Parameterization of Neutron Yields for the First Chance Photofission Fragments

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1. INTRODUCTION

The number of prompt neutrons is important nuclear-physical parameters necessary for practical calculations. This value is determined in detail and accurately for neutron-induced reactions for the most nuclides. At the same time, the experimental data and evaluation of prompt neutron yield in the case of photofission are much scarcer. With the increasing interest in the methods of nuclear fuel burning and long-lived actinide decontamination the need in precise values of nuclear constants. This is especially true for photonuclear constants. Therefore, the search was focused on a search the methods that can be used to estimate the value of averaged number $\bar{\nu}(A_F)$ and number of neutrons $\nu(A)$, emitted by corresponding fission fragments of atomic mass for photofission of arbitrary actinide.

The same data are also used to obtain the mass and charge distribution of actinide nuclei fission products and to convert the secondary fragments (products) yields into the primary ones. Typically, to estimate the number of neutrons $\nu(A)$, emitted by corresponding fission fragments of atomic mass A the phenomenological Wahl method [1] is applied, which is widely used till present for estimation of $\nu(A)$ during neutron- and gamma-induced fission. However, this method does not reflect the complex structure of sawtooth-like $\nu(A)$ dependence due to nuclear shells effect.

So far there is only information about the average number of neutrons emitted by two conjugated fragments and there is no information on photofission neutron emission curves. Primarily, this is due to experimental difficulties of time-span or direct neutron measurements in photofission experiments. However, in principle, neutron emission curves can be obtained combining the measurement of fragments mass distribution and post-fission neutrons.

The question is whether it is possible to identify some of the basic laws using the results of calculations depending on such parameters of actinides photofission as charge, mass, photon energy, shell characteristics. According to the results, one can parameterize neutron yields depending on the mass of actinide photofission fragments and other parameters that would reproduce the complex structure of saw-tooth behavior and allow to predict the dependence of $\nu(A)$ for the photofission of a wide class of actinide nuclei. This is the task of extreme interest.

2. PARAMETERIZATION OF ν(A) AND RESULT OF CALCULATION

A recent analysis of experimental data on neutron yields from fragments of thermal neutron fission of ^{233}U, ^{235}U, ^{239}Pu and spontaneous fission of ^{252}Cf showed that for a detailed account of “saw-tooth” particularity of dependence of fission neutron yield from a mass, an efficient tool is the value of model function $R(A)$, introduced by Wahl [1], which is defined as

$$R(A) = {\nu_{L,H}(A)}/{\bar{\nu}(A)}, \qquad (1)$$

where $\nu_{L,H}(A)$ – prompt neutron yield of light and heavy fragment mass respectively, $\bar{\nu}(A)$ – total neutron yield, A – fragment mass and consists of several segments to reflect the observed features, depending on the complexity of the experimental behavior of $R(A)$. Therefore, the whole range of fragments mass was divided for more than 2×4 segments. "Experimental" values of $R(A)$ (with errors) can be determined using formula (1) from experimental values of $\nu_{L,H}(A)$ and $\bar{\nu}(A)$.

Model function $R(A)$ is chosen as a linear function for each segment:

$$R_i^L(A) = a_i^L + b_i^L(A - A_L), \qquad (2)$$

$$R_i^H(A) = 1 - R_i^L(A - A_H) \qquad (3)$$

for light and heavy fragments, respectively, i - number of the segment, a_i^L, b_i^L A_L – parameters, $A_H = A_f - A_L$, A_f – mass of compound nucleus.

To parameterize neutron emission and identify general prediction patterns we will use the results of $\nu(A)$ calculation for the photofission ^{235}U and ^{238}U at bremsstrahlung boundary energies of 12 ÷ 30 MeV (E* = 9.7–14.1 MeV) [2]. Here we simulate the behavior of neutrons from photofission of ^{235}U and ^{238}U actinides depending on the energy and nucleon composition in the giant dipole resonance energy range. As a result, we get the following picture (Fig. 1).

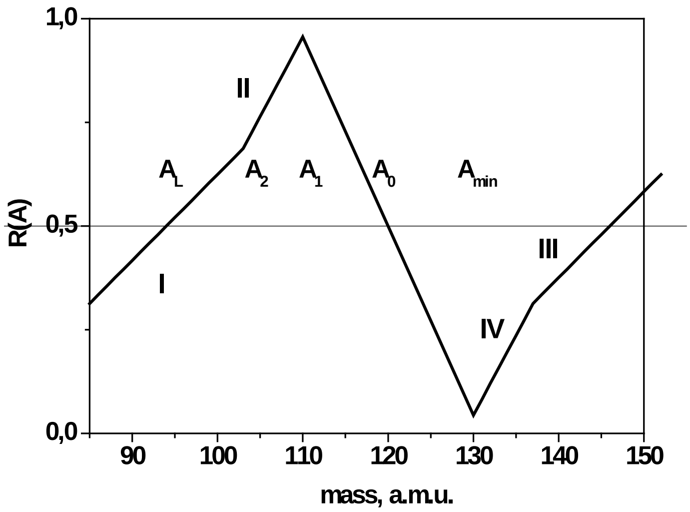

Fig. 1. An example of $R(A)$ function. The segments I-II and III-IV correspond to our parameterization.

Let us consider some features of $R(A)$ function. At the point of symmetric fission $A_0 = A_F/2$; $R(A_0) = 0.5$ and $R(A_L) = 0.5$, where A_L is determined from fitting. Kink points A_{min}, A_1 and A_2 are chosen from physical considerations and experiment: $A_{\rm min} = 130$ corresponds to the mass of nearly magic nucleus fragment associated with spherical shells $Z = 50$ and $N = 82$, where fission neutron yield is minimal. Then maximum neutron fission yield for light fragments will match point A_1, which is symmetrical to $A_{\rm min}$ relative to A_0,

$$A_1 = 2A_0 - A_{min}. \qquad (4)$$

The kink point A_2 corresponds to the average fragment mass of light fragments,

$$A_2 = \langle A_L \rangle = A_F - \langle A_H \rangle \qquad (5)$$

and related to the intermediate deformation of the fissioning nuclide.

The parameters a_i, b_i and A_L are determined by calculating the function $R(A)$ for 4 segments I – IV (see Fig.1). The number of free parameters can be reduced using the conditions.

$$a_1 = R(A_L) = 0{,}5 \qquad (6)$$

$$a_2 = a_1 + (b_1 - b_2)(A_2 - A_L) \qquad (7)$$

The dependence of b_i slopes on excitation energy is noticed on the Fig. 1 for $R(A)$, so we have chosen $b_i = x_i + yE_\gamma$. The value of A_L varies significantly with changes of actinide mass, at least for neutron-induced actinide fission. Therefore, we chose a similar parameterization [3]:

$$A_L = 90 + B * A_0 \qquad (8)$$

B=1.45. To take even-even and even-odd effect into account we introduce the factor

$$P(N_F) = 2 - c[(-1)^{N_F} - (-1)^{Z_F}], \qquad (9)$$

$$N_F = A_F - Z_F.$$

As a result b_i slopes will look as

$$b_i = (x_i + yE_\gamma)P(N_F), \qquad i = 1{,}2 \qquad (10)$$

We calculate the function $R(A)$ for the photofission of actinides ^{235}U and ^{238}U according to (1) – (10) by fitting of 356 "experimental" values of $R(A)$.

Using the least squares method the five parameters x_1, x_2, c, B and y were defined to satisfactorily describe the characteristic "saw-tooth" behavior of prompt neutrons from the photofission of actinides with A = 235 a.m.u. and A=238 a.m.u.

Table 1. Calculated parameters of *R(A)* function.

Parameter	Value	Error
x_1	$1.66 \cdot 10^{-2}$	$0.35 \cdot 10^{-2}$
x_2	$1.08 \cdot 10^{-2}$	$0.29 \cdot 10^{-2}$
y	$-0.384 \cdot 10^{-3}$	$0.212 \cdot 10^{-3}$
c	-0.282	0.159
B	$1.26 \cdot 10^{-2}$	$0.52 \cdot 10^{-2}$

The results of $R(A)$ calculation at the bremsstrahlung maximum energy 12 MeV are shown in Fig. 2.

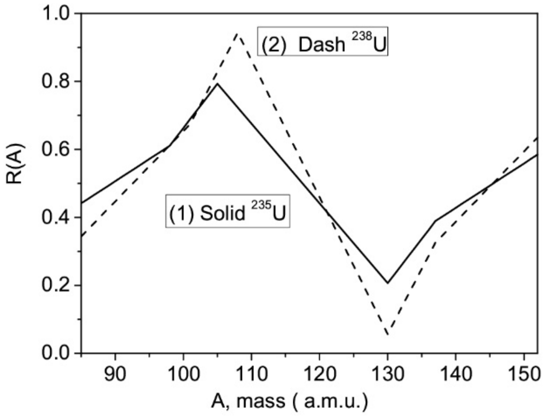

Fig. 2. The result of $R(A)$ functions calculation for ^{235}U ((1), solid line), ^{238}U ((2), dashed line) photofission at the bremsstrahlung maximum energy 12 MeV.

The curves for prompt neutrons yield $\nu_{L,H}(A)$ can be calculated with help (1), if the value of the prompt neutrons averaged number for photofission of the actinides is known. Otherwise instead the experimental values of $\bar{\nu}(A)$ in (1) one may use the results of empirical calculations of $\bar{\nu}(A_F)$, presented in [4] and described below.

The initial formula has been chosen:

$$\bar{\nu}(A_F, Z_F, E_\gamma) = \bar{\nu}_0(A_F, Z_F) + a(A_F, Z_F) * (E_\gamma - E_S) \tag{11}$$

where the slope $\bar{\nu}_0(A_F, Z_F)$ and the intercept $a(A_F, Z_F)$ are *:*

$$\bar{\nu}_0(A_F, Z_F) = C_1 + C_2(Z_F - Z_0) + C_3(A_F - A_0) + C_4 P(A_F, Z_F), \tag{12}$$

$$a(A_F, Z_F) = C_5 + C_6(Z_F - Z_0) + C_7(A_F - A_0) + C_8 P(A_F, Z_F), \tag{13}$$

where $P(A, Z)$ – parity factor, E_S – nucleon separation.

Coefficients C_i were calculated by the least-square method. The final formula for calculating the averaged number of prompt neutrons for photofission of actinides was:

$$\begin{gathered}\bar{\nu}_0(A_F, Z_F) = (1{,}97 \pm 0{,}05) + (0{,}165 \pm 0{,}028)(Z_F - 90) + \\ +(0{,}0341 \pm 0{,}0093)(A_F - 232) - (0{,}0853 \pm 0{,}0094) * P(A_F, Z_F)\end{gathered} \tag{14}$$

$$\begin{gathered}a(A_F, Z_F) = (0{,}0963 \pm 0{,}75 * 10^{-2}) + (0{,}0371 \pm 0{,}43 * 10^{-2})(Z_F - 90) - \\ -(0{,}566 \pm 0{,}138) * 10^{-2} * (A_F - 232)\end{gathered} \tag{15}$$

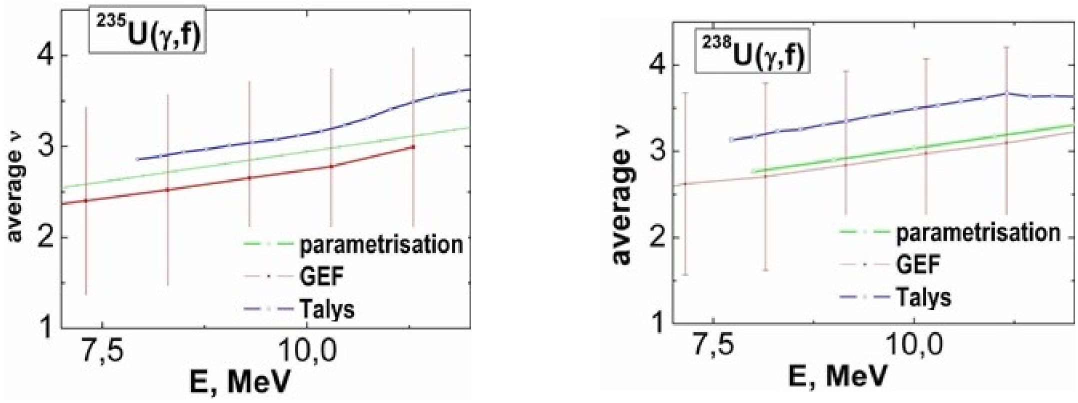

Fig.3. Energy dependence of the average number of prompt neutrons in photofission of ^{235}U and ^{238}U actinides.

The Fig. 3 shows the results of calculations of the energy dependence of the average number of prompt neutrons in the photofission of actinides ^{235}U and ^{238}U. Simulation results using the "GEF" [5] and "Talys" [6] codes are also presented, which are consistent with our data.

The results of the $\nu_{L,H}(A)$ calculation using (1) – (11), (14), (15) and Table 1 are shown in Fig. 2 and Fig. 4 (solid curve). As can be seen from the figures the calculated values for prompt neutrons yield are everywhere within the errors. For comparison we repeated our calculations, confining ourselves to two 2×1 segments in Fig. 4. Both calculations are indistinguishable from the x^2 criterion, but we prefer the first variant of the approximation $\nu(A)$, as such, in which physical considerations are taken into account. These observations allow to estimate the possible values of fission neutrons yield from light and heavy fragments with known total yields just through the mass distributions of fission fragments using the modified Terrell method [7].

The obtained results of the estimation of the dependence of the prompt neutron yields from light and heavy fragments for the first chance of actinide photofission are compared with the results of calculations (modeling) by the program codes "GEF" [5] and "Talys1.9" [6] (Fig. 5, 6).

With $R(A)$ function parameterization, which fairly well reproduces the characteristic features of its behavior and parameterization of average number of prompt neutrons, we can calculate the expected values of prompt neutrons yield for another actinides. Thus, to determine the photofission yield for neutron yield on fragment mass $\nu(A)$ of arbitrary actinides we need to know the value $\nu_F(A)$, which is determined by the general empirical formulas (11), (14–15). The resulting formulas of prompt neutron yield on fragment mass $\nu(A)$ of photofission of actinide nuclei can be used as initial (seed) for solving the integral equations [7] in these processes. The calculations are in qualitative agreement with the results of modeling by "GEF" [5] and "Talys1.9" [6].

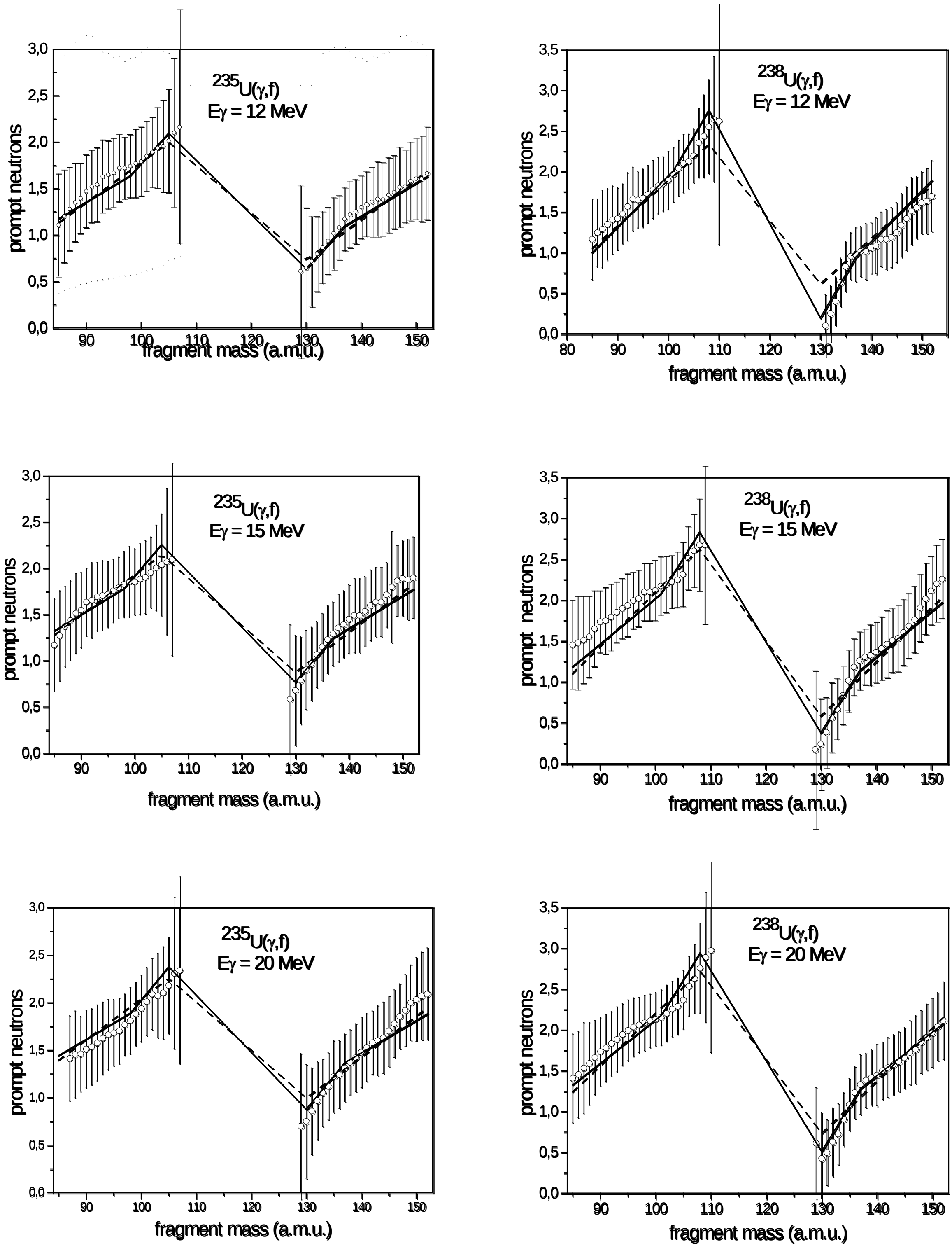

Fig. 4. Results of 2×2-segments calculation of $\nu_{L,H}(A)$ (solid lines) of ^{235}U (left) and ^{238}U (right) photofission with bremsstrahlung maximum energy of 12, 15, 20 MeV. Dashed lines correspond to 2×1-segment variant. Circles – experimental points of $\nu_{L,H}(A)$.

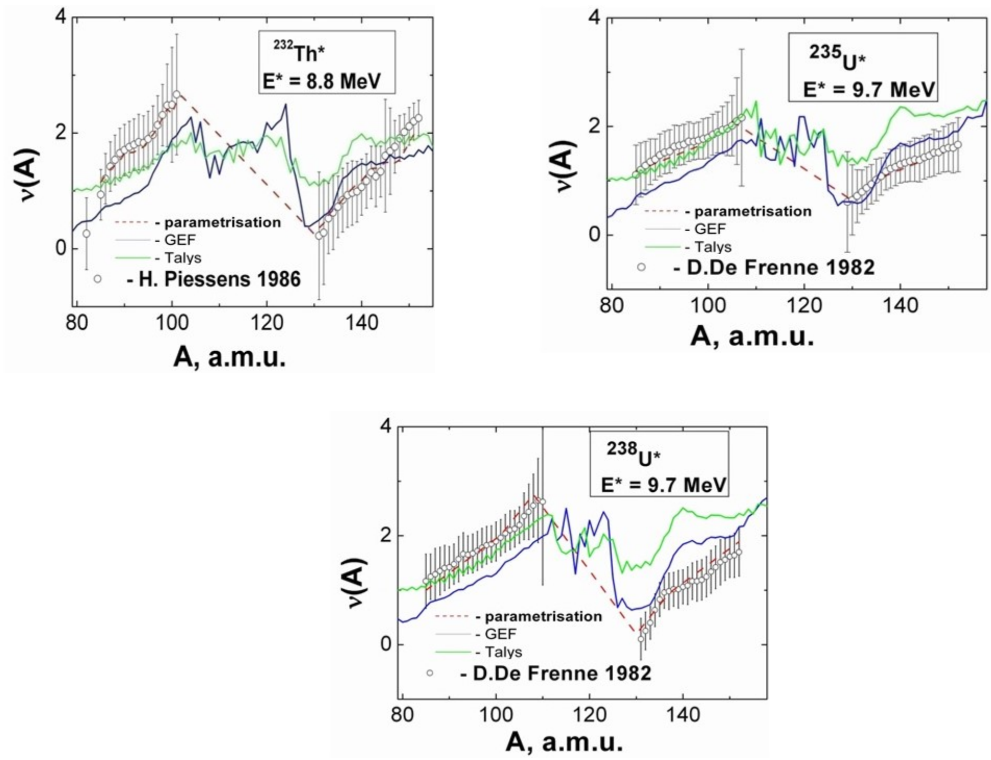

Fig. 5. Dependence $\nu_{L,H}(A)$ at close excitation energies for fissile nuclei ^{232}Th, ^{235}U, and ^{238}U.

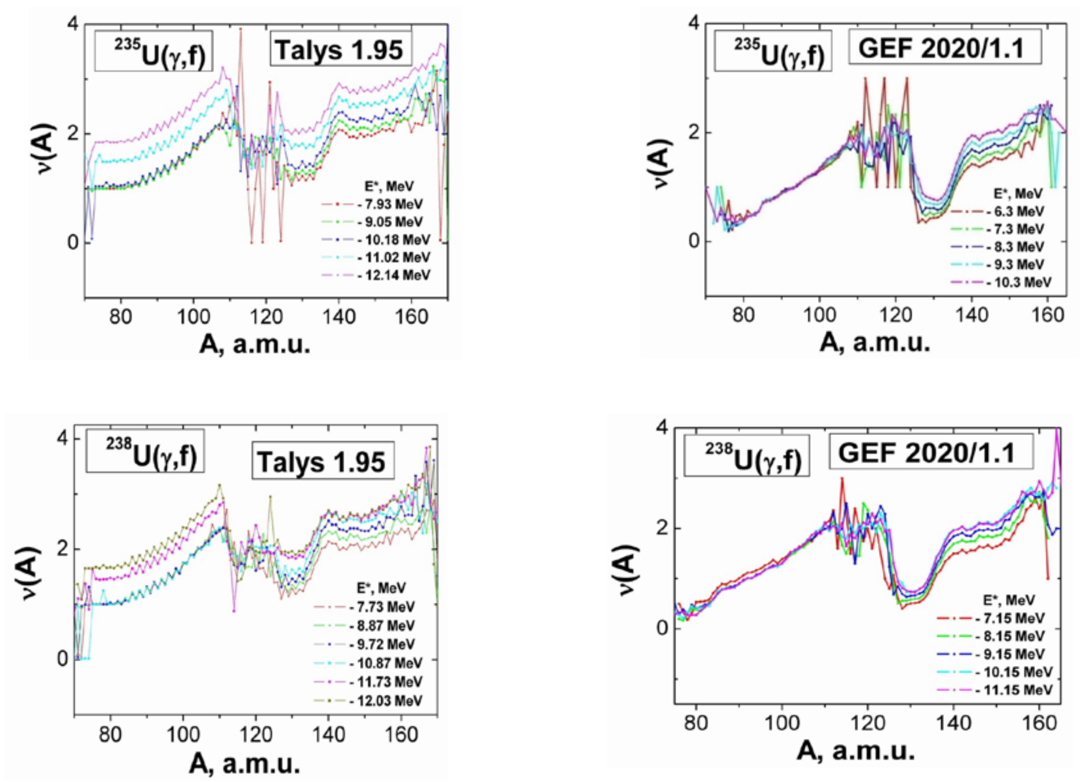

Fig. 6. Dependence of ν(A) on the excitation energy for fissile ^{235}U and ^{238}U nuclei.

3. CONCLUSIONS

The dependence of the averaged number of prompt neutrons emitted from fission products for each mass number (*A*) of photofission of actinide nuclei ^{232}Th, ^{235}U, and ^{238}U in the giant dipole resonance energy range have been parameterized. This approach allowed to describe the observed changes of "saw-tooth" behavior of neutron yield from the light and heavy fragments using few energy and nucleon composition-dependent free parameters and to predict (*A*) for other actinide isotopes. To estimate the number of neutrons emitted by corresponding fission fragments of atomic mass, *A*, the phenomenological Wahl method was applied. The total averaged number of prompt neutrons needs to construct the parametrizing function, which reasonably reproduces the characteristic features of its behavior and parameterization of averaged number of prompt neutrons. This method can calculate the expected values of prompt neutrons yield for arbitrary neighboring actinides, such as ^{237}Np or ^{239}Pu. It allows estimating the possible values of fission neutrons yield from light and heavy fragments with known total yields just through the mass distributions of fission fragments using the modified Terrell method. The obtained results of estimating the dependence of the prompt neutron yields from light and heavy fragments for the first chance of actinide photofission are compared with the results of calculations (modeling) by the program codes “GEF” and “Talys1.9”. Both approaches give the results qualitatively consistent. The energy dependence of neutron multiplicity is the same in both cases. Quite another, so called “Many Ensembles Method” to estimate the neutron multiplicity on bases the mass-charge spectra of fission fragments was used [8] with the same conclusion.

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