NEUTRON RESONANCES IN THE GLOBAL CONSTITUENT QUARK MODEL

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1. Introduction

Neutron resonance spectroscopy is part of nuclear physics based on the Standard Model (SM) as a theory of all interactions. In this work, we present the symmetry motivated and electron-based empirical approach to SM development. It was shown in [1], that nonstatistical effects, observed by different authors in the positions and spacing distributions of neutron resonances in many isotopes are systematic.

The high accuracy in determining the neutron resonance energy achieved by the time-of-flight method allowed us to consider together empirical correlations in nuclear data, namely the existence of fine and superfine structures, respectively, with periods of ε'=1.2 keV and ε''=1.4 eV=5.5 eV/4, which are equal to the first and second QED radiative corrections to the empirically found period 1022 keV=$\varepsilon_\circ$=2m$_e$ in few-nucleon excitations and the differences of the nuclear binding energies. The factor $\alpha/2\pi$ between them corresponds to the influence of the physical QED condensate known for the magnetic moment. The distinguished character of the electron mass m_e=511 keV was found in many empirical data: 1) The frequent appearance of stable nuclear intervals related to the values of m_e and $\delta m_N = m_n - m_p$=1293 keV, shown as maxima in Fig. 1, and as 2) the period $16m_e$ in the empirical relations between the nucleon and electron masses [2-4]

$$m_n = 115 \cdot 16m_e - m_e - \delta m_N/8 \qquad m_p = 115 \cdot 16m_e - m_e - 9(\delta m_N/8). \tag{1}$$

3) The discreteness in the nucleon separation energies shown in Figs. 2-4 and discussed later, as well as in many other effects with the parameter m_e.

The neutron mass shift $\delta_n = 161.6491(6)$ keV from $k \cdot 16m_e$ coincides with the nuclear tensor forces parameter $\Delta TF = 161$ keV, which corresponds to the one-pion exchange dynamics [3,4], and is equal to the radiative correction $\alpha/2\pi$ to the pion mass.

Nucleons and the electron are stable particles that determine the visible mass of the universe. They belong to the first and last components of the Standard Model, respectively, QCD and QED in the representation

$$SU(3)_{col} \otimes SU(2)_L \otimes U(1)_Y. \tag{2}$$

The masses of the neutron and the electron are in a ratio that is very accurately estimated by the CODATA commission as m_n/m_e=1838.6836605(11), see unexpectedly simple "CODATA relations" (1). Y. Nambu noted [5]: "a) When we discover new phenomena which we do not understand, the first thing to do is to collect data and try to find some empirical regularities among them, b) one next tries to build concrete models, c) finally there emerges a real theory ... Standard Model ...is theoretically unsatisfactory... a) the unification of forces is only partially realized, and b) there are too many input parameters. The nature can be at the same time more complicated than we think, and simpler in a way we do not know yet... ."

According to S. Weinberg [6], d: we are "still seeking a solution" for these problems.

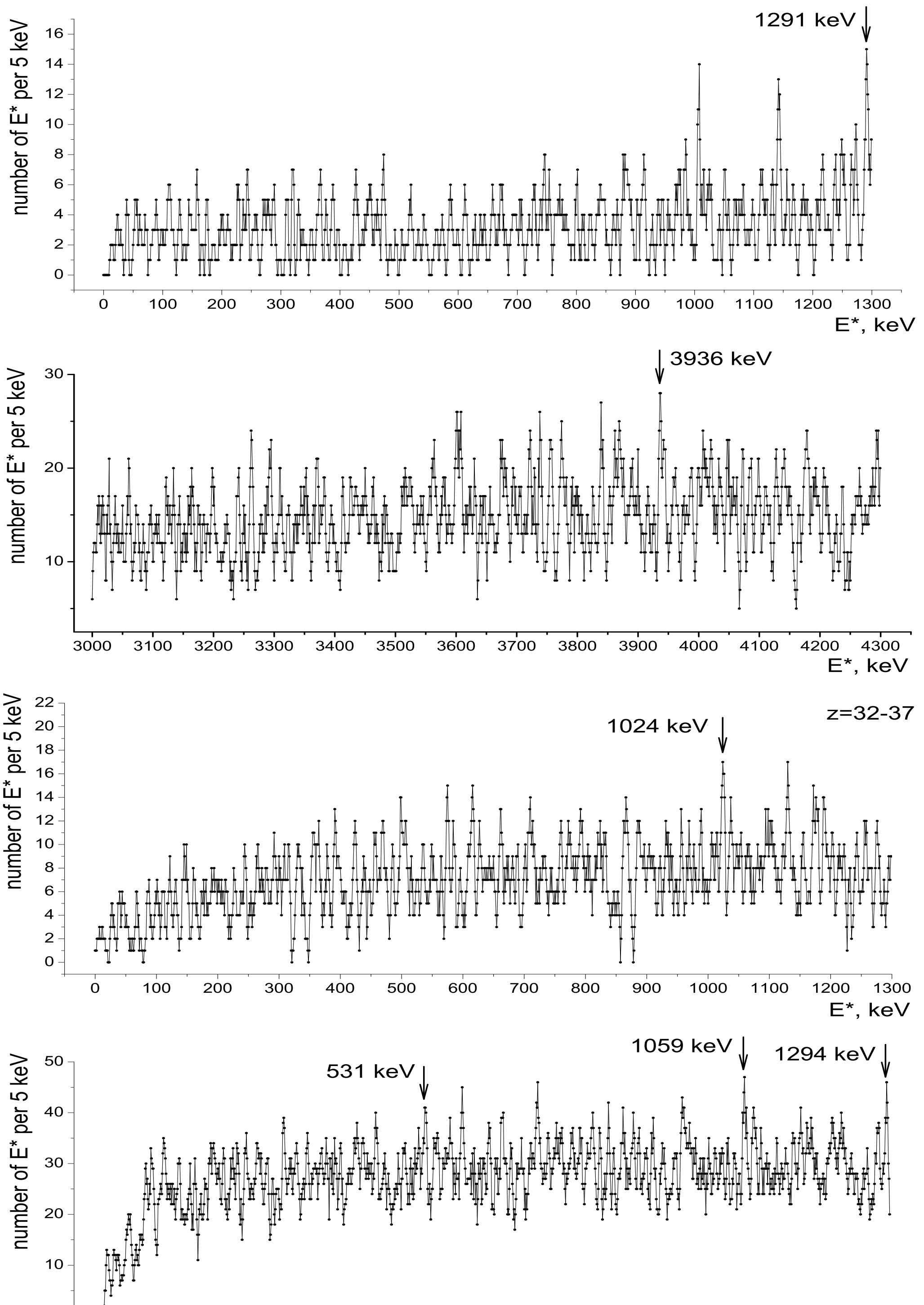

Figure 1: *Top and 2nd line*: E^*-distribution in nuclei with Z=4-29 for E^*<1300 keV and 3000-4300 keV. Arrows mark δm_N and $4\times 8\times 13\delta'$=3936 keV. Schematic diagram of the nuclear level system is shown in Fig. . The equidistant maxima at E^*=1008 keV=$2\times 17\delta'+4\times 18\delta'$, 1142 keV=$5\times 17\delta'$ + $2\times 18\delta'$ and 1291 keV=$8\times 17\delta'$ are explained in the text. $S_{2\mathrm{p}}$ (Z) for Z=40-84. *Center:* The E^*-distribution in all nuclei with Z=32-35. The maximum at 1024 keV=$6\times 18\delta'$. *bottom:* The excitation energy distribution in all nuclei with Z=61-73. Maxima at 531 keV=$4\times 7\delta'$, 1059 keV=$8\times 7\delta'$ and 1294 keV=$8\times 17\delta'=\delta m_N$.

Fine and superfine structures observed in neutron resonances with parameters $\varepsilon' = 1.2\,\mathrm{keV}$ and $\varepsilon''=1.4\,\mathrm{eV}=5.5\,\mathrm{eV}/4$ were introduced together with the parameters of "stabilizing effects of nuclear shells" [7] and empirical observations of many authors on the exact relations between the particle masses, namely the relations $m_N=m_\mu+6m_\pi$ and $m_\Lambda=8m_\pi$ (by Y. Nambu), $m_\omega/2=m_K\text{-}m_\mu=m_N\text{-}m_\eta=391\,\mathrm{MeV}=M_q^\omega$ (by G. Wick), $m_\eta\text{-}m_\mu= m_N\text{-}m_K=m_\Xi/3=M_q=441\,\mathrm{MeV}$ (by R. Sternheimer and P. Kropotkin), $m'_\eta-m_\eta = m_\eta-m_\pi$ (by T. Takabayasi) and $m_{\pi^\pm} - m_{\pi^\circ} = 9m_e$ (by the first author [1,8]).

This allowed us to introduce a common period of the mass discreteness $\delta = 16m_e$, close to the doubled value of the pion β-decay energy. Particle masses ($m_\mu = 13\delta, m_\pi = 17\delta$ and $m_N = 115\delta$), as well as large intervals $391\,\mathrm{MeV}=M_q^\omega = 48\delta$ and $M_q = 54\delta=441\,\mathrm{MeV}$, later used as constituent quark masses in the Nonrelativistic Constituent Quark Model (NRCQM) [9,10]) are shown in Fig. 2 from [1].

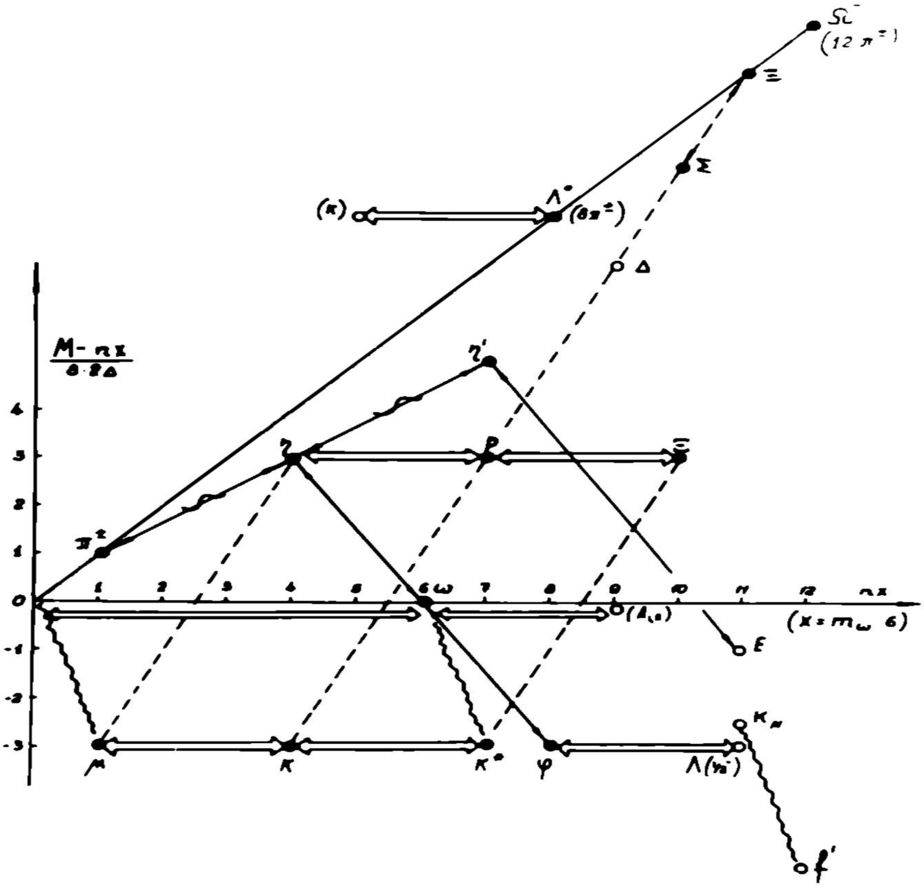

Figure 2: Stable intervals in particle masses found by G. Wick (double arrows) R. Sternheimer (dotted lines) and T. Takabaiasi (arrows). For convenience the integer number $16\delta = 16 \times 16m_e = m_\omega/6$ was subtracted from masses.

The presence of the main CODATA relations (1) for the nucleon masses allows one to consider the previously observed dependence of the two protons separation energies on Z for Z=40-58 and Z=76-84, shown in Figs. 3-4, marked with arrows at the ends of neutron shells at N=50, 82, 126 (see n$\varepsilon_o = 2m_e$ on the right axis and the reference value $S_{2\mathrm{p}}=10183\,\mathrm{MeV}$ in ^{212}Po close to $10\varepsilon_o=10220\,\mathrm{MeV}$).

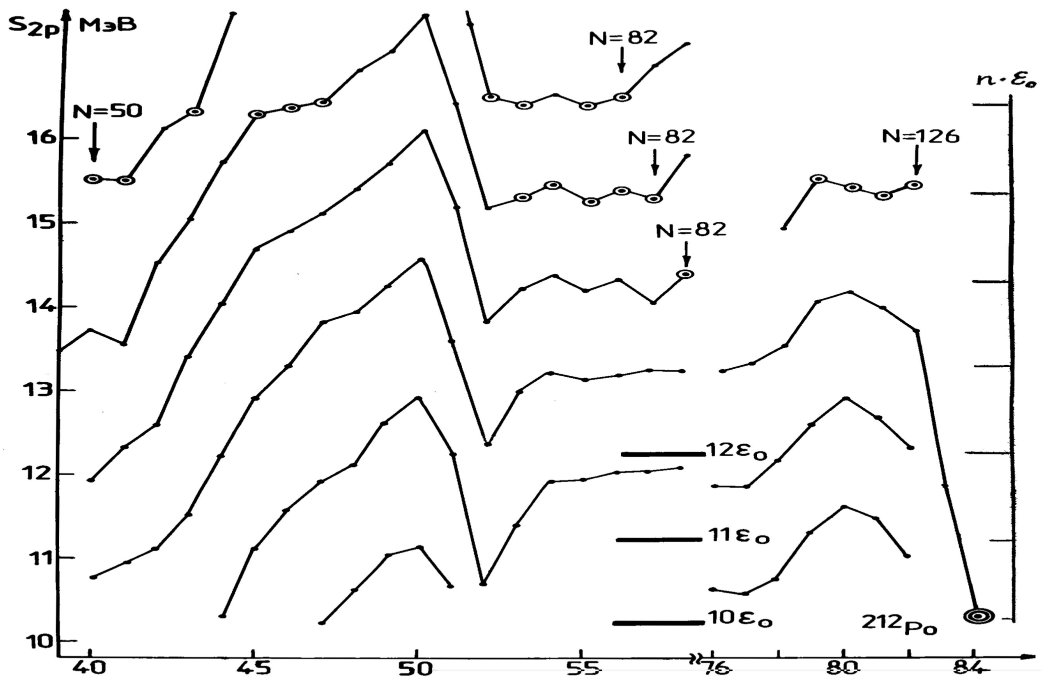

Figure 3: $S_{2\mathrm{p}}$ (Z) for Z=40-84. Double circled is the reference point value $S_{2\mathrm{p}}$=$10\varepsilon_o$ in ^{212}Po. Lines mark groupings in deformed region.

Grouping $S_{2\mathrm{p}}$ values by the number of periods n=10-16 is shown in Fig 3. Near constancy at the end of the closed shell takes place in nuclei differing by ΔZ=1,ΔN=2, and in nuclei differing by ΔZ=2, ΔN (or ^{4}He cluster), ΔE_B turns out to be close to integers k Δ=$9m_e$=4.6 MeV. At N=82 for k = 20 ΔE_B=46 MeV (see Fig. 5, left). A similar ΔE_B=147 MeV=32Δ = 18 $\times$ 16m_e = 18δ grouping was found in light nuclei (Z $\leq$26), differing by 4^{4}He cluster (Fig. 5, right [11]). In Table 1 it is shown that the unexpected proximity of these empirical values ΔE_B to the symmetry motivated multiple values $k \times m_e$ is not contained in the existing theoretical models. Three empirical observation are used during production Electron-based Constituent Quark Model (ECQM):
1. Leptons are considered together with the parameters of a very successful Nonrelativistic Constituent Quark Model (NRCQM), namely pion parameters f_π = 130 MeV, m_π=140 MeV and the constituent quark masses M_q = $m_\Xi/3$=441 MeV, M_q^ω = $m_\omega/2$=391 MeV. The mass of τ-lepton is equal to $2m_\mu$+$4M_q^\omega$.
2. Leptons and hadrons are forming the observed correlations in the mass spectrum with a common period 8.176 MeV=δ = 16m_e (Fig. 1 [3,4]), where it is shown that the masses of the fundamental fields $M_Z = m_\mu(\alpha/2\pi)^{-1}$ and $M_{H^\circ} = m_e/3(\alpha/2\pi)^{-2}$, as well as the main parameter of the ECQM and NRCQM models, $M_q = m_e(\alpha/2\pi)^{-1}$, are interconnected by symmetry motivated relations and the common QED correction.
3. In this work, we consider additional empirical observation of the particle mass spectrum and nuclear data, including the important role of neutron resonance data in confirming the QED correction, which is a very important factor in the SM development.

In at least three cases: in the masses of leptons, in the masses of nucleons (CODATA relations), and in the masses of hadrons containing bottom quarks, unexpectedly accurate empirical relations with the electron mass m_e are observed.

Table 1: Proximity $\Delta E_{\rm B}$ (keV) to $(45{=}5{\times}9)\varepsilon_o$=45.99 MeV in nuclei differing by $2\Delta Z{=}\Delta N{=}4$, N=82 *center* and to $144\varepsilon_o = 8 \times 18\varepsilon_o$=147.2 MeV, N=20 *right* In nuclei differing by 4α cluster. Small deviations from $k \times \varepsilon_\circ = 2m_e$=1022.0 keV in real values and large deviations in $\Delta E_{\rm B}$ drom Finite Range Droplet Model *bottom*, are boxed [10].

Nucl.	^{137}Cs	^{135}La	^{137}La	^{139}La	^{136}Ce	^{138}Ce	^{140}Ce	^{139}La	^{39}K
Z	55	57	57	57	58	58	58	57	19
N	$\boxed{82}$	78	80	$\boxed{82}$	78	80	$\boxed{82}$	$\boxed{82}$	$\boxed{20}$
$\Delta E_{\rm B}$	45970	46018	45927	46024	46087	45997	45996	91975	147160
$n\varepsilon_o$	45990	45990	45990	45990	45990	45990	45990	91980	147168
diff.	$\boxed{-20}$	28	-63	$\boxed{34}$	97	$\boxed{7}$	$\boxed{6}$	$\boxed{-5}$	$\boxed{8}$
Theo.	46340	45950	46820	46970	45960	46850	47160	93200	147450
diff.	$\boxed{350}$	-40	830	$\boxed{980}$	-30	860	$\boxed{1170}$	$\boxed{1220}$	$\boxed{282}$

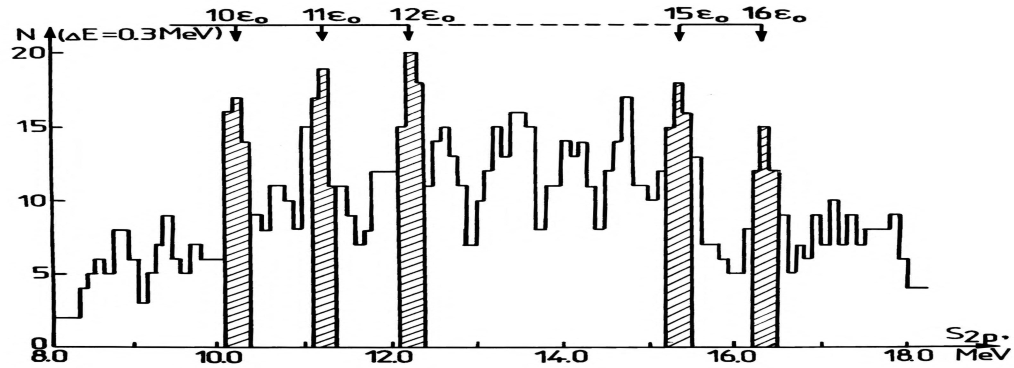

Figure 4: Distribution of $S_{2\rm p}$ (Z) (averaging 300 keV, step of the ideohistogram - 100 keV). The regions of groupings at multiples of ε_o=1022 keV are shaded.

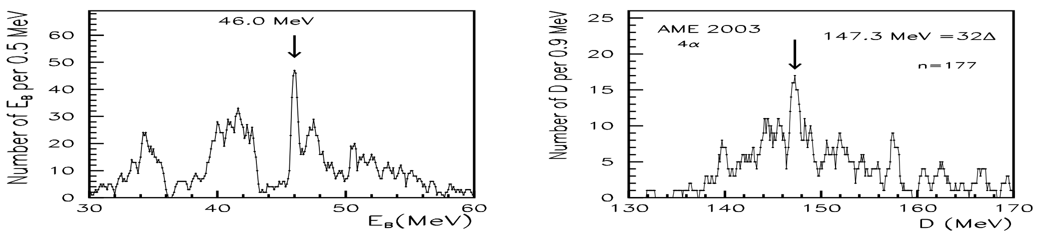

Figure 5: Grouping of ΔE_B in nuclei differing by ^{6}He-cluster (*left*) and by 4 ^{4}He-clusters.

Observed in Fig. 1 discreteness (fine structure) with numbers k=13,14,17 and 18 of the common period δ'=9.5 keV was similar to other empirically found discreteness in nuclear data and particle masses. In particle masses, k=13 corresponds to the masses of the muon and Z-boson (Table 2, periods $\delta = 16m_e$=8.176 MeV and δ°=3.50 GeV), while k=18 (with the same periods) corresponds to the difference in baryon masses due to the appearance of strangeness (m_s ≈150 MeV in NRCQM) and due to the residual quark interaction 147 MeV=$(m_Delta - m_N)/2$, as well as the scalar mass 125 GeV.

Here we show that discreteness with k=13 also exists in neutron resonance data. Symmetry motivated relations with k=13, 16-18 in addition to the proximity of the ratio $1/(32 \times 27)115.74 \times 10^{-5}$ to $\alpha/2\pi = 115.96 \cdot 10^{-5}$ provide the production of Table 2.

2. Electron-based Constituent Quark Model

The muon and tau-lepton masses, together with the mass of the constituent quark $M_q^\omega = m_\omega/2$=391.33 MeV, take part in the exact relation (3). There is a proximity of the ratio m_τ/m_{K^*}=1776.9 MeV/891.7 MeV=1.99 to 2.0.

$$m_\tau = 2m_\mu + 2m_\omega \approx 2m_\mu + 4 \cdot 48 \cdot 6m_e = 2m_{K^*} \tag{3}$$

It was found [3,8] that besides the m_μ/m_e=105.658 MeV/0.510.999 MeV=206.768 proximity to an integer (lepton ratio) L=207=16×13-1, the same ratio exists between the vector bosons and constituent quarks masses M_Z/M_q=206.8 and M_W/M_q''=207.3 for $M_q'' = m_\rho/2$=387.6 MeV. For an exactly integer value L one can get an estimate of the respective mass value 441.0 MeV close to $54\delta - m_e$. The initial mass of the baryon constituent quark in NRCQM is three times greater than ΔM_Δ, namely M_q=441 MeV=$m_\Xi/3$. The mass of the meson constituent quark M_q^ω in NRCQM is derived as half of the masses of ω-meson close to 780 MeV=$6f_\pi$, where f_π=130.7 MeV=16δ is the pion β-decay parameter.

Table 2: Presentation of particle masses (3 top sections) and nuclear data (bottom) by the expression n·16m$_e$$(\alpha/2\pi)^X$M with QED correction $\alpha/2\pi$. Boxed values m$_\mu$, M_Z, M_{H°, $\delta^\circ, \delta, \delta', \delta''$ and ΔM_Δ=m_s, m$_e$/3 are considered in [3.4,7,8]. Intervals in nuclear binding energies (X=0) and fine structure in nuclear states are considered elsewhere.

X	M	n = 1	n = 13	n = 16	n = 17	n = 18
-1	3/2			m$_t$=173.2		
GeV	1	$\boxed{16\mathrm{M}_q=\delta^\circ}$	$\boxed{\mathrm{M_Z}=91.2}$	M'_{H}=115		$\boxed{M_{\mathrm{H}^\circ}=125}$
0	1	$16m_e$=$2m_d$-$2m_e$	$\boxed{\mathrm{m}_\mu=106}$	f$_\pi$=130.7	m$_\pi$, Λ_{QCD}	$\boxed{\Delta M_\Delta=147}$
MeV	1	ΔE_B	106	130	140	147
	2	Figs. 6,7	212	262		296
	3	NRCQM		$\boxed{\boxed{\mathrm{M}_q^\omega=391}}$		$\boxed{\boxed{\mathrm{M_q}=441}}$
	4	Radial excit.			$(\mathrm{b}\tilde{\mathrm{b}})$=563.0	$(\mathrm{c}\tilde{\mathrm{c}})$=589.1
	6			m_ω=782		2M$_q$=882
	9				m_c=1270(20)	
	10	$m_\Lambda = 19m_\pi$			1390-1407	
	12	$m_\Omega = 12m_\pi$			1671-1688	
	60	Fig.? in [3]				8848
	64	$\eta_b(1S), \Upsilon(1S)$				9399-9460
1	1	$\boxed{16m_e=\delta=8\varepsilon_\circ}$			$k\delta$-m$_n$-m$_e$=	$\boxed{170 = \mathrm{m_e}/3}$
keV	1,8,8 · 4	CODATA, Fig.1	3936	δm_N=1293.3	=161.651	
1	1	$\boxed{9.5=\delta'=8\varepsilon'}$	123	152	Δ^{TF}=161	170 (Sn)
keV	2		247 (^{91}Zr)		322 (^{33}S)	340 (^{100}Mo)
2	1, 4	$\boxed{11=\delta''=8\varepsilon''}$	143 (As)		749 (Br, Sb)	Neutron
eV	4, 8		570 (Sb)		1500 (Sb, Pd)	reson.

The above mentioned CODATA relations (1) and the empirical observation used for production of the Electron-based Constituent Quark Model (ECQM) are in agreement with direct observation of the parameters of the very successful Nonrelativistic Constituent Quark Model (NRCQM), namely, the pion parameters $f_\pi = 130\,MeV, m_\pi$=140 MeV and the constituent quark masses $M_q = m_\Xi/3$=441 MeV (maxima at 445 MeV, 3504-3962-4427 MeV), as well as $M_q^\omega = m_\omega/2$=391 MeV and 781 MeV on the total spacing distribution of masses all 198 particles from the PDG-2020 compilation shown in Fig. 6.

The muon and τ-lepton (equal to $2m_\mu+4M_q^\omega$ [3,8]) masses together with hadrons form the observed correlations in the mass spectrum with a common period of 8.176 MeV=$\delta = 16m_e$. The masses of the fundamental fields $M_Z = m_\mu(\alpha/2\pi)^{-1}$ and $M_{H^\circ} = m_e/3(\alpha/2\pi)^{-2}$ in symmetry motivated relations (1:3) with leptons and common QED correction. The important role of neutron resonance data in confirming QED correction and symmetry motivated relations will be seen in the further development of the Standard Model.

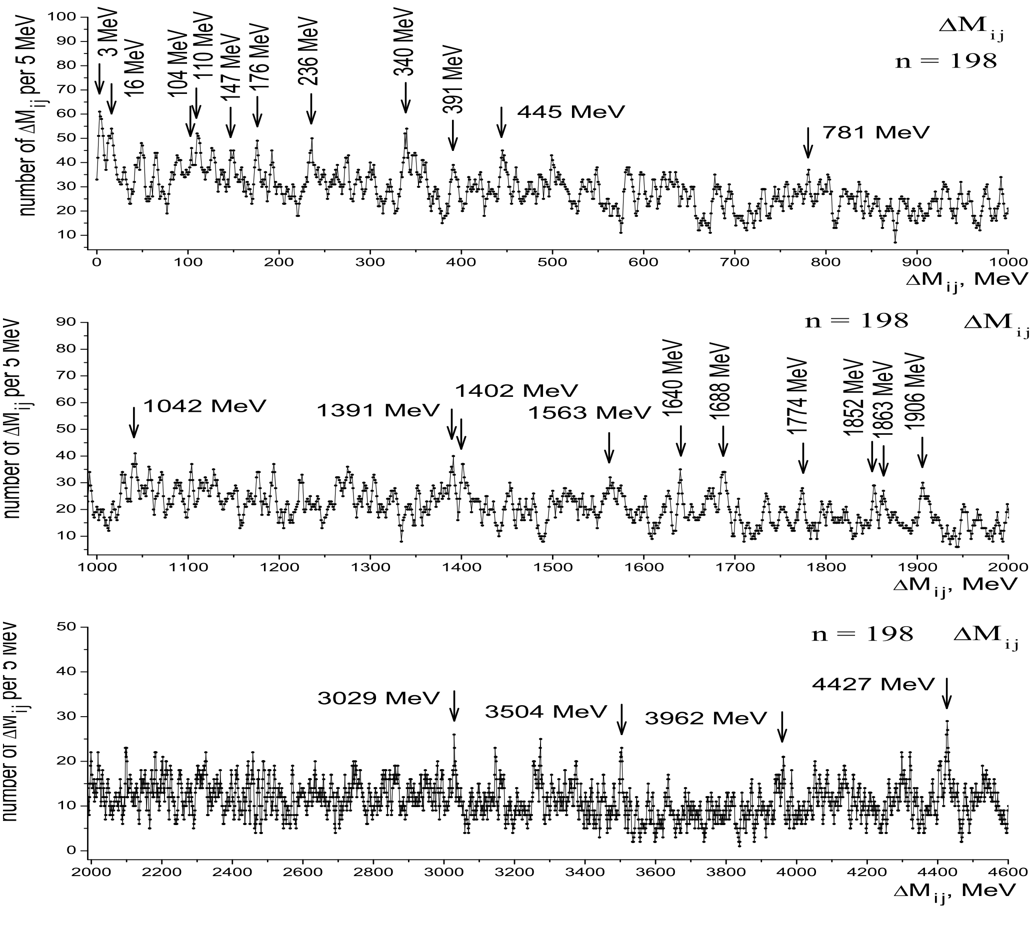

Figure 6: *Top:* ΔM distribution of all differences between particle masses from compilation PDG-2020 (averaging 5 MeV) for the energy region 0–1000 MeV. Maxima at 16 MeV= $2\delta = 2 \times 16m_e$, 391 MeV=$m_\omega/2$, 445 MeV=M_q, 781 MeV=m_ω. *Center:* The same for energy region 2000–4600 MeV. Maxima are at 1774 MeV≈ m_τ=1777 MeV. *Bottom:* The same for energy region 2000–4600 MeV. Maxima are at 3504 MeV≈ $8M_q = \delta^\circ/2$, 3962 MeV≈ $9M_q$ and 4427 MeV≈ $10M_q$.

3. Analysis of neutron resonance data

Resonance parameters, which are investigated within neutron resonance spectroscopy demonstrate the same symmetry motivated relations observed between stable nuclear intervals and in particle masses.

In [11] it was found that for some monoisotopic odd-odd targets, stable intervals can be observed (143 eV in As, 43 eV in Nb and 594 eV in Cs). The same values were found in the positions of strong resonances of many nuclei (43 eV in Nb, 570 eV in Th etc.). In the distribution of relatively strong neutron resonances in Z=33-56 nuclei, maxima at 44 eV and 572 eV were observed (Fig. 7), and in nuclei Z=51-94, maxima at 22 eV and 286 eV. We show here confirmation of the distinguished character of the 4:13 relation between stable intervals in neutron resonances.

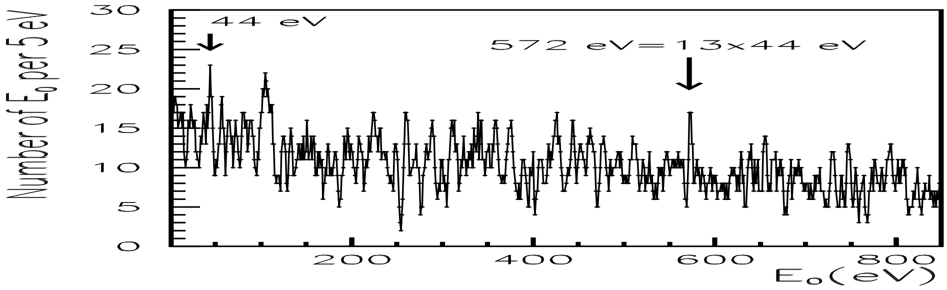

Figure 7: Distribution of positions of relatively strong neutron resonances of all nuclei with Z=33-56 [11]. There is strong resonance in ^{233}Th at 570 eV with $g\Gamma_n^\circ$=1.1 meV, which means, that neutron separation energy is correlated with the period 573 eV under consideration.

We use here the data for the ^{233}Th and $^{234-236}$U isotopes to check the 1:4:13 relation previously found between the stable intervals in the neutron resonances of many other isotopes. Some results of an earlier analysis of neutron resonance data for ^{233}Th were also given in [13]. These data for structurally important isotopes contain the most numerous lists of resonance parameters (evaluated by F. Gunsing and L. Leal).

Thorium isotopes have 90 protons, corresponding to the filled $f_{7/2}$ subshell. It was noted long ago that the spacing distribution of its L=0 resonances is clearly nonstatistical. On the histogram with the averaging parameter 5 eV in Fig. 8 (top), the equidistancy of the maxima at k=1, 2, 3, 5 of the estimated period 11 eV corresponds (as k=288/11=26) to the strongest maximum at D=288 eV (marked with an arrow). Fixing all such intervals (x=288 eV) in the spectrum of all s-wave resonances (see Fig. 8, center), we obtain maximum at a doubled value of 576 eV. Such an interval corresponds to the distance between strong neutron resonances (maximum at 573 eV in Fig. 2, bottom, with the selection of resonances with a reduced neutron widths greater than 1 meV, deviation from the random level $\approx 3\sigma$). A small maximum at 42 eV on the same distribution (Fig. 8, bottom) corresponds to a 1:13 ratio between strong resonances (between states with a relatively large single-particle component in the wave function). Similar correlations were found earlier in strong resonances of many different isotopes (Fig. 7).

The spectrum of highly excited ^{236}U states contains 3164 states with the spacing distribution shown in Fig. 9, top (all states have L=0). Neutron resonances were selected according to spin J (n=1438 for J=3 and n=1734 for J=4), and respective spacing distributions are given in Figs. 9 and 10.

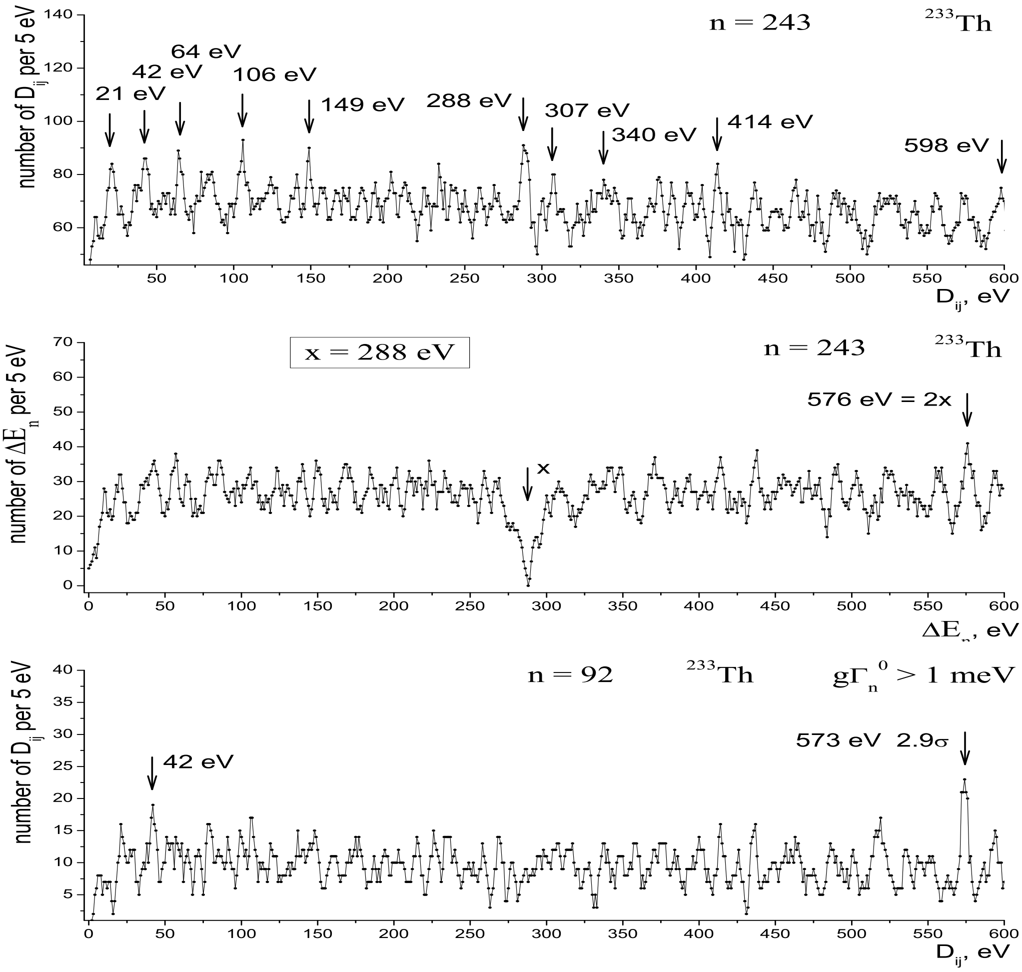

Figure 8: *Top:* Spacing distribution of all L=0 neutron resonances in ^{233}Th. *Center:* Spacing distribution of L=0 neutron resonances in ^{233}Th adjacent to intervals D'=x=288 eV. *Bottom:* Spacing distribution of all L=0 strong neutron resonances in ^{233}Th.

In spacing distribution of ^{236}U resonances with J=3 (Fig. 9, 2nd line) a grouping of 4 maxima at 55 eV=$5\delta''$ is situated at the doubled position of the maximum at 27.8 eV (see Fig. 9 center). Resonances forming the maximum at =393.5 eV (close to 396 eV=$4 \times 9 \times 11$ eV) are frequently appear together with another resonance situated 288 eV away from them (see maximum at 287.9 eV, close to 2×143 eV=$2 \times 13\delta''$). This interval, coinciding with D'=288 eV in ^{233}Th (Fig. 8, top), has exactly twice the value of the interval 143.4 eV in independent spacing distribution for J=4 resonances (Fig. 10, top). This interval appears together with the interval 44.2 eV=$4\delta'' = 4 \times 11$ eV.

In spectrum of ^{236}U (Fig. 9, top) there is a small maximum at 43 eV, but only by selecting resonances according to their spin and by using correlation analysis of their decay properties one can hope to obtain further fundamental information.

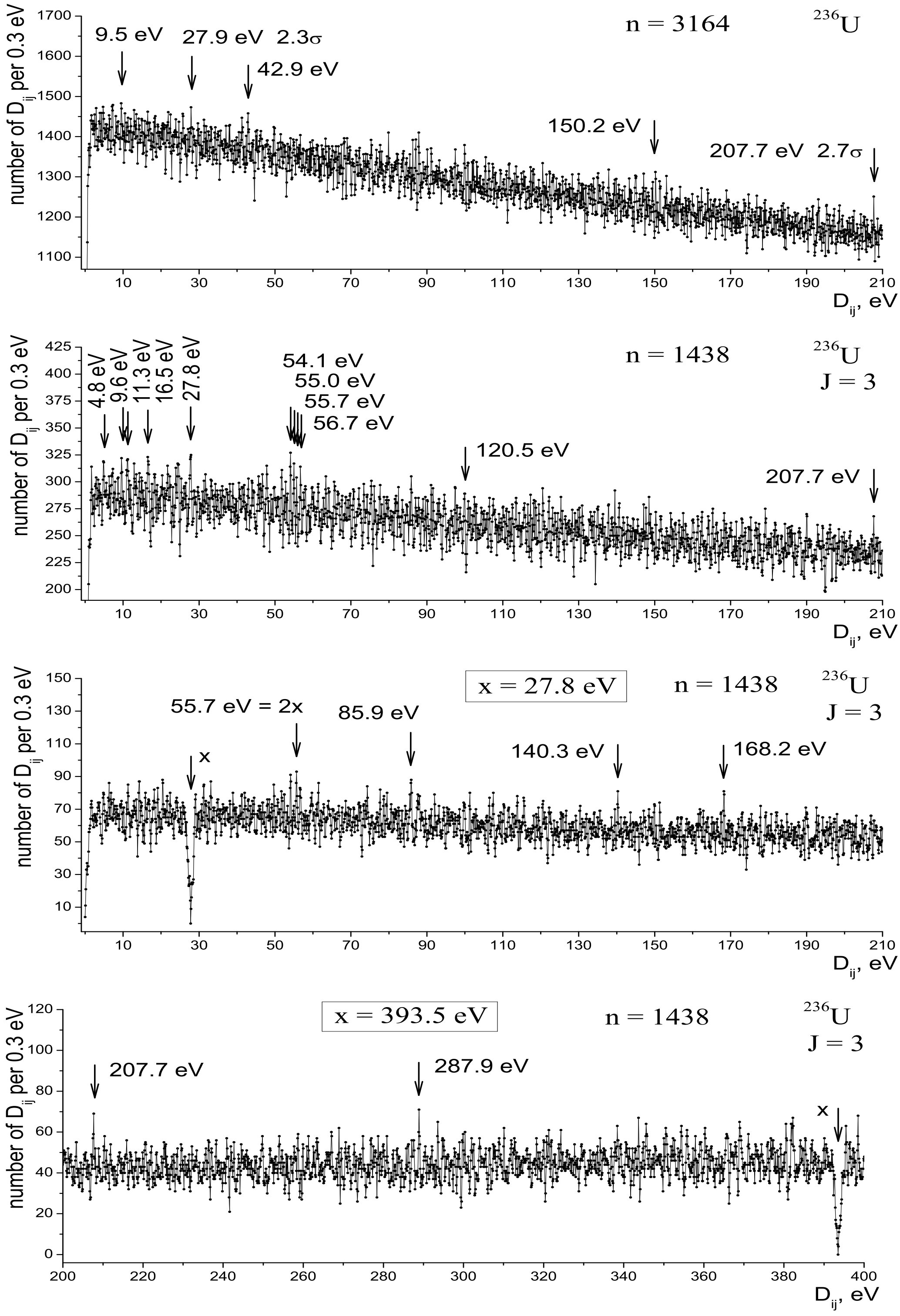

Figure 9: *Top*: Total spacing distribution in all ^{236}U resonances. *2nd line*: The same for resonances with J=3. *Center:* Adjacent interval distribution for J=3 ^{236}U resonances for x=27.7 eV=(55 eV=$5\delta''$)/2. *Bottom*: The same for x=393.5 eV (close to 396 eV=$4 \times 9 \times 11$ eV). Maximum at 288 eV (close to 2×143 eV=$2 \times 13\delta''$) coincides with D'=288 eV in ^{233}Th (Fig. 8, top).

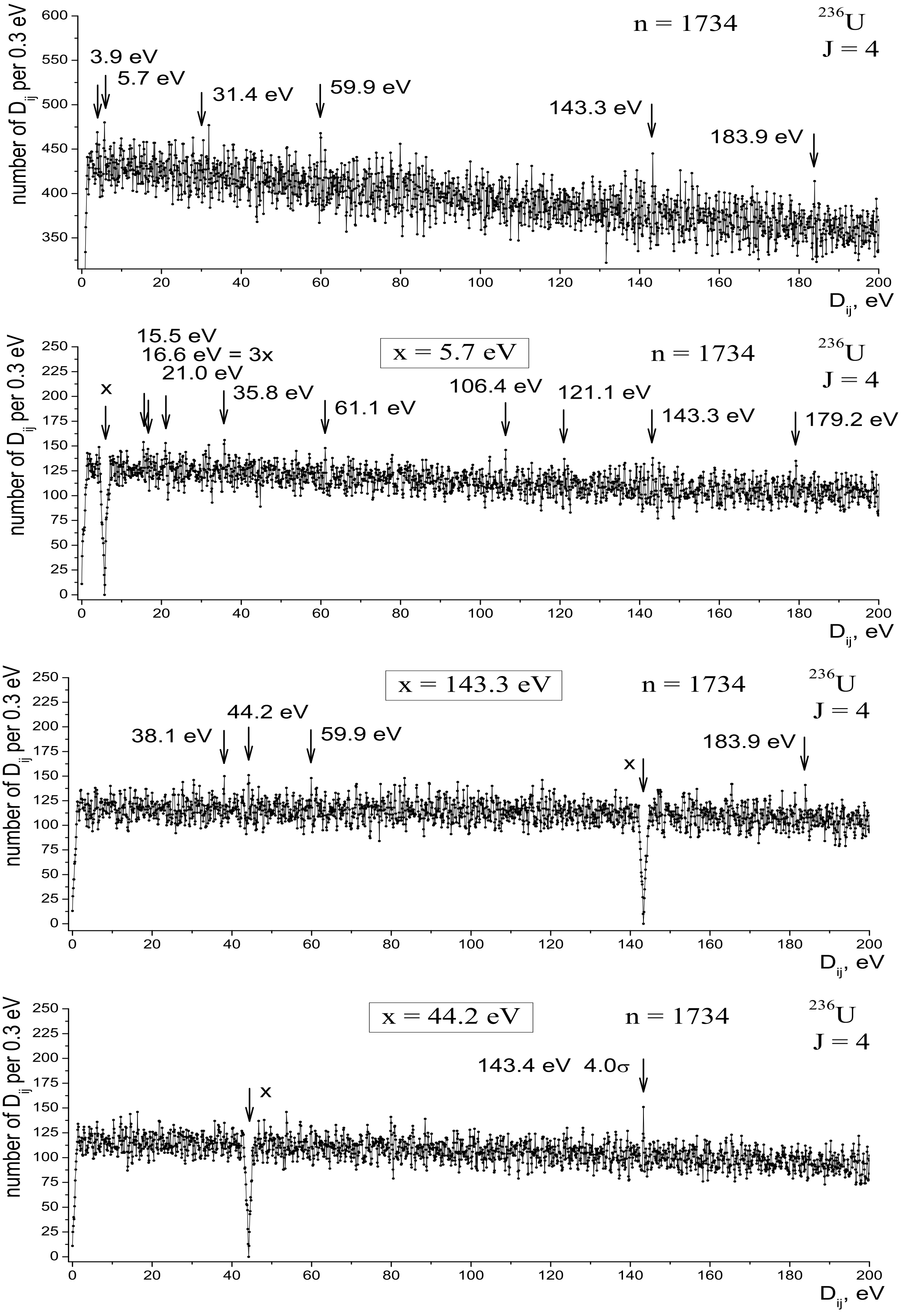

Figure 10: *Top*: Spacing distribution of all ^{236}U neutron resonances with J=4. *2nd line*: Adjacent interval distribution of ^{236}U resonances for x=5.7 eV. *Center:* Adjacent interval distribution for J=4 ^{236}U resonances for x=143.3 eV. *Bottom*: The same for x=44.2 eV. 4Σ deviation is marked. An exact ratio 13:4 = 143.4:44.2 is considered in the text.

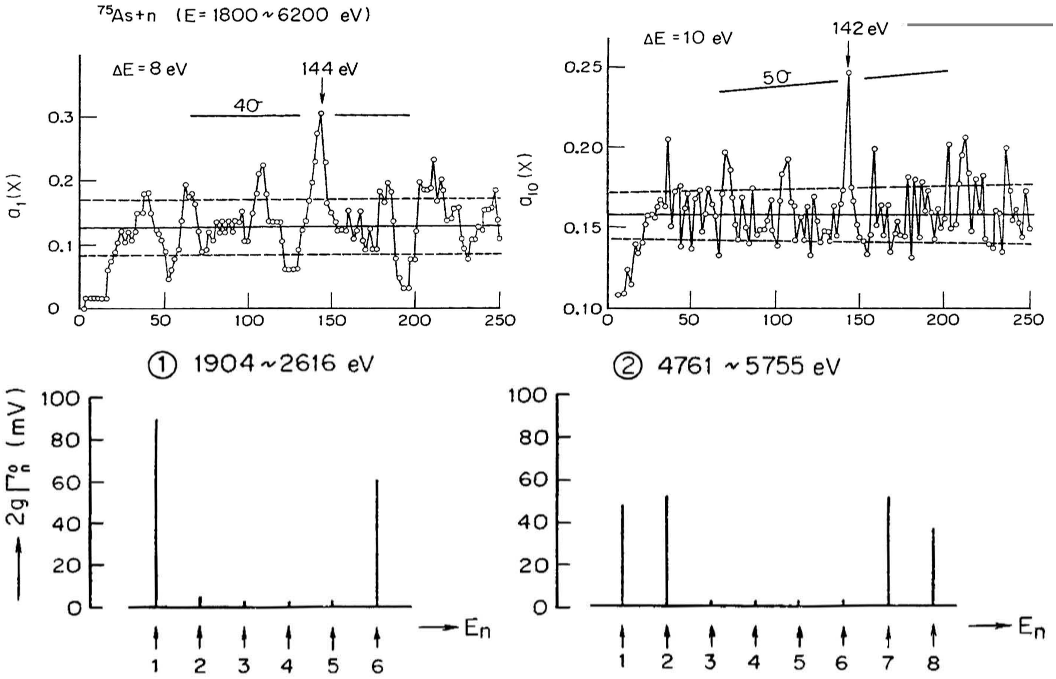

Figure 11: *Top:* The spacing distribution in neutron resonances of the compound nucleus ^{76}As with averaging interval ΔE. The response function of the program "Period" means the number of resonances coinciding with the periodic system (k is the number of periods x, one for the left distribution and ten for the right distribution) [14]. *Bottom:* The periodicity in the positions of neutron resonances of the target ^{75}As, found in [14].

Considering the role of the ratio k=13 in particle masses [8,12], discussed in the Introduction, we could mention observations of this ratio in neutron resonance spectra of other isotopes [13,14]. Observation by K. Ideno and M. Ohkubo of the periodicity in resonances of As [14] is shown in Fig. 11 (maxima at $D = k \times 13\delta'' = k \times 143\,\mathrm{eV}$ are given as deviation in units of σ). Two examples of long range correlations are presented.

Discreteness with k=10,12,14 of the period of $13\delta'$=123 keV in $E^*(0^+)$ of ^{108,110,112}Cd was noticed.

4. Conclusions

Symmetry motivated relations 1:9:13:16:17 between particle masses and stable nuclear intervals of the few-nucleon-, fine- and superfine-structures effects are considered here as an indirect check of the ECQM model with the parameter $\alpha/2\pi$ corresponding to the QED correction due to influence of the condensate. This factor can be indirectly studied in nuclear data. Important application of this factor can be seen in empirical analysis of particle mass data. In Table 3 from [8] a coincidence of two estimates of the mass of the constituent quark $M_q \approx$441 MeV used in NRCQM and ECQM models are presented. The first estimate is derived from the coincidence of the ratio m_μ to M_Z with $\alpha/2\pi$ and $L = 13 \times 16 - 1$. The second estimate is obtained from empirical mass of the scalar (b$\tilde{\mathrm{b}}$) meson ($m(\eta_b) = 64M_q$).

We see that any confirmation of different aspects of CODATA relations as multiple relations with the real mass of the electron, its symmetry and QED correction could be

Table 3: Comparison of the SM and NRCQM parameters m_μ=105.659375(35) MeV and m_e=510.998328(11) keV with QED radiative correction $\alpha/2\pi$=116.0·10^{-5} [8].

	Values	Mass and ratios
1	Ratio=m_μ/m_e	206.768
2	(L - ratio)/L	112.08·10^{-5}
3	m_μ/M_Z	115.9·10^{-5}
4	m_e/M_q	115.7·10^{-5}
5	$M_q = m_{\Xi^-}/3$	441.5 MeV=$54 \cdot 16 m_e$
6	M_Z/L=$M_q(1-\alpha/2\pi)-m_e$	$\boxed{440.5\,\mathrm{MeV}{=}M_q^{red}}$
7	$(3/64)m(\eta_b)$	$\boxed{440.6\,\mathrm{MeV}}$

used for SM development. The unexpectedly universal relations in the neutron resonance positions and spacings signal fundamental aspects of nuclear physics [16,17].

The symmetry motivated relations with k=13 in the nuclear data, manifested in the stability of the intervals, are associated with the same discreteness in the particle masses and result in the lepton ratios and masses of both heavy leptons.

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