_{\gamma\gamma}^{cal}(E_1)$ and $I_{\gamma\gamma}^{\exp}(E_1)$ are model-parametrized and experimental intensities, and σ^2 is a dispersion of their difference. At χ^2-redundancy recognition the initial parameters of iteration process are corrected.

As determination of the nuclear parameters surely demands a usage of parametrical functions $\rho(E_{ex})$ and $\Gamma(E_\gamma)$, so there are no problem of degeneracy of the system (1). Usually, for a given model there is the only solution.

In Dubna technique there is no need to use any hypothesis untested experimentally. Now our analysis of the experiment is carried out on a base of a modern model of density of *n*-quasi-particle levels [16], on balance between an entropy change and a change of the energy of quasi-particle states [18] and on tested representations [19] about a shape of energy dependence of the radiative strength functions. Of course, our results (the nuclear parameters obtained simultaneously) have a systematic error connected with inaccuracy of used models, and only by improvement of these models it is possible to decrease the error. A comparison of the results obtained using different modern models would promote a clarification of intranuclear processes.

The other sources of systematic errors of $\rho(E_{ex})$ and $\Gamma(E_\gamma)$ functions are limited interval of spins of excited levels for TSCs (this interval is determined by spins of initial and final levels and dipole type of gammas) and incomplete number of energy-resolved transitions considered in the experiment.

4. Results of reanalysis of 4π-C experiment

To evaluate an influence of all possible errors on a value and a shape of the experimental distribution $I_{\gamma\gamma}$, which was obtained for ^{164}Dy nucleus with a calorimeter [8], we analyzed the Los-Alamos data for M=2 using likelihood method taking into account anti-correlation between required nuclear parameters, ρ and Γ.

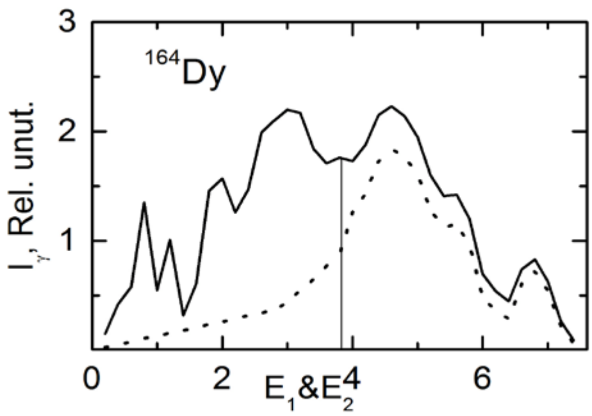

Fig. 2. Intensity distribution $I_{\gamma\gamma}$ (E_γ) of the TSCs in ^{164}Dy. Solid line – experimental-intensity distribution of all transitions for M=2 from [8]; dashed line – intensity distribution of only primary transitions estimated according to the spectroscopy data from [14].

The shape of $I_\gamma(E_1)$-distribution shown in Fig.2 by dashed line was obtained using spectroscopy data from [14]. As we have no experimental and theoretical bases for identification of the parameters (the number of quantum in the cascade, the level life-time) for primary and secondary gamma-transitions in 4π-C experiment, we cannot determine the quanta sequence in the framework of our procedure. Nevertheless, evaluating (on a base of the results from [14]) the part of intensity contained mainly primary gamma-transitions and keeping an equality $I_{\gamma\gamma}(E_1) = I_{\gamma\gamma}(E_2)$ (an area under dash line is equal to a difference of areas under solid line and dash line) we can minimize an error of our analysis of 4π-C experiment.

As there are no published values of the intensities for TSC spectra per one decay in [8], so analyzing the experimental spectra of 4π-C experiment we made calculations for the intensity $I_{\gamma\gamma}$ = 45%, which correspond to the value from Dubna [14] experiment for the same nucleus, and, for a comparison, for an underrated value $I_{\gamma\gamma}$ = 22% (see. Fig. 3).

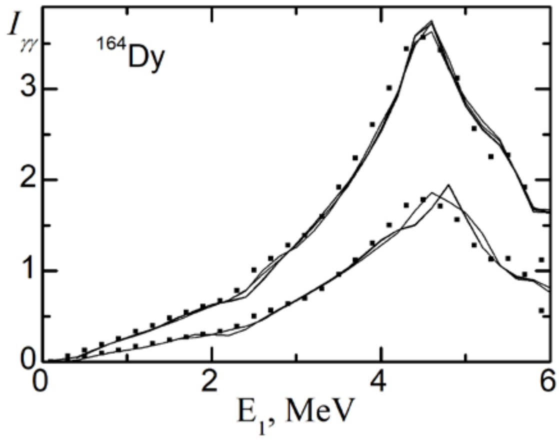

Fig.3. Intensity distributions of part of primary transitions of the TSCs in ^{164}Dy: upper solid line and points are the best fit and a shape of expected distribution for a sum intensity $I_{\gamma\gamma}(E_1)$=45% per one decay, bottom line and points – the same for $I_{\gamma\gamma}(E_1)$ =22% per a decay (the energy bin is 200 keV).

From the experimental two-step gamma-spectra of [8], after its division according to procedure [7] into two parts (of mostly primary or secondary transitions of the cascades), the part of secondary transitions was eliminated (see. Fig. 1).

Results of analysis of the 4π-C experiment carried out by our technique are shown in Figs. 3–7, and they distinguish from the data of analysis of our experiment [20] (see Figs. 8, 9). In our reanalysis of 4π-C experiment the breaking thresholds of the second and the third Cooper pairs of nucleons in ^{164}Dy were obtained at the energies of 3.05(6) and 5.0(1) MeV. In Dubna experiment [20] with a capture of thermal neutrons, the breaks of the same pairs are happened at 2.57(1) and 5.48(5) MeV, correspondingly. It is seen in Figs. 3–7 that a difference in the intensities of primary transitions (Fig. 3), which are approximated by strong-correlated functions $\rho(E_{ex})$ and $\Gamma(E_1)$, has a various influence on the obtained nuclear parameters: a step-wise level-density distribution changes much less (Figs. 4, 5) than an energy dependences of the strength functions of electrical and magnet dipole transitions (Figs. 6, 7).

As just $\rho(E_{ex})$-function has mainly influence on description of the spectrum of the cascades intensity, it seems reasonable to conclude that, without taking into account anti-correlation between the nuclear parameters, approximating data by smooth functions the authors of [8] lose a possibility to discover a dependence of the nuclear parameters on the structure of wave functions of excited levels of nucleus.

Modern theoretical representations both about a co-existence in a nucleus of quasi-particle and vibrational levels [15] and about a fragmentation of states [22] of a certain type at increasing of the excitation energy of nucleus point to a presence of various structure of the wave-functions of excited nuclear levels, which eliminates a smooth energy dependence of the level density and radiative strengths.

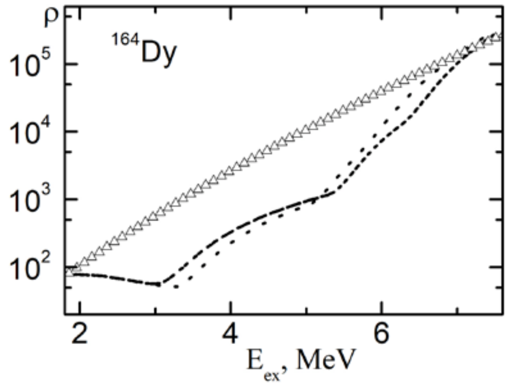

Fig.4. The most probable level density for ^{164}Dy (spin J=2). Lines are different approximations at $I_{\gamma\gamma}(E_1)$=22%. Triangles is a calculation using the model [21].

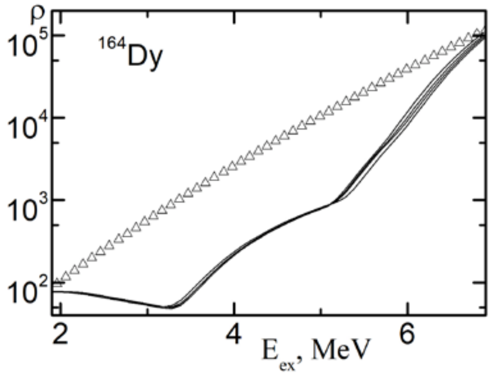

Fig.5. The most probable level density for ^{164}Dy (spin J=2). Lines are different approximations at $I_{\gamma\gamma}(E_1)$=45%. Triangles is a calculation using the model [21].

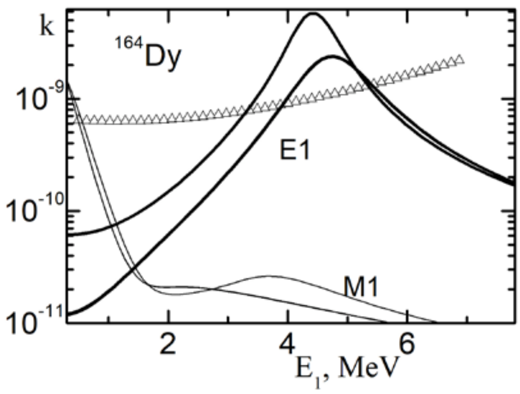

Fig.6. The most probable values of radiative strength functions of $E1$- and $M1$-transitions for cascades of M=2 in ^{164}Dy at $I_{\gamma\gamma}(E_1)$=22% (lines). Triangles is a calculation using the model [19] in a sum with $k(M1)$=const.

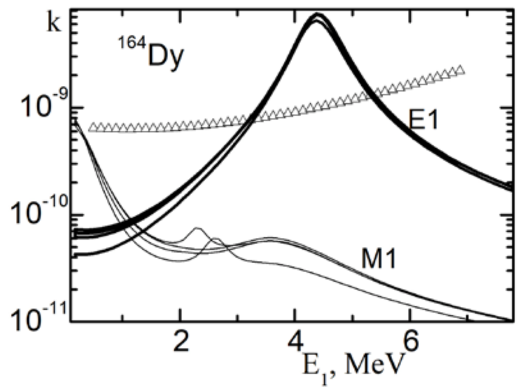

Fig.7. The most probable values of radiative strength functions of $E1$- and $M1$-transitions for cascades of M=2 in ^{164}Dy at $I_{\gamma\gamma}(E_1)$=45% (lines). Triangles is a calculation using the model [19] in a sum with $k(M1)$=const.

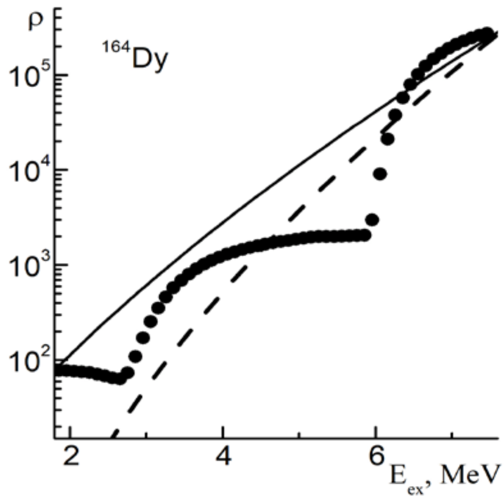

Fig.8. Distribution of ^{164}Dy level density from our analysis of the data of [20]. Points is the best fit, solid and dotted lines are calculations using the model [21] without taking into account a correction on the shell inhomogenities and with this correction, correspondingly.

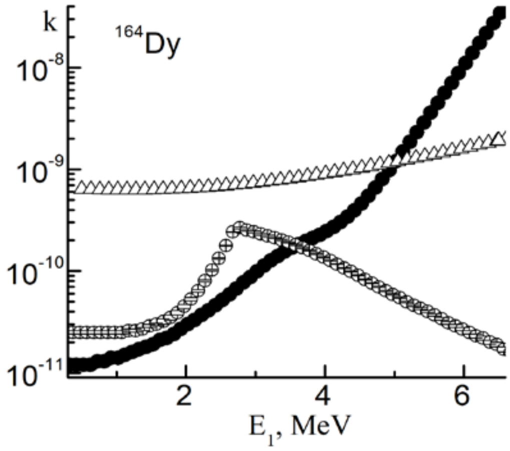

Fig.9. Dependences of the radiative strength functions on the energy of primary transitions of cascades in ^{164}Dy (analysis of the data of [20]): for *E*1-transitions – close points, for *M*1-transitions – open points. Triangles is calculation using the model [19] in a sum with k(*M*1)=const.

5. Gamma-spectrum simulation

There is good reason to think that in the analysis using a technique with spectra simulation [8], just as in the paper [10] of the same experimenters' group, the criterion was used:

$$\chi^2 = \sum_{i=1}^{N} \frac{\left(\overline{A}_{\exp}(E_i) - \overline{A}_{sim}(E_i)\right)^2}{\varepsilon^2_{\exp}(E_i) + \varepsilon^2_{sim}(E_i)}, \qquad (3)$$

where $\overline{A}_{\exp}(E_i)$ and $\overline{A}_{sim}(E_i)$ are the counts in E_i energy bin of measured and simulated spectra, and $\varepsilon^2_{\rm exp}$ and $\varepsilon^2_{\rm sim}$ are corresponding variances. The spectra consisted of *N* bins.

When gamma-spectrum simulation using, determining $\rho(E_{\rm ex})$ and $\Gamma(E_1)$ it is necessary to distinguish primary transitions in TSC spectrum which isn't done by the authors in the analysis of 4π-C experiment. Emission widths of second and the followed transitions are unknown in the considered experiment. In the experiment with ^{164}Dy nucleus presented in [23], the partial widths of secondary transitions were obtained only for decay of 44 levels in excitation-energy region of 1675 – 3984 keV (these data are not considered in the simulation [8]), and the experimental data for the rest levels are absent. Unknown secondary gamma-transitions and the cascades with multiplicity M ≥ 3 are useless for a search of the most probable values of the level density and partial widths.

Authors of [8] suppose that energy distribution for the level density is «a priori known», so they attempt to find the radiative strengths *k*, which are acceptable for measured gamma-spectrum, calculating *k* in simulation under different existent models. But testing of different radiative strength functions makes sense to do only at simultaneous testing of used models for the level density.

Conclusion

By the technique developed in Dubna it is possible to approximate precisely any experimental gamma-spectrum of TSC intensities by functions $\rho(E_{\rm ex})$ and $\Gamma(E_1)$ parametrized with the help of models of the required nuclear parameters [16–19], without using Porter-Tomas distribution and Axel-Brink hypothesis. In such analysis the nuclear parameters are obtained from the experiment simultaneously. By now the best approximation of the experimental data was obtained with the use of the model of density of *n*-quasi-particle levels [16].

The best χ^2 value from a set of ones obtained solving the system (1) using the Dubna technique guarantees a maximal accuracy of the nuclear parameters extracted in the experiment if to compare with alone χ^2 (3) from the model with gamma-spectrum simulation [8].

The step-wise structure which are observed in the obtained energy dependence $\rho(E_{ex})$, at a distance between "steps" in the energy scale of about 2Δ, where Δ is the pairing energy of the last nuclear nucleon, can be explained as points of breaks of the Cooper pairs of nucleons.

Approximation of experimental TSC-intensities using a smooth function $\rho(E_{ex})$ postulated by the authors of [8], which excludes step-wise effect, results in appreciable increase in χ^2.

Because of an absence of the data of the intensities of individual primary transitions of the cascades for a spectrum of quanta multiplicity of M=2 (and a large number of possible quanta arrangements for cascades with $M \geq 3$), in the considered 4π-C experiment there is no possibility to study intranuclear processes at nuclear excitations as well as to test existing models of the nuclear parameters.

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