AN INVERSE-PROBLEM SOLVING BY THE EXAMPLE OF $^{238}U(n,2\gamma)^{239}U$ REACTION ANALYSIS

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1. Introduction

In spite of preserved representation that a nucleus is a system of non-interactive Fermi-particles, a modern theory supposes that the wave function of any excited level includes both quasi-particle and phonon components. So an experimental obtaining of the nuclear-physical parameters of ^{239}U nucleus is needed not only for practical application, but also for investigation of fundamental interaction between fermion and boson states of nuclear matter. Information about the nuclear-matter behavior can be received if only the strong-correlated nuclear-physical parameters (the nuclear level density ρ and partial widths Γ of reaction-products emission) are obtained simultaneously at the nuclear-reaction investigation.

As the required strong-correlated parameters enter into the measured spectrum as a product $\rho\times\Gamma$, their extraction from indirect experiment is a complicated problem of search for inverse solution. Analysis of data of indirect experiment in order to obtain the strong-correlated parameters ρ and Γ always leads to inevitability of unknown sizeable systematical errors. By the highest standards, reliable nuclear-physical parameters can be obtained in two independent experiments only.

When γ-quanta coincidences of the two-step cascades are recording, indeterminateness of the nuclear-physical parameters obtained from measured intensities of $\gamma\gamma$-cascades can be diminished in the presence of information about initial, final and intermediate cascade levels. And with all this going on, it is very important to ascertain the quanta sequence in the cascades in order to determine more reliable nuclear-parameters. But an ambiguity of the obtained nuclear parameters always exists in $\gamma\gamma$-coincidence experiment, even at essential difference of the $\rho(E_{ex})$ and $\Gamma(E_\gamma)$ functions, where E_{ex} and E_γ are energies of nuclear excitation and of γ-quantum.

The empirical method was created and developed in Dubna in order to enable an investigation of dynamics of nuclear-structure change below the neutron binding energy in a nucleus. The Dubna method allows simultaneous extraction of the nuclear level density and partial widths of γ-quanta emission from approximation of the experimental intensities of only primary transitions of the two-step γ-cascades, without using experimentally-untested hypothesis.

The Dubna empirical method was applied to analyze presented in [1] experimental γ-spectrum from the $^{238}U(n,2\gamma)^{239}U$ reaction, which has been measured using nearly 4π γ-ray calorimeter DANCE composed of a spherical array of 160 BaF_2 crystals.

2. Opening remarks concerning analyzable data

The multistep γ-cascade spectra at the decay of isolated *s*-wave ($J^\pi = 1/2^+$) resonances in compound-nuclei of three uranium isotopes are presented in [1] for multiplicities up to M=4. For reliability of extraction of the nuclear-physical parameters from experimental γ-cascade spectra it is necessary to determine the quanta sequence in the cascades. As for M-cascade quanta there is a possibility of M! placing variants in the decay scheme, it makes the quanta-sequence determination impossible in the cascades with multiplicities M>2.

An uncertainty in the quanta sequence exists even for the cascades with multiplicities M=2, but the presence of energy-resolved cascades' peaks in the experimental spectrum allows the use of available information about known intense transitions. As far as an energy resolution of coincidences recording enables, the experimental spectra are composed of isolated peaks and continuum of unresolved ones. In the two-step cascade the γ-quanta sequence can be unambiguously determined in the experimental-intensity spectrum only for a part of energy-resolved cascades corresponded to available spectroscopic data. The part of such cascades in the experimental spectrum can be very sizeable if detectors of high energy resolutions (now HPGe-detectors only) are used to record γγ-coincidences. For example, used in our experiments HPGe-detectors of greatly high intensity allowed us to increase a part of energy-resolved cascades in the experimental γ-spectra up to 40–60% at statistics about of 40000 events (or more).

But energy-resolving power of scintillation counters, which are used in experiments with 4π-calorimeter, unfortunately doesn't allow a separation of individual intense transitions, so the decay-scheme application is impossible in the analysis. And what is more, in the experiment with 4π-calorimeter a sharp increase in a number of counts was always observed for experimental γ-spectra of the cascades of M=3 multiplicity as compared with ones of M=2. Underestimation of the intensities of the cascades of M=2 multiplicity is quite possible owing to redistribution of annihilation quanta between detector crystals at low energy resolution of the spectrometer.

In spite of insufficient-detailed spectrum recorded by scintillation detectors, which make impossible a separation of spectrum of primary γ-transitions with a fair degree of confidence, in spite of everything, we try to analyze presented in [1] experimental spectrum of M=2 multiplicity from the ^{238}U(n,2γ)^{239}U reaction.

3. The basis of the Dubna empirical method

A key concept of the Dubna empirical method is obtaining the $\rho(E_{ex})$ and $\Gamma(E_\gamma)$ functions from the fitting of the $I_{\gamma\gamma}(E_1)$-intensities of only primary transitions of the cascades (E_1 is energy of primary γ-quantum of the two-step γ-cascade), calculated when Monte-Carlo solving a system of nonlinear equations (1), to the experimental intensities of primary transitions. The use of high-aperture HPGe-detectors in the experiments allows a determination of the part of primary transitions of the two-step cascades with an uncertainty of 10–20% without distortion of the spectrum normalization, which was confirmed by negligibility of the effect of the systematic errors of $I_{\gamma\gamma}(E_1)$-spectrum separation, for example, for ^{172}Yb nucleus [2].

Each of the equation (1) connects $I_{\gamma\gamma}(E_1)$-intensities with the partial widths of γ-transitions between neutron resonance λ and a group of final levels f via all possible intermediate levels i in a small energy interval ΔE_j:

$$I_{\gamma\gamma}(E_1) = \sum_{\lambda,f}\sum_{i}\frac{\Gamma_{\lambda i}}{\Gamma_\lambda}\frac{\Gamma_{if}}{\Gamma_i} = \sum_{\lambda,f}\sum_{j}\frac{\Gamma_{\lambda j}}{<\Gamma_{\lambda j}> M_{\lambda j}} n_j \frac{\Gamma_{jf}}{<\Gamma_{jf}> m_{jf}}. \quad (1)$$

In interval ΔE_j there are n_j intermediate levels i, to which $M_{\lambda j}=\rho\Delta E_j$ transitions go from initial level λ, and m_{jf} secondary transitions go to final level f ($n_j=\rho\Delta E_j$ and $m_{jf}=\rho\Delta E_j$ at all intermediate-levels energies). At the excitation energy $E_{ex}< E_d$ (E_d is maximal excitation energy of "discrete" level area) the system (1) contains only experimental data on energies and quantum numbers of known to date levels and their decay modes.

In spite of the nonlinearity inevitably leads to false likelihood maxima, the system of non-linear equations (1) could have a definite solution (hypothetical limit) if the experimental

$I_{\gamma\gamma}(E_1)$-intensities would be known in each energy point. But as there are no high-aperture spectrometers of gammas with an electron-volt resolution, solving of the system of the equations (1) is impossible without using model representations of the nuclear level density and the strength functions $k(E_\gamma)=\Gamma/(A^{2/3}\cdot E_\gamma^{3}\cdot D_\lambda)$, where A – nuclear mass number, D_λ – average distance between nuclear levels. However, available in RIPL-file [3] models of the required nuclear parameters (based at the representation of a nucleus by a pure fermion system) don't allow a description of real experimental spectra. So the problem of a choice of the most realistic model representations for the $\rho(E_{ex})=\varphi(p_1, p_2, ...)$ and $\Gamma(E_1)=\psi(q_1, q_2, ...)$ functions of some fitted parameters p and q, is as great as ever.

It is important that deformed at every iteration fitted parameters would lead to the best experimental-spectra description. That is guaranteed by the Dubna method, as the model representations of the nuclear-physical parameters are tested and modified in the course of the analysis. Average amplitudes of changing of correction-vector components (no more than 1% of current values of the set of fitted parameters) decrease at each iteration to guarantee χ^2 minimum:

$$\chi^2=\sum_{n_j}\frac{(I_{\gamma\gamma}^{cal}(E_1)-I_{\gamma\gamma}^{\exp}(E_1))^2}{\sigma^2}, \qquad (2)$$

where $I_{\gamma\gamma}^{cal}(E_1)$ and $I_{\gamma\gamma}^{\exp}(E_1)$ are model-parametrized and experimental intensities, and σ^2 is a dispersion of their difference. With all this going on there is no need to use any hypothesis untested experimentally.

A systematical uncertainty of the obtained nuclear parameters is mainly determined by inexactness of model-phenomenological representations about energy dependences of the $\varphi(p_1, p_2, ...)$ and $\psi(q_1, q_2, ...)$ functions.

4. The nuclear-parameters representations in the Dubna empirical method

At given $\rho(E_{ex})=\varphi(p_1, p_2, ...)$ and $\Gamma(E_1)=\psi(q_1, q_2, ...)$ functions parametrized according to definite models there is only one solution of the system of equations (1).

Now for representation of the $\rho(E_{ex})$ function in the Dubna empirical analysis a modern Strutinsky model [4] (which is able to describe successfully pre-equilibrium nuclear reactions) as well as a balance between changes of entropy and energy of quasi-particles' states [5] are being applied.

In a framework of the model [4] an expression for density ρ_l of levels of fermion type is written as:

$$\rho_l=\frac{(2J+1)\exp(-(J+1/2)^2/2\sigma^2)}{2\sqrt{(2\pi)}\sigma^3}\Omega_n(E_{ex}), \qquad \Omega_n(E_{ex})=\frac{g^n(E_{ex}-U_l)^{n-1}}{((n/2)!)^2(n-1)!}. \qquad (3)$$

Here Ω_n is of n-quasi-particle states, a cut-off factor σ of spin J of excited state of compound-nucleus above the energy E_d was taken from back-shifted Fermi-gas model [6], U_l is the energy of l-th Cooper pair breaking threshold, and density $g=6a/\pi^2$ of single-particle states near Fermi-surface was also taken from the [6] model). An influence of the shell inhomogeneities of a single-particle spectrum was taken into account by a definition of the level-density dependence on excitation energy in a value:

$$a(A, E_{ex})=\tilde{a}\,(1+((1-\exp(\gamma\, E_{ex}))\,\delta E/E_{ex})). \qquad (4)$$

An asymptotic value $\tilde{a}=0.114A+0.162A^{2/3}$ and coefficient $\gamma=0.054$ were taken from [5]. A shell correction δE calculated from the data of mass defect in a liquid-drop nuclear model [3] is lightly changed to keep an average distance D_λ between resonances of an investigated nucleus.

Generally accepted now phenomenological coefficient C_{col} of enhancement of collective level density imitates well an increase in vibrational level density and for given excitation energy $E_{\rm ex}$ on the basis of a theoretical description of [5] is written as:

$$C_{col} = A_l \exp\left(\sqrt{(E_{\rm ex} - U_l)/E_\nu} - (E_{\rm ex} - U_l)/E_\mu\right) + \beta. \qquad (5)$$

Here A_l are fitted independently parameters of densities of vibrational levels above the breaking point of each l-th Cooper pair, E_μ is a change in the nuclear entropy, E_ν is a change of quasi-particles excitation energies, parameter $\beta \geq 1$ can differ from 1 for deformed nuclei.

The smooth parts of the energy dependences of the strength functions, $k(E1,E_\gamma)$ and $k(M1,E_\gamma)$, of dipole electrical and magnet γ-transitions are expressed similarly as in [7]:

$$k(E1,E_\gamma) = w_E \frac{1}{3\pi^2\hbar^2c^2A^{2/3}} \frac{\sigma_{GE}\Gamma_{GE}^2(E_\gamma^2 + \kappa_E 4\pi^2 T_E^2)}{(E_\gamma^2 - E_{GE}^2)^2 + E_{GE}^2\Gamma_{GE}^2},$$

$$k(M1,E_\gamma) = w_M \frac{1}{3\pi^2\hbar^2c^2A^{2/3}} \frac{\sigma_{GM}\Gamma_{GM}^2(E_\gamma^2 + \kappa_M 4\pi^2 T_M^2)}{(E_\gamma^2 - E_{GM}^2)^2 + E_{GM}^2\Gamma_{GM}^2}, \qquad (6)$$

where parameters $T_{\rm E}$ (or $T_{\rm M}$) are varied thermodynamic parameters, and $w_{\rm E}$ (or $w_{\rm M}$) and $\kappa_{\rm E}$ (or $\kappa_{\rm M}$) are added parameters of weight and of a change of derivatives of the strength function, correspondingly (indices E and M refer to E1- or M1-transitions). In equations (6) $E_{\rm G}$, $\Gamma_{\rm G}$ and $\sigma_{\rm G}$ are location of the center of giant dipole resonance, its width and cross section in maximum.

If $E_1 \approx B_{\rm n}$ ($B_{\rm n}$ is the nucleon binding energy), fitted ratios Γ_{M1}/Γ_{E1} of E1- and M1-strength functions are normalized to known experimental values, and their sum Γ_λ is normalized to the full radiation width of the resonance.

5. Analysis of the data from ^{238}U(n,2γ)^{239}U reaction

All multistep γ-cascade spectra measured in [1] were analyzed using DICEBOX-code [8]. DICEBOX algorithm, taking into account the levels from ENSDF-file up to critical energy (830 keV for ^{239}U), above this energy generates levels according to increase in of their number in strict correspondence to statistical theory of a nucleus. Probabilities of individual transitions between each pair of levels are simulated using partial-widths formulae, which includes both the level density and a random number taken from a normal Porter-Thomas distribution. As it is mentioned by the experimenters themselves, in their analysis an extremely large number of different artificial “nuclear realizations” appeared due to Porter-Thomas fluctuations. In those calculations several forward-modelling approaches for the E1 and M1 photon strength functions as well as for the nuclear level density were used. However, the practical-applied Strutinsky model of the nuclear-level density was absent among the most available models used by the experimenters of [1]. By the way, modern theory about dynamics of intra-nuclear processes at the excitation-energy increase (see [9], for example) points to existence of different wave-function structure of the excited levels which excludes smoothness of the energy dependences both the level density and radiative strength functions. It is reasonable also to note that testing of the different types of the strength functions makes sense only with simultaneous testing of the models of the nuclear level density.

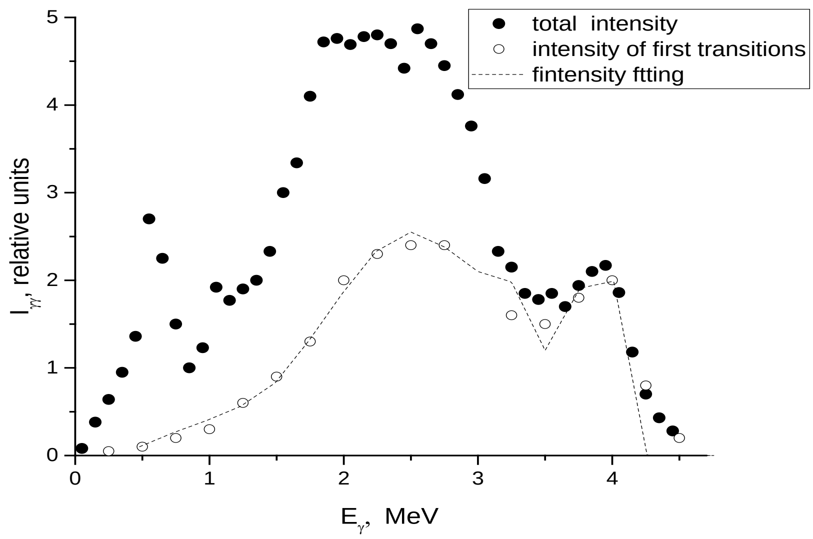

Fig.1. The dependence of intensities of the two-step γ-cascades (relative to their total area) on γ-quanta energies at the decay of compound-state in ^{239}U nucleus (36 eV): close points are the total $I_{\gamma\gamma}(E_{\gamma})$-intensities of the two-step cascades; open points are $I_{\gamma\gamma}(E_1)$-intensities of first transitions only; dashed line is the best fit of $I_{\gamma\gamma}(E_1)$-intensity in a framework of the Dubna empirical method.

Unfortunately, the experimental spectrum of the intensities of the two-step γ-cascades in ^{239}U nucleus is shown in [1] by plotted points only. Nevertheless, we separated primary γ-transitions spectrum (open points in Fig.1), as it is necessary for our subsequent analysis. We used a valid assumption that, as a rule, primary transition in the two-step γ-cascade has more energy than secondary one, and removed secondary transitions from the total spectrum, taking into account mirror-symmetry of energy distributions of primary and secondary transitions relative to a central point of the $I_{\gamma\gamma}(E_{\gamma})$-intensity spectrum (at a half of total energy of the cascades). Experimental points of the total-intensity of the two-step γ-cascades are shown in Fig.1 as close ones.

In spite of impossibility to get a shape of $I_{\gamma\gamma}(E_1)$-distribution accurately from examinee experimental data, at least, a qualitative evaluation of the energy dependences of the level density and strength functions for ^{239}U nucleus has been accomplished.

The authors of [1] have explained an unsatisfactory description of measured $I_{\gamma\gamma}(E_{\gamma})$-intensities by impossibility of taking into account of all parameters of the γ-decay in the used models. We agree with their reasonable assertion completely, but our calculations showed that experimental spectrum of $I_{\gamma\gamma}(E_1)$-intensities cannot be described successfully by any smooth dependence of the nuclear level density on excitation energy.

If a nucleus is imagined as a system of non-interactive nucleons, an assumption about consecutive breaks of Cooper pairs of nucleons in decayed nucleus at an increase in its excitation energy is soundly enough. The step-wise dependence of the level density of nucleus on its excitation energy can be well founded by existence of energy gaps in the spectrum of excited nucleus at discontinuity of a number of pair of excited nucleons.

The dependence of the level density of ^{239}U nucleus on its excitation energy (see Fig.2) obtained from the best $I_{\gamma\gamma}(E_1)$-intensity fits in a framework of the Dubna empirical method, have an evident step-wise behavior. Approximation of the experimental data using a smooth $\rho(E_{\mathrm{ex}})$ function postulated by the authors of [1] results in appreciable increase in χ^2.

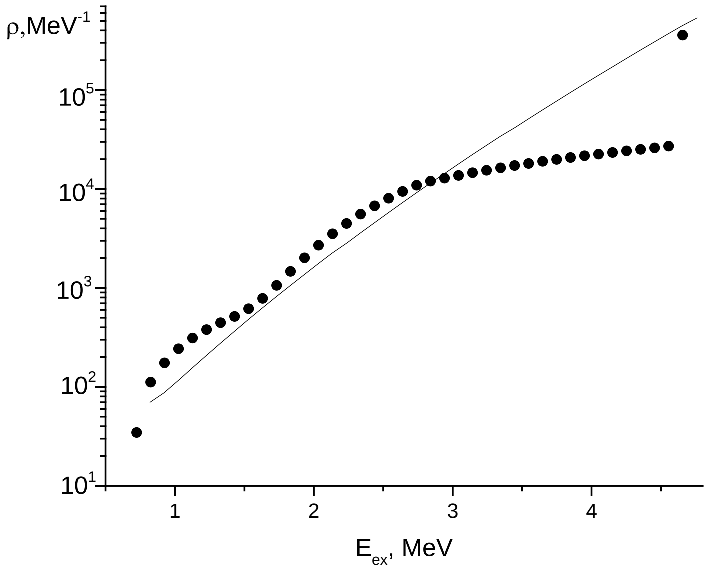

Fig.2. The expected level density of ^{239}U nucleus: close points is our calculation (using fitted parameters from the best $I_{\gamma\gamma}(E_1)$-distribution fit); line is calculation according to the back-shifted Fermi-gas model predictions.

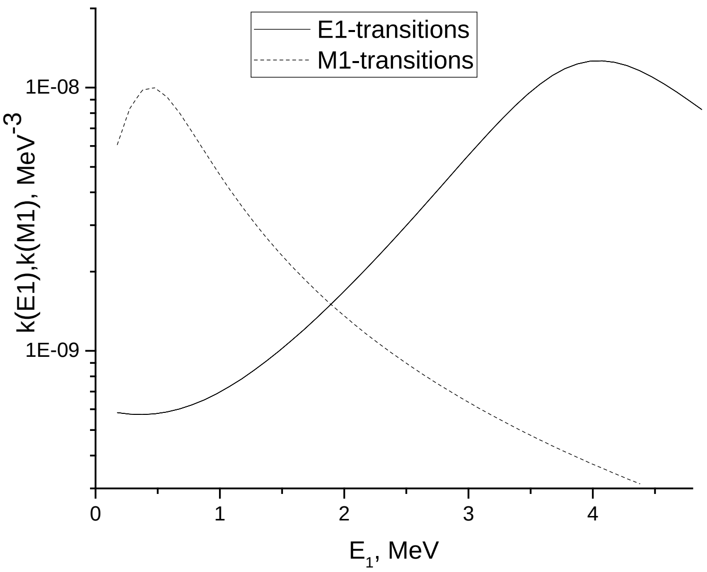

Fig.3. The radiative strength functions for ^{239}U nucleus (our calculations for the $I_{\gamma\gamma}(E_1)$-distribution best fit): solid line is our calculation of k(E1) for E1-transitions in the cascades; dashed line is k(M1)-calculation for M1-transitions.

And a rate of the level-density rise essentially differs from that predicted by the back-shifted Fermi-gas model.

It must be noted that in all nuclei analyzed earlier by the Dubna empirical method (see, for example, [10–12]), the experimental γ-spectra of which were far better suitable for analysis than spectrum from ^{239}U nucleus, the breaks in nuclear level-density dependences on the excitation energy are positioned with a step $\approx 2\Delta_0$ (Δ_0 is the pairing energy of the last nuclear nucleon). Our fittings of $I_{\gamma\gamma}(E_1)$-intensities of the cascades in ^{239}U nucleus also do not exclude for this nucleus a possibility of nucleons' pairing and consecutive breaking of these pairs.

The expected behavior of energy dependences of the radiative strength functions for E1- and M1-transitions in the two-step cascades of ^{239}U nucleus obtained from our calculations on a base of fitted parameters at the $I_{\gamma\gamma}(E_1)$-intensities description are presented in Fig.3.

6. Conclusion

An experimental obtaining of the parameters of γ-decay of any compound-state is exclusively important to understand processes which take place in an excited nucleus.

In spite of insufficient energy-resolving power of scintillator detectors of DANCE calorimeter, in the absence of individual cascade peaks in measured γ-spectrum, we have been successful in description of the experimental spectrum of intensities of primary transitions of the two-step γ-cascades in ^{239}U nucleus with subsequent simultaneous obtaining of its, at least, evaluative $\rho(E_{ex})$ and $\Gamma(E_1)$ functions.

Unfortunately, an absence of experimental individual energy-resolved cascades in total γγ-spectra from 4π-experiment doesn't allow a clarification of the intra-nuclear processes in ^{239}U nucleus. A process of breaking of Cooper pairs is not been experimentally investigated until now. Nevertheless, our calculations visually demonstrate that successful description of the experimental intensities of the two-step γ-cascades in ^{239}U nucleus is possible if only the nuclear-level density has not smooth dependence on the excitation energy.

Only high-transmission spectrometers of gammas and testing of suitable model representations of required strong-correlated nuclear parameters can provide their reliability.

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