Analysis of Scattering Phase Shifts for Two-Cluster Systems

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I. INTRODUCTION

Study of resonances in the scattering problem of light nuclei has been carried out using various methods, one of which is the complex scaling method [1–2]. It is possible to investigate the resonance contributions and to obtain a deep understanding of resonance structure by separation of a scattering quantity. Suzuki et al. [3] showed that scattering phase shifts can be calculated from the continuum level density, which is expressed using the complex scaled Green's function.

We apply the complex scaling method to the calculation of scattering phase shifts and extract the contributions of resonances in a phase shift. The decomposition of the phase shift is shown to be useful in understanding the roles of resonant and non-resonant continuum states. We apply this method to the two-body alpha+nucleon systems. We discuss the explicit relation between the scattering phase shifts and complex-energy eigenvalues in the complex scaling method via the continuum level density. The results provide us with deeper understanding of the role of resonant states characterized by the widths described as an imaginary part of the eigen-energy.

II.THEORETICAL FRAMEWORK

A. Complex Scaling Method

In the complex scaling method the relative coordinate is rotated as $r \longrightarrow re^{i\theta}$ in the complex coordinate plane. Therefore, the *Schrödinger* equation

$$\hat{H}|\Psi\rangle = E|\Psi\rangle \tag{1}$$

is rewritten as

$$\hat{H}(\theta)|\Psi^{\theta}\rangle = E^{\theta}|\Psi^{\theta}\rangle, \tag{2}$$

where $\hat{H}(\theta)$ and Ψ^{θ} are the complex scaled Hamiltonian and wave function, respectively. The θ is scaling angle being a real number, $U(\theta)$ operate on a function Ψ^{θ}, that is

$$\Psi^{\theta} = U(\theta)\Psi(r) = e^{\frac{3}{2}i\theta}\Psi(re^{i\theta}). \tag{3}$$

The eigenvalues and eigenstates are obtained by solving the complex scaled *Schrödinger* equation Eq.(2). The eigenvalues of resonance states are found as $E^{\theta} = E_r - i\Gamma_r/2$, where E_r is resonance energy and Γ_r is width of the resonant state. More detailed explanation of the complex scaling method is given in Ref.[2].

To solve the eigenvalue problem of Eq. (2), we employ the Gaussian basis functions given as:

$$\phi_i(r) = N_l(b_i) r^l \exp\left(-\frac{1}{2b_i^2} r^2\right) Y_{lm}(\hat{r}), \tag{4}$$

where the range parameters are given by a geometric progression as $b_i = b_0 \gamma^{i-1}$; $i = 1 \cdots, N$, and $N_l(b_i)$ is the normalization factor. We take $N = 60$ and employ the optimal values of $b_0$and γso as to obtain stationary solutions. All results are obtained with $\theta = 15°$.

B. Continuum Level Density and Phase Shift

The continuum level density $\Delta(E)$ is given as

$$\Delta(E) = -\frac{1}{\pi} \mathrm{Im}\{\mathrm{Tr}[G^{+}(E) - G_0^{+}(E)]\}, \tag{5}$$

where

$$G^{+}(E) = (E + i\epsilon - H)^{-1},$$
$$G_0^{+}(E) = (E + i\epsilon - H_0)^{-1}$$

are the full and free Green's functions, respectively. In this study, the Hamiltonian H and H_0 are transformed using the complex scaling method.

The continuum level density is related to the scattering phase shift $\delta(E)$, it can be expressed in the following form in the single channel case:

$$\Delta(E) = \frac{1}{\pi} \frac{d\delta(E)}{dE}. \tag{6}$$

Using this relation, we can obtain the phase shift as a function of the eigenvalues in the complex scaled Hamiltonian by integrating the continuum level density.

When we expand the wave functions in terms of the finite number N of the basis states, the discretized eigenstates are obtained with number N and the level density can be approximated as in [4]:

$$\Delta(E) \approx \Delta_{\theta}^{N}(E) =$$
$$-\frac{1}{\pi} \mathrm{Im}\left\{\sum_{b=1}^{N_b} \frac{1}{E+i0-E_b} + \sum_{r=1}^{N_r^{\theta}} \frac{1}{E-E_r^{res}+\frac{i\Gamma_r}{2}} + \sum_{c=1}^{N_c^{\theta}} \frac{1}{E-\epsilon_c^{r}+i\epsilon_c^{i}} - \sum_{k=1}^{N} \frac{1}{E-\epsilon_k^{0r}+i\epsilon_k^{0i}}\right\}, \tag{7}$$

where $N = N_b + N_r^{\theta} + N_c^{\theta}$ is the total number of N_b (bound states), N_r^{θ} (resonance states) and N_c^{θ} (continuum states) solutions. Then, we can obtain the phase shift:

$$\delta_{\theta}^{N}(E) = N_b\pi + \sum_{r=1}^{N_r^{\theta}}\left\{-\cot^{-1}\left(\frac{E-E_r^{res}}{\Gamma_r/2}\right)\right\} + \sum_{c=1}^{N_c^{\theta}}\left\{-\cot^{-1}\left(\frac{E-\epsilon_c^{r}}{\epsilon_c^{i}}\right)\right\} - \sum_{k=1}^{N}\left\{-\cot^{-1}\left(\frac{E-\epsilon_k^{0r}}{\epsilon_k^{0i}}\right)\right\}, \tag{8}$$

where $E \geq 0$. When we define δ_r, δ_c and δ_k as

$$\cot\delta_r = \frac{E_r^{res}-E}{\Gamma_r/2},\ \cot\delta_c = \frac{\epsilon_c^{r}-E}{\epsilon_c^{i}},\ \cot\delta_k = \frac{\epsilon_k^{0r}-E}{\epsilon_k^{0i}}, \tag{9}$$

respectively, we can write the phase shift as

$$\delta_\theta^N(E) = N_b\pi + \sum_{r=1}^{N_r^\theta}\delta_r + \sum_{c=1}^{N_c^\theta}\delta_c - \sum_{k=1}^{N}\delta_k. \quad (10)$$

The geometrical indications for δ_r, δ_c and δ_k are given for two energy cases, larger or smaller than the real parts of the eigen-energies E_r, ε_c and ε_k, as shown in Fig. 1. The phase shift δ_r for the resonances is the angle of the r-th resonant pole measured at the energy E on the real energy axis. At $E = E_r^{res}$, we have $\delta_r = \pi/2$ for every resonant pole. In addition, $\delta_r = \tan^{-1}(\Gamma_r/2E_r^{res}) > 0$ at $E = 0$ and $\delta_r = \pi$ at $E = \infty$, for each resonance. Similarly phase shifts from continuum terms including the asymptotic part, δ_k, are given by the angles of the discretized continuum energies. At $E = \infty$, the continuum terms of the phase shifts go to $-(N_b + N_r^\theta)\pi$ because of the relation $N = N_b + N_r^\theta + N_c^\theta$.

III. RESULTS AND DISCUSSIONS

In our previous works [5-6], we proposed a new method to get information of the pole position of the virtual state applying the continuum level density, the phase shift obtained in the complex scaling method. Based on the proposed method, we discuss the contribution of each state into the scattering phase shifts calculating the decomposed phase shifts. In this work, we choose two-body two mirror nuclei for calculation of decomposed phase shifts. The KKNN (Kanada, Kaneko, Nagata, Nomoto) [7] potential is used for the effective nucleon-nucleon interaction of α+n and α+p systems.

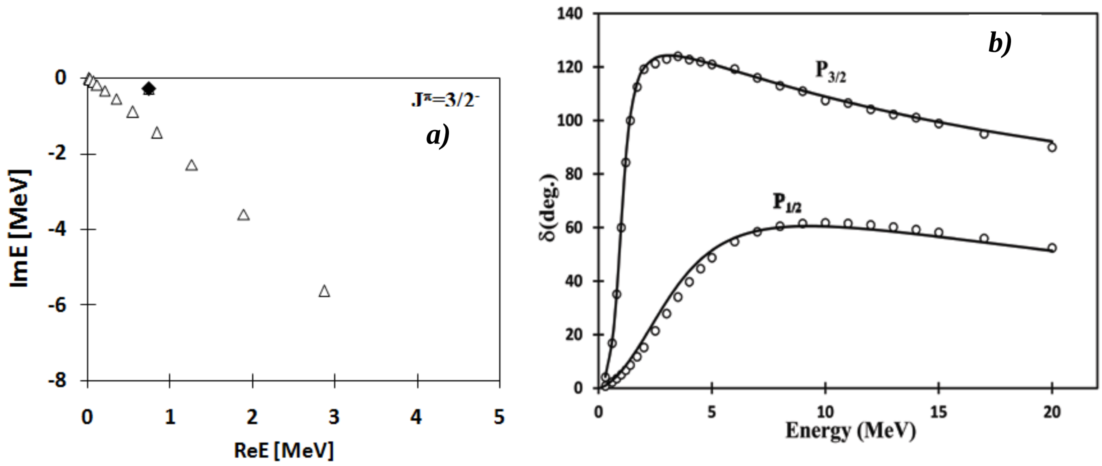

Fig. 1. In the left hand side: the calculated eigenvalue distribution on the complex energy plane for $J^\pi = 3/2^-$ of ^{5}He. In the right hand side: the scattering phase shifts of ^{5}He at the $J^\pi = 3/2^-$ and $1/2^-$ states for $\theta = 20^0$. The calculated phase shifts are displayed as solid curves and the experimental data are given by open circles.

At the first step, we calculate the scattering phase shifts and compared it with measured data. The calculated eigenvalue distribution on the complex energy plane and scattering phase shifts for the low-lying $3/2^-$ and $1/2^-$ states of α+n system are shown in Fig. 1. We can see from Fig. 1, our calculated scattering phase shifts well reproduce the measured data [8] in the low-lying states. The same trend is obtained on its mirror α+p system, too.

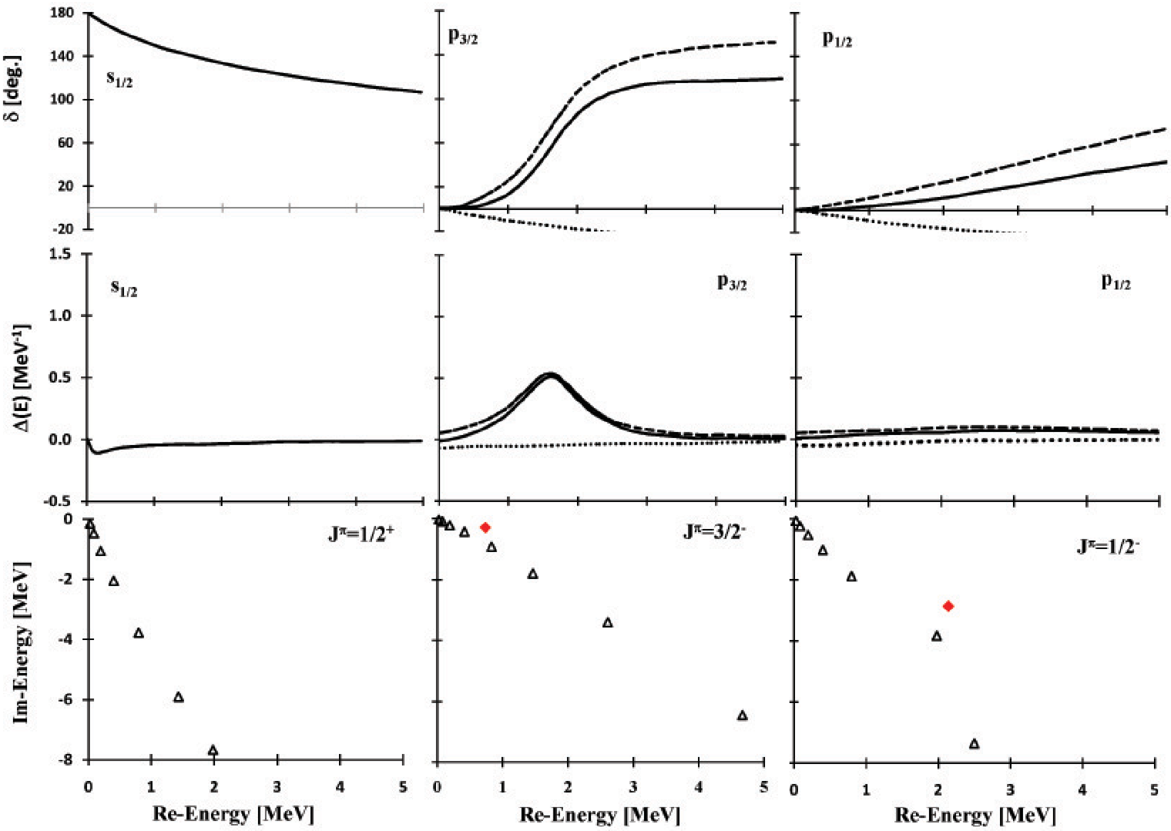

Fig. 2. Upper panel: the decomposition of scattering phase shifts of *α–p* (^{5}Li) system for the $J^\pi = 1/2^+; 3/2^-; 1/2^-$ states and middle panel: the decomposition of continuum level densities. The dashed and dotted lines represent the contributions of resonance and continuum terms, respectively. The solid lines display total scattering phase shifts or total continuum level densities. Lower panel: the distributions of eigenvalues are displayed in the complex energy plane. The diamond displays the resonance pole.

In the next step, we calculate the decomposed phase shifts by selecting the energy states and analyze its contribution into the scattering phase shifts. In Fig. 2, the energy eigenvalue distribution on the complex energy plane, decomposed phase shifts and continuum level density for the J^π=3/2$^-$ state of ^{5}Li system is shown. The resonance phase shift of 3/2$^-$ increases rapidly due to the small decay width. Although 1/2$^-$ has a larger width, the phase shift of 1/2$^-$ shows a clear resonance behavior beyond $\pi/2$. The continuum phase shifts of both states are very similar. This trend seems due to the same *p*-wave scattering and a small effect of the $\ell \cdot s$ force to the background states. The property of the scattering phase shifts is determined from a sum of resonance and continuum terms. Therefore, the observed resonances depend on not only resonant states as poles but also the contribution from the non-resonant continuum states.

IV. SUMMARY

Applying Green's function, we can precisely extract the contributions of resonance and continuum terms from the total continuum level density. This analysis clarifies the physical role of resonances and non-resonant continuum states in the observables. We have also shown the application of the complex scaling method to the calculation of the decomposed continuum level density and the decomposed phase shifts. The role of resonance poles on the phase shifts and continuum level densities are discussed.

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