Systematical Analysis of (n,2n) Reaction Cross Sections for 14–15 MeV Neutrons

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1. Introduction

Fast neutron induced nuclear reaction cross section data are necessary for both nuclear energy technology and the study of nuclear structure and reactions. The information of (n,2n) cross sections is quite essential in nuclear reactor technology as a significant portion of the fission neutron spectrum lies above the threshold of (n,2n) reaction for most of the structural materials. These cross section data are required in radiation shielding and nuclear fuel breeding calculations. Furthermore, the application of the fast neutron induced nuclear reaction cross section data have been increasing in the fields of biomedical applications, accelerator driven transmutation, material irradiation experiments concerning research and development for fusion reactor technology. On the other hand, systematics of fast neutron induced reaction cross sections is useful to clarify nuclear reaction mechanisms. Also, it is often necessary, in practice, to use the systematical analysis for evaluation of the neutron cross sections of the nuclides, for which no experimental data are available.

A systematic dependence of (n,2n) reaction cross sections on the neutron number excess parameter (N–Z)/A for target nuclei was studied in a number of works [1–11]. Purely empirical formulae [1–4,10] and semi-empirical formulae based on the statistical model [5–9,11], pre-equilibrium [7] and direct reaction [5] mechanisms were suggested to explain this regular behaviour of the (n,2n) cross sections. However, a strict and unified theoretical validity no up to now is available for explanation of the neutron cross section regularity which in the literature is termed as the isotopic effect.

In this work new formulae for the (n,2n) cross section are deduced using the concepts of statistical and non-statistical nuclear reaction mechanisms. In the calculation of the statistical part of the cross section the constant nuclear temperature approximation, evaporation model and Weizsäcker's formula for binding energy were utilized. Non-statistical part was obtained by subtraction of statistical model cross section from total one. The total cross section was determined by geometrical cross section of target nuclei as an upper limit of interaction of neutrons with nuclei.

2. Theoretical background

In the framework of the statistical model based on the Bohr's assumption of a compound mechanism the cross section formula for (*n,x*) reaction is expressed as [12]:

$$\sigma(n,x)=\sigma_c(n)\frac{2S_x+1}{2S_n+1}\frac{M_x}{M_n}e^{\frac{Q_{n,x}-V_x}{\Theta}}\left\{\frac{1-\frac{W_{n,x}}{\Theta}e^{-\frac{W_{n,x}}{\Theta}}-e^{-\frac{W_{n,x}}{\Theta}}}{1-\frac{E_n}{\Theta}e^{-\frac{E_n}{\Theta}}-e^{-\frac{E_n}{\Theta}}}\right\}, \qquad (1)$$

where: $\sigma_c(n) = \pi(R+\lambda\!\!\!^{-}_n)^2$ is the compound nucleus formation cross section; R is the target nucleus radius; $\lambda\!\!\!^{-}_n$ is the wavelength of the incident neutrons divided by 2π; S_n and S_x are the spins of the incident neutron and emitted *x*-particle, respectively; M_n and M_x are the masses of the neutron and *x*-particle, respectively; $Q_{n,x}$ is the reaction energy; V_x is the Coulomb barrier for *x*-particle; Θ is the thermodynamic temperature; E_n is the incident neutron energy; $W_{n,x} = E_n + Q_{n,x} - V_x$.

According to our evaluations, term in the curly brackets of the formula (1) approximately equals to 1 for the fast neutron induced (n,2n) reaction cross sections except a few very light nuclei. Then, from the formula (1) can be obtained following formula, which is similar to Cuzzocrea's *et al*. [13] and Ericson's [14] formulas:

$$\sigma(n,2n)=\sigma_c(n)\frac{2S_{2n}+1}{2S_n+1}\frac{M_{2n}}{M_n}e^{\frac{Q_{n,2n}-V_{2n}}{\Theta}}. \qquad (2)$$

Using the Weizsäker's formula for binding energy we can obtain following expressions for the target and residual nuclei:

$$E_i=\alpha A-\beta A^{2/3}-\gamma\frac{Z^2}{A^{1/3}}-\xi\frac{(N-Z)^2}{A}\pm\frac{\delta_i}{A^{3/4}} \qquad (3)$$

and

$$E_f=\alpha(A-1)-\beta(A-1)^{2/3}-\gamma\frac{Z^2}{(A-1)^{1/3}}-\xi\frac{(N-1-Z)^2}{A-1}\pm\frac{\delta_f}{(A-1)^{3/4}}. \qquad (4)$$

Then, we get the (n,2n) reaction energy $Q_{n,2n}$ as following:

$$Q_{n,2n}=-\alpha-\beta\left\{(A-1)^{2/3}-A^{2/3}\right\}-\gamma\left\{\frac{Z^2}{(A-1)^{1/3}}-\frac{Z^2}{A^{1/3}}\right\}-\xi\left\{\frac{(N-1-Z)^2}{A-1}-\frac{(N-Z)^2}{A}\right\}\pm\frac{\delta_f}{(A-1)^{3/4}}\mp\frac{\delta_i}{A^{3/4}}. \qquad (5)$$

where: α = 15.7 MeV; β = 17.8 MeV; γ = 0.71 MeV; ξ = 23.7 MeV; δ_i and δ_f depend on either odd or even number of neutrons and protons $|\delta|$ = 34 MeV or 0.

So, taking into account the Coulomb barrier for neutrons $V_{2n} = 0$, the spin and mass of neutrons, and reaction energy for formula (2) we get the (n,2n) cross section formula:

$$\sigma(n,2n) = 4\pi(R+\lambdabar_n)^2 \exp\left\{\frac{-\alpha-\beta\left((A-1)^{2/3}-A^{2/3}\right)-\gamma\left(\frac{Z^2}{(A-1)^{1/3}}-\frac{Z^2}{A^{1/3}}\right)-\xi\left(1-\frac{4Z^2}{A(A-1)}\right)\pm\frac{\delta_f}{(A-1)^{3/4}}\mp\frac{\delta_i}{A^{3/4}}}{\Theta}\right\}. \quad (6)$$

In the case of A >> 1 can be obtained the following formulae for systematical analysis of the (n,2n) cross sections:

$$\frac{\sigma(n,2n)}{\pi(R+\lambdabar_n)^2} = C\exp(-K\frac{Z^2}{A^2}) \quad (7)$$

where: Z and A are proton and mass numbers of the target nuclei; the parameters K and C are expressed as:

$$K = \frac{4\xi}{\Theta}; \quad (8)$$

$$C = 4\exp\left\{\frac{-\alpha-\beta\left((A-1)^{2/3}-A^{2/3}\right)-\gamma\left(\frac{Z^2}{(A-1)^{1/3}}-\frac{Z^2}{A^{1/3}}\right)\pm\frac{\delta_f}{(A-1)^{3/4}}\mp\frac{\delta_i}{A^{3/4}}+\xi}{\Theta}\right\}. \quad (9)$$

It can be, as a first approximation, assumed that for (n,2n) reactions induced by 14–15 MeV neutrons the pre-equilibrium and direct mechanisms predominate. In this case, we find out the non-statistical share of cross section by subtracting the (n,2n) cross section defined by compound mechanism from the total one as following:

$$\sigma^{nonstat}(n,2n) = \sigma^{tot}(n,2n) - \sigma^{comp}(n,2n) \quad (10)$$

If the total cross section is determined by geometrical cross section of the target nuclei as an upper limit of interaction of neutrons with nuclei:

$$\sigma^{tot}(n,2n) = \pi(R+\lambdabar_n)^2, \quad (11)$$

then the non-statistical part can be obtained as following:

$$\sigma^{nonstat}(n,2n) = \pi(R+\lambdabar_n)^2\left(1-C\exp(-K\frac{Z^2}{A^2})\right). \quad (12)$$

Then, the reduced (n,2n) cross section is expressed as:

$$\frac{\sigma^{nonstat}(n,2n)}{\pi(R+\lambda\!\!\!^{-}_{n})^2} = \left(1 - C\exp(-K\frac{Z^2}{A^2})\right). \tag{13}$$

3. Systematics of (n,2n) reaction cross sections and Discussions

We have analyzed 139 experimental (n,2n) cross section data for the neutron energy of 14–15 MeV from EXFOR [15]. The dependence of the reduced (n,2n) cross section on the parameter Z^2/A^2 is shown in the Fig. 1.

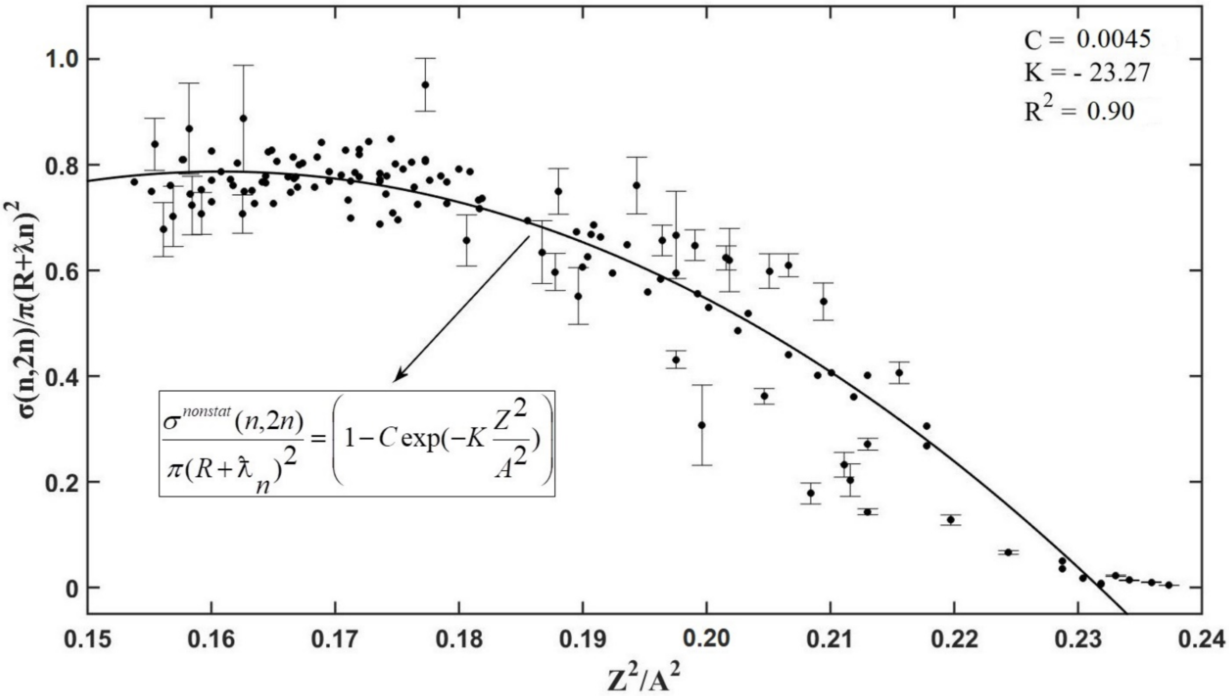

Fig. 1. The dependence of the reduced (n,2n) cross sections on parameter Z^2/A^2

It is seen from Fig. 1 that the theoretical curve is in satisfactorily agreement with the experimental data. Also, in Fig. 1 experimental errors of the cross sections for some isotopes are given, as example.

4. Conclusions

1. In the framework of the statistical model a theoretical formula for the (n,2n) reaction cross section was deduced. In addition, a non-statistical share of the total neutron cross section was obtained.
2. Known experimental data of the (n,2n) cross sections for 14 – 15 MeV neutrons were analyzed using the obtained formulae. It was shown that the non-statistical share of the total cross section is in satisfactorily agreement with experimental data.

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