

Experimental Study of the Gamma-Decay of Compound-States of ^{56}Mn and ^{94}Nb Nuclei in the $(n_{\text{th}}, 2\gamma)$ -Reaction

Anatoly M. Sukhovoj¹, Liudmila V. Mitsyna¹, Nikola Jovančević², David Knežević³, Aleksandar Dragić³, László Szentmiklósi⁴, Tamás Belgya⁴, Zsolt Revay⁵, Christian Stieghorst⁵, Stephan Oberstedt⁶, Miodrag Krmar², Dimitrije Maletić³, and Dejan Joković³

¹ *Joint Institute for Nuclear Research, 141980 Moscow region, Dubna, Russia*

² *University of Novi Sad, Faculty of Science, Department of Physics, Trg Dositeja Obradovica 3, 21000, Novi Sad, Serbia*

³ *University of Belgrade, Institute of Physics Belgrade, Pregrevica 118, 11080 Zemun, Serbia*

⁴ *Centre for Energy Research, Hungarian Academy of Sciences, Budapest, Hungary*

⁵ *Technische Universität München, Forschungsneutronenquelle Heinz Maier-Leibnitz (FRM II), Lichtenbergstr.1, D-85747 Garching, Germany*

⁶ *European Commission, Joint Research Centre, Directorate G Nuclear Safety and Security*

When the results obtained from the reactions $^{55}\text{Mn}(n_{\text{th}}, 2\gamma)$ and $^{93}\text{Nb}(n_{\text{th}}, 2\gamma)$ [1, 2] analyzing, the experimental-data array on the intensities of two-step gamma-cascades (TSC) at a decay of compound-nuclei ^{56}Mn and ^{94}Nb after radiative capture of thermal neutrons was enlarged, which already contain the data of for 44 nuclei in mass region $28 \leq A \leq 200$. The experimental analysis was carried out with the use of the empirical Dubna model of the cascade gamma-decay, approximating of the $I_{\gamma\gamma}(E_1)$ -intensities by parametrical functions of the nuclear level density, $\rho = \varphi(p_1, p_2, \dots)$, and partial radiative widths, $\Gamma = \psi(q_1, q_2, \dots)$, when the most probable nuclear parameters were fitted simultaneously. The Dubna model provides a high precision of experimental-data description.

1. Introduction

The basic parameters of any nucleus at a changing of its excitation energy can be determined only if spectra of products of investigated nuclear reactions are experimentally measured. And the spectrum of total gammas at the decay of any high-lying nuclear level (compound-state) is one of the most convenient for the nuclear-parameters extracting. But such spectrum cannot be completely separated into individual γ -transitions because the level density in the absolute majority of nuclei is too high for modern-spectrometers' resolution. Therefore, in spite of an existence of the complete system of equations, which connect the nuclear level density, ρ , and partial widths, Γ , of the γ -decay of excited nuclear states in each point of the excitation energy, E_{ex} , ρ and Γ values cannot be calculated exactly. Thus, the nuclear parameters (ρ and Γ or radiative strength functions $k = \Gamma / (A^{2/3} \cdot E_{\gamma}^3 \cdot D_{\lambda})$, where A is a mass of nucleus, D_{λ} is an average space between its levels, and E_{γ} is an energy of emitted γ -quantum) are always determined from the experiment indirectly.

For indirect experiments, situation is always strongly hindered by a inevitability of large coefficients of transfer of errors of the experimental spectrum to the errors of the

required nuclear parameters – relative errors of extracted ρ and k values can be several orders greater than the relative error of measured TSC intensity, $I_{\gamma\gamma}$.

For γ -transition identification in the measured total gamma-spectrum, it is necessary to determine the sequence of γ -quanta in the cascade, the excitation energy of nucleus, the number of decaying levels for each interval of excitation energy, etc. In other words, there are too many values, which must be extracted in experiment directly. Because of a deficiency of experimental information, a simultaneous determination of ρ and Γ values from the experimental data is possible only using different appropriate model representations about $\rho(E_{ex})$ - and $\Gamma(E_{\gamma})$ -functions when different acceptable models of the gamma-process are tested.

One should take into account that the partial widths and nuclear level density are strong anti-correlated values, so they cannot be determined independently. This fact is practically disregarded when the modern models of these parameters are tested. With the regard for this anti-correlation, in the experiment with recording of total-cascades energy (if a difference between the summarized energy of the cascade quanta and the energy of decaying compound-state isn't more than 0.5–1 MeV), the Fermi-gas model of the level density [3] and a representation that a nucleus is a system of non-interactive particles are not apparent for using.

A development of realistic models of a nucleus, as, for example, “generalized model of superfluid nucleus” created in Obninsk [4] and a fact of an existence of a collective-excitation spectrum in any nuclei (with excitations of both vibrational and rotation types) also demand to develop representations about nucleus which would be alternative to the statistical model. First of all, it is need to establish a set of models which take into account the data of modern experiments.

This way was just proposed in Dubna [5, 6, 7], where, for the first time, a possibility was shown to use experimental data on the TSCs of γ -transitions recorded by Ge-detectors for simultaneous obtaining of the gamma-decay parameters. The experiment gives a spectrum of primary transitions, but principally different spectrum of secondary transitions is unknown – only branching coefficients are obtained experimentally.

In order to reduce an absolute anti-correlation between required nuclear parameters, ρ and Γ (or k), in the Dubna analysis we transform the experimental mirror-symmetrical quanta-energy dependences of the TSCs, $I_{\gamma\gamma}(E_1, E_2)$, where E_1 and E_2 are energies of primary and secondary quanta of the cascades, to two energy dependencies $I_{\gamma\gamma}(E_1)$ and $I_{\gamma\gamma}(E_2)$. And only the $I_{\gamma\gamma}(E_1)$ distribution is described with the use of the maximum likelihood method by appropriate parametrized functions $\rho = \varphi(p_1, p_2, \dots)$ and $\Gamma = \psi(q_1, q_2, \dots)$ at all excitation energies of nucleus. Fitted parameters p and q of the most probable functions $\rho = \varphi(p_1, p_2, \dots)$ and $\Gamma = \psi(q_1, q_2, \dots)$ were determined fitting the model description of the cascade intensity $I_{\gamma\gamma}(E_1)$ to the experimental one. A necessity of description of just the $I_{\gamma\gamma}(E_1)$ -distribution and a procedure of its separation from the mirror-symmetrical $I_{\gamma\gamma}(E_1, E_1)$ distribution, which composed of isolated energy-resolved intense gamma-transitions and continuum of unresolved ones with zero average and small dispersion, are described in [8].

The reliable information about the nucleus can be obtained when comparing several model representations. The maximum likelihood method excludes using of Porter-Thomas and Axel-Brink hypothesis. An accuracy of approximation of the experimental distributions of the cascades intensities is determined by a total error of the $I_{\gamma\gamma}(E_1)$ -distribution and varied shapes of functions $\rho = \varphi(p_1, p_2, \dots)$ and $\Gamma = \psi(q_1, q_2, \dots)$, which are explicitly defined for different ways of iteration process at small variations of p and q parameters.

As in coincidence experiments the energies of initial state and of final levels of the cascade are unambiguously defined, it allows a determination (using additional spectroscopic data) of the energies of intermediate levels with a not great systematic error in the energy region near the center of $I_{\gamma\gamma}(E_1, E_2)$ -distribution, where the energies of primary and secondary transitions are preliminary equal [8]. It means that the use of a technique of decay-scheme constructing [5] gives a possibility to include in analysis all accumulated spectroscopic data [9].

Moreover, a simultaneous determination of the nuclear parameters promotes an understanding of the intranuclear processes. Thanks to the fact that the TSCs connect low-lying weakly-fragmented nuclear levels and high-lying energy region of levels with strong fragmentation, a possibility to study nuclear superfluidity appears. The principal problem for a study of superfluidity in the excited nucleus is a choice of model representations for reliable description of the investigated process. In a majority of the world's experiments, in order to obtain the level density and the partial widths of the products of the nuclear reaction, the models are used which are based on the calculations of different spectra and cross-sections at the large excitation energies [3, 10, 11]. But experiments on many nuclei [12, 13] showed that the intensities of the γ -cascades cannot be exactly described by energy dependences of the nuclear parameters if they are represented with the use of predictions of the conventional models [14]. Most probably, this is because of unremovable difference in the wave-functions structures of excited levels of various nuclei.

2. Representation of the nuclear parameters in empirical Dubna model

In order to extract reliable experimental information about a behavior of a superfluid phase of the nuclear matter it is necessary:

- 1) to measure an intensity of gamma-cascades to the low-lying levels of investigated nucleus (both the ground state and a group of levels with small energies);
- 2) to ensure the best description of measured spectra at a simultaneous fitting of the parameters of both the level density and the partial radiative widths.

In the absence of credible theoretical models we created our empirical model, inclusive of the different realistic phenomenological representations, without even generally-accepted representations, which are not tested experimentally.

For the level-density description in the present analysis, the model of density of n -quasi-particle nuclear excitations [3, 15], which is commonly used in a study of the pre-equilibrium reactions, was parameterized. The density ρ_l of levels of fermion type above the expected breaking threshold of the l th Cooper pair of nucleons in nucleus was written as:

$$\rho_l = \frac{(2J+1) \cdot \exp(-(J+1/2)^2 / 2\sigma^2)}{2\sqrt{2\pi}\sigma^3} \Omega_n(E_{ex}),$$

$$\Omega_n(E_{ex}) = \frac{g^n (E_{ex} - U_l)^{n-1}}{((n/2)!)^2 (n-1)!}. \quad (1)$$

Here Ω_n is a density of n -quasi-particle states, σ is a spin cutoff-factor parameter, J is a spin of the compound-state of nucleus, g is a density of single-particle states near Fermi-surface, and U_l is the breaking energy of the l th Cooper pair of nucleons (or the energy of an excitation of pair of quasi-particles).

For a description of the coefficient C_{coll} of an increase in a density of collective levels, a phenomenological relation between entropies of phases of the nuclear matter was used [10] with taking into account a cyclical break of Cooper pairs:

$$C_{col} = A_l \exp(\sqrt{(E_{ex} - U_l)/E_u} - (E_{ex} - U_l)/E_u) + \beta. \quad (2)$$

Here A_l are fitting parameters of vibrational level density above the breaking point of each l th Cooper pair, parameter $\beta \geq 0$ can differ from 0 for deformed nuclei. It turned out, that parameter E_u (a rate of a change in densities of quasi-particle and phonon levels) is practically equal to the average pairing energy of the last nucleon in the majority of investigated nuclei [12, 13, 16].

As was experimentally determined earlier [17], a closest fit to the intensities of the TSCs is possible only if to add one or two peaks to the smooth energy dependence of the radiative strength functions of $E1$ - and $M1$ -transitions. The smooth parts of the energy dependences $k(E1, E_\gamma)$ and $k(M1, E_\gamma)$ were described just as in the model [11], but with addition fitted parameters of weight w_E (or w_M) and of a change of derivatives of the strength function κ_E (or κ_M), where indexes E and M refer to $E1$ - or $M1$ -transitions, correspondingly. And in order to take into account the local peaks, one or two summands were added to the smooth parts of the strength functions. And at a description of the shape of each local peak by the asymmetric Lorentzian curve, $k(E1, E_\gamma)$ and $k(M1, E_\gamma)$ strength functions were expressed similarly as:

$$k(E1, E_\gamma) = w_E \frac{\Gamma_{GE}^2 (E_\gamma^2 + \kappa_E 4\pi^2 T_E^2)}{(E_\gamma^2 - E_{GE}^2)^2 + E_{GE}^2 \Gamma_{GE}^2} + \sum_i W_{Ei} \frac{(E_\gamma^2 + (\alpha_{Ei} (E_{Ei} - E_\gamma)/E_\gamma)) \Gamma_{Ei}^2}{(E_\gamma^2 - E_{Ei}^2)^2 + E_\gamma^2 \Gamma_{Ei}^2}, \quad (3)$$

$$k(M1, E_\gamma) = w_M \frac{\Gamma_{GM}^2 (E_\gamma^2 + \kappa_M 4\pi^2 T_M^2)}{(E_\gamma^2 - E_{GM}^2)^2 + E_{GM}^2 \Gamma_{GM}^2} + \sum_i W_{Mi} \frac{(E_\gamma^2 + (\alpha_{Mi} (E_{Mi} - E_\gamma)/E_\gamma)) \Gamma_{Mi}^2}{(E_\gamma^2 - E_{Mi}^2)^2 + E_\gamma^2 \Gamma_{Mi}^2}. \quad (4)$$

Here E_{GE} (or E_{GM}) and Γ_{GE} (or Γ_{GM}) are location of the center and width of the maximum of the giant dipole resonance, T_E (or T_M) is varied nuclear thermodynamic temperature, for $E1$ - (or $M1$ -) transitions. And for each i th peak ($i \leq 2$) of the strength functions of $E1$ - (or $M1$ -) transition: E_{Ei} (or E_{Mi}) is a center position, Γ_{Ei} (or Γ_{Mi}) – width, W_{Ei} (or W_{Mi}) – amplitude, and α_{Ei} (or α_{Mi}) $\sim T^2$ is an asymmetry parameter. A necessity of taking into account a local-peak asymmetry in the radiative strengths follows from theoretical analysis of features of the fragmentation of single-particle states in the nuclear potential [18]. In the fitting process the functions (3) and (4) vary in wide range of parameters.

The shell inhomogeneties of a single-partial spectrum were also taken into account in our analysis (see detailed description in [10]).

3. Analysis of the experimental data

In fig. 1 the spectra of sums of amplitudes of coincident pulses are shown for the cascade gamma-decay of compound-nuclei ^{56}Mn (left picture) and ^{94}Nb (right picture). For ^{56}Mn nucleus, 5 cascades were recorded at $E_1 + E_2 = 6930, 7057, 7157, 7243,$ and 7270 keV. The rest of peaks in this spectrum corresponds to recording of energy sums of primary γ -quanta and third (or fourth, etc.) cascade quanta as well as to recording of primary quanta of the cascades in their single-escape modes. And for ^{94}Nb compound-nucleus, 7 two-step cascades were recorded with total energies of $6831, 6916, 7087, 7114, 7168, 7186,$ and 7227 keV.

Absolute intensities of all cascades were determined with the use of experimental data from the cites [9, 19] on intense primary gamma-transitions to low-lying levels of investigated nuclei, and branching coefficients for intermediate levels were obtained from experimental data array of collected coincidences. Procedure [20] of resolution improvement of intense cascade gamma-transitions essentially increases an accuracy of determination of their intensities.

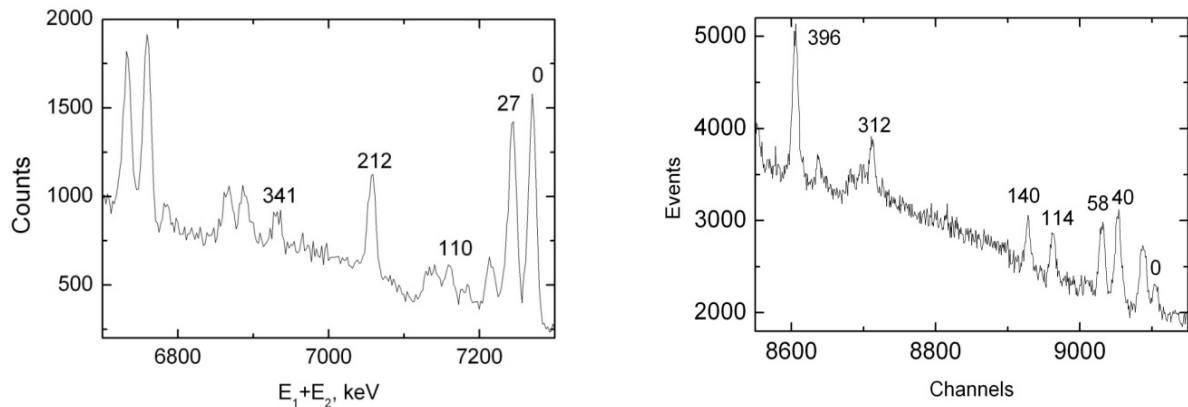


Fig. 1. The spectra of the cascades intensity at radiative capture of thermal neutrons by ^{55}Mn (left picture) and ^{93}Nb (right picture) nuclei. Peaks of the full capture of the TSC energy are marked by energies of their final levels.

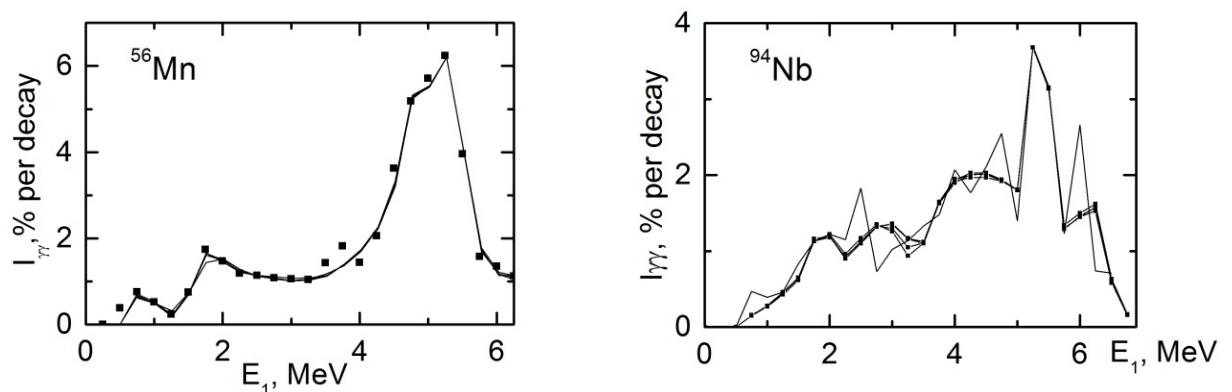


Fig. 2. Intensity distributions for the TSCs to final levels with the energies $E_f \leq 341$ keV for ^{56}Mn and $E_f \leq 396$ keV for ^{94}Nb as a function of energy of primary transitions. Solid lines – several fits for $I_{\gamma\gamma}(E_1)$ summarized over energy intervals $\Delta E = 250$ keV.

The total intensity of abovementioned TSCs of ^{56}Mn is 50.9(18)%, and for ^{94}Nb nucleus it is 35.2(40)% per a capture. The parts of intense energy-resolved peaks of γ -transitions for the cascades of these nuclei are 0.597 and 0.56.

Error of normalization of the experimental $I_{\gamma\gamma}(E_1)$ spectra are 4% for ^{56}Mn and 10% for ^{94}Nb [9]. And in each energy bin of $I_{\gamma\gamma}(E_1)$ distributions, a difference between experimental and approximated values is a few times less. In spite of an uncertainties of the $I_{\gamma\gamma}(E_1)$ -distributions for a majority of nuclear cascades are largish with respect to modern nuclear-data requirements, such accuracy is acceptable for obtaining of the reliable nuclear parameter. An accuracy of bipartition of the total spectrum of the TSC intensity, on spectra of

primary and secondary transitions, increase with increasing in statistics of recorded coincidence events.

As for investigated earlier light spherical nuclei (^{40}K , ^{52}V , ^{60}Co , ^{64}Cu) [13], $I_{\gamma\gamma}(E_1)$ -distributions for nuclei ^{56}Mn and ^{94}Nb decrease at low energies of primary transitions and increase at their bigger energies. Functions $I_{\gamma\gamma}(E_1)$ for nuclei ^{56}Mn and ^{94}Nb have similar general trend and mostly vary from each other in energy region near $0.5B_n$ due to a difference in the intensities of their cascades there. At the same time, these $I_{\gamma\gamma}(E_1)$ -distributions nothing like ones for heavy odd-odd nuclei in the energy region of 2–4 MeV [13], i.e. there is a certain factor, which provides a principal difference of the cascade-decay spectra for light and heavy nuclei.

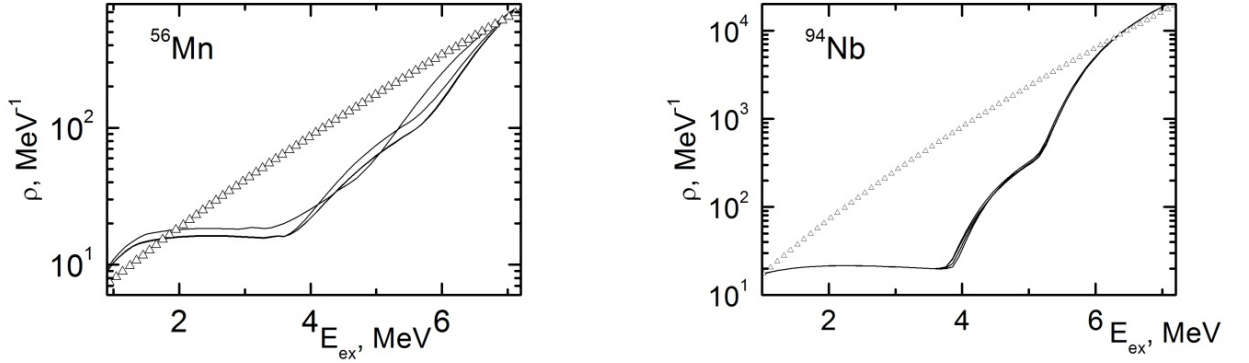


Fig. 3. Dependences of the level densities on the excitation energy of ^{56}Mn and ^{94}Nb nuclei. Lines are several fits, triangles are calculations using the back-shift Fermi-gas model [3].

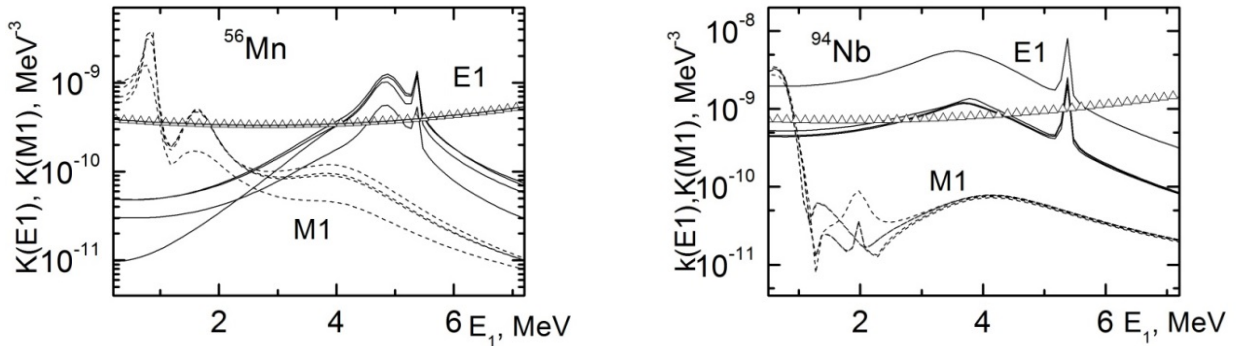


Fig. 4. The radiative strengths as functions of the energies of primary cascade quanta obtained for ^{56}Mn and ^{94}Nb nuclei. Solid and dotted lines are the best fits for $E1$ - and $M1$ -transitions, correspondingly. Triangles are expected values calculated using the model [11] for electrical transitions in a sum with a constant radiative strength for magnet transitions.

As for all investigated earlier light odd-odd nuclei, in ^{56}Mn and ^{94}Nb nuclei the $\rho(E_{\text{ex}})$ -distributions have the very strong deviations from their calculations using the model of [3] at excitation energies $E_{\text{ex}} \approx 4$ MeV (for investigated heavy odd-odd nuclei it is at $E_{\text{ex}} \approx 3$ MeV) [13]. And like the other light odd-odd nuclei, a step-wise behavior of the fitted parametrical $\rho(E_{\text{ex}})$ -functions was also discovered for ^{56}Mn and ^{94}Nb nuclei which can be explained by a decrease of the density of the levels of vibrational type between the breaking thresholds of Cooper pairs of nucleons.

The probable values of $E1$ - and $M1$ -radiative strength functions for ^{56}Mn and ^{94}Nb nuclei in dependence on the energies of primary transitions of the cascades are presented separately in fig. 4. A noticeable difference in the radiative strengths for $E1$ - and $M1$ -transitions for various energies of primary transitions can be qualitatively explained by diverse structures of their wave-functions. An existence of fluctuations (appearance of some sharp deviations) in the best fits of functions $k(E1)$ and $k(M1)$ can be explained by residual anti-correlation of these functions with $\rho(E_{\text{ex}})$ -functions.

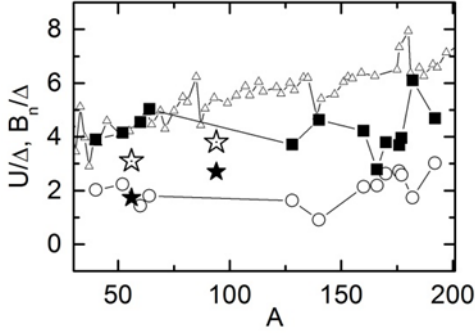


Fig. 5. The breaking thresholds U_2 and U_3 of the second (circles) and the third (squares) Cooper pairs of nucleons obtained for odd-odd nuclei [13]. Black and open stars – obtained in present analysis U_2 and U_3 for ^{56}Mn and ^{94}Nb , correspondingly. Triangles – the neutron binding energy in units of pairing energy Δ_0 .

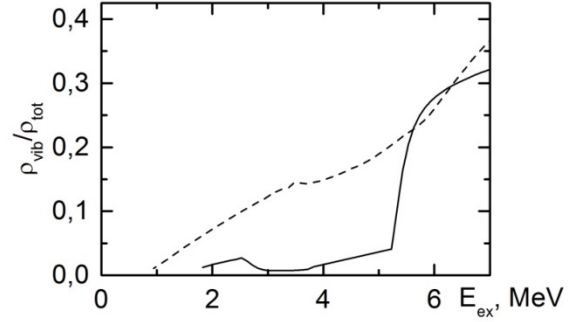


Fig. 6. The ratio of the density of vibrational levels to the total density of nuclear levels in ^{56}Mn (dashed line) and in ^{94}Nb (solid line).

The obtained in the analysis ratios U_2/Δ_0 and U_3/Δ_0 for ^{56}Mn and ^{94}Nb are shown in fig.5 among these ratios for all investigated earlier odd-odd nuclei, and in fig. 6 the ratios $\rho_{\text{vib}}/\rho_{\text{tot}}$ of the density of vibrational levels to the total density of nuclear levels in ^{56}Mn and in ^{94}Nb are also presented. Like the others investigated light odd-odd nuclei (^{40}K , ^{52}V , ^{60}Co , ^{64}Cu), the ^{56}Mn and ^{94}Nb nuclei differ from odd-odd heavy and near-magic odd-odd nuclei by a smaller number of Cooper pairs which break below the neutron binding energy. A part of vibrational levels in ^{56}Mn and ^{94}Nb nuclei in the region of the neutron binding energy is close to its average [13].

Conclusion

In the analysis of experimental data on the intensity of the TSCs in ^{56}Mn and ^{94}Nb nuclei, any principal difference of the gamma-decay process, which occurs in them and in the others light nuclei (^{40}K , ^{52}V , ^{60}Co , ^{64}Cu) investigated earlier, is not discovered.

The gamma-decay process is really determined by two dipole strength functions. As for all investigated nuclei, quadruple transitions were observed neither in ^{56}Mn nor in ^{94}Nb .

A noticeable difference in the radiative strengths for $E1$ - and $M1$ -transitions for various energies of primary transitions cannot be described by a model of the strength functions with the same parameters for all investigated nuclei.

The best fits of the level densities contradict completely the representation about an excitations of non-interacted Fermi-particles in nuclear.

The reliable information about the nucleus can be obtained when comparing several model representations of the required nuclear parameters.

References

1. L. Szentmiklósi, T. Belgya, Z. Révay, Z. Kis, *Journal of Radioanalytical and Nuclear Chemistry* **286** (2), 501 (2010).
2. L. Szentmiklósi, Z. Kasztovszky, T. Belgya, Z. Révay, Z. Kis, B. Maróti, K. Gméling, V. Szilágyi, *Journal of Radioanalytical and Nuclear Chemistry* **309** (1), 71 (2016).
3. W. Dilg, W. Schantl, H. Vonach, and M. Uhl, *Nucl. Phys. A* **217**, 269 (1973).
4. A.V. Ignatyuk, G.N. Smirenkin and A.S. Tishin, *Yad. Fiz.* **21**, 485 (1975).
5. Yu. P. Popov et al., *Izv. Acad. Nauk SSSR, Ser. Fiz.*, **48**, 1830 (1984).
6. S.T. Boneva et al., *Sov. J. Part. Nucl.* **22**, 232 (1991).
7. S.T. Boneva et al., *Sov. J. Part. Nucl.* **22**, 698 (1991).
8. S.T. Boneva, A.M. Sukhovej, V.A. Khitrov, and A.V. Voinov, *Nucl. Phys.* **589**, 293 (1995).
9. <http://www-nds.iaea.org/ENDSF>.
10. A.V. Ignatyuk, Report INDC-233(L), IAEA (Vienna, 1985).
11. S.G. Kadenskij, V.P. Markushev and W.I. Furman, *Sov. J. Nucl. Phys.* **37**, 165 (1983).
12. A.M. Sukhovej and L.V. Mitsyna, in *Proceedings of XXII International Seminar on Interaction of Neutrons with Nuclei, Dubna, May 2014*, Preprint № E3-2015-13 (Dubna, 2015), p. 245; <http://isinn.jinr.ru/past-isinns.html>.
13. D.C. Vu et al., *Phys. Atom. Nucl.* **80**, 237 (2017).
14. *RIPL Reference Input Parameter Library RIPL-2, Handbook for calculations of nuclear reaction data*, IAEA-TECDOC (2002).
15. V.M. Strutinsky, in *Proceedings of the International Congress on Nuclear Physics, Paris, France, 1958*, p. 617.
16. A.M. Sukhovej, L.V. Mitsyna, N. Jovancevich, *Phys. Atom. Nucl.* **79**, 313 (2016).
17. A.M. Sukhovej, W.I. Furman, V.A. Khitrov, *Phys. Atom. Nucl.* **71**, 982 (2008).
18. L.A. Malov, V.G. Soloviev, *Sov. J. Nucl. Phys.* **26**, 384 (1977).
19. <http://www-nds.iaea.org/EGAF>.
20. A.M. Sukhovej, V.A. Khitrov, *Instrum. Exp. Tech.*, **27**, 1071 (1984).