

The Covariance Analysis of $^{nat}\text{Sn}(\alpha, x)^{122}\text{Sb}$ Nuclear Reaction Cross Sections

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Introduction

In nuclear medicine, a range of radioactive isotopes are employed for therapy and diagnosis. Several types of radioisotopes are produced by alpha-induced reactions with different types of targets. In this study, we have used ^{nat}Sn as a target material and alpha particle as a projectile. The radioisotopes $^{116, 117, 118, 119, 121, 123}\text{Te}$, $^{117, 120, 122, 124, 126}\text{Sb}$, ^{117}Sn and ^{111}In are produced from $^{nat}\text{Sn}(\alpha, x)$ nuclear reactions. In this work, we have obtained the production cross sections for $^{nat}\text{Sn}(\alpha, x)^{122}\text{Sb}$ nuclear reaction in the incident alpha energy range of about 24 – 40 MeV. This isotope has a broad spectrum of uses, ranging from medical imaging and cancer therapy to industrial radiography. The uncertainty propagation in the measured cross sections was calculated using covariance analysis by taking into account the micro-correlation between various variables such as particle number density, efficiency of the HPGe detector, decay constants and counts etc. [1–3]. The covariance analysis is a statistical technique employed in nuclear physics to quantify and manage uncertainties associated with experimental data, nuclear reaction models and nuclear data evaluations. It is particularly valuable in the study of nuclear reactions, where precise and reliable data are essential for numerous applications. The primary objective of covariance analysis in this context is to provide a comprehensive understanding of the uncertainties in nuclear reaction cross sections and other relevant parameters.

The comparison of measured cross sections for the $^{nat}\text{Sn}(\alpha, x)^{122}\text{Sb}$ nuclear reaction along with previous experimental results from EXFOR and theoretical calculations from the TALYS nuclear code is also presented. The theoretical prediction of the excitation function of nuclear reaction $^{nat}\text{Sn}(\alpha, x)^{122}\text{Sb}$ has been carried out using the TALYS nuclear reaction code. The TALYS is a Fortran-based nuclear reaction model code which is used to calculate different physical observables related to nuclear reactions. In the TALYS, six level density models were used to calculate the theoretical predictions [4–9]. The ldmodel-1, ldmodel-2, and ldmodel-3 are classified as phenomenological level density models, while ldmodel-4, ldmodel-5, and ldmodel-6 are classified as microscopic level density models. We have compared the results of the theoretical calculations with the experimental measurements of the nuclear reaction cross section.

Experiment Details

The experiment was performed at K-130 cyclotron, VECC, Kolkata, India for this study. The stacked foil activation technique followed by the offline gamma-ray spectrometry was used to measure the reaction cross sections for the $^{nat}\text{Sn}(\alpha, x)^{122}\text{Sb}$ nuclear reaction. We irradiated two stacks to determine the excitation function of $^{nat}\text{Sn}(\alpha, x)^{122}\text{Sb}$ nuclear reaction

in the energy range from threshold energy for the nuclear reaction up to 40 MeV. The first stack consisted of five sets of foils arranged in ^{nat}Cu - ^{nat}Sn - ^{nat}Al order. We irradiated this stack with a 40 MeV alpha-particle beam for approximately 5 hours. The second stack consisted of three sets of foils arranged in ^{nat}Cu - ^{nat}Sn - ^{nat}Al order. We irradiated this stack with a 28 MeV alpha-particle beam for about 5 hours. Table 1 provides information about beam energy used for both stacks in the present experiment.

Table 1. The details of stacks used in this experiment

Stack	Beam Energy (MeV)	Beam Current (nA)	Irradiation Time (h)
1	40	~150	5
2	28	~150	5

In this experiment, ^{nat}Cu was used as a monitor to measure the intensity of the alpha-particle beam, ^{nat}Sn acted as the target foil for the nuclear reaction $^{nat}\text{Sn}(\alpha,x)^{122}\text{Sb}$, and ^{nat}Al acted as an energy degrader and catcher foil. The thickness of the ^{nat}Cu , ^{nat}Sn , and ^{nat}Al foils was 9 mg/cm², 11 mg/cm², and 6.75 mg/cm², respectively. The arrangement of foils is shown in Figure 1. The activity of the irradiated samples was measured by using the HPGe detector. In the previous articles [10-12], we discussed in detail the calibration of the HPGe detector's efficiency as well as the coincidence summing effect.

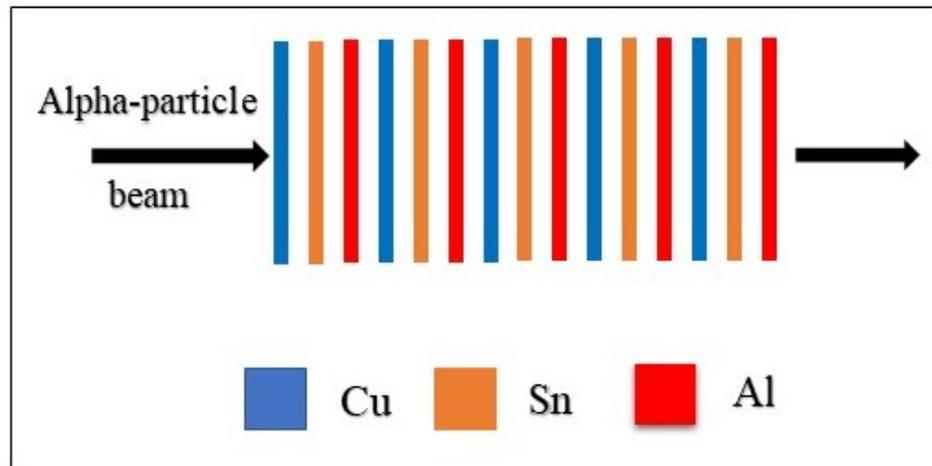


Figure 1. The foil arrangement used in the present experiment.

Data Analysis

The nuclear reaction cross-sections were calculated using the following standard activation formula

$$\sigma = \frac{C_{\gamma}\lambda}{\varepsilon_d I_{\gamma} N_t \Phi (1 - e^{-\lambda t_b}) e^{-\lambda t_c} (1 - e^{-\lambda t_m})} \quad (1)$$

In the above equation, σ is the nuclear reaction cross sections, C_{γ} is the peak area counts for the irradiated sample foils, λ is the decay constant for the nuclear reaction, N_t is the particle

density in the target material, ϕ is the incident particle flux per unit time (sec^{-1}), I_γ represents the gamma-ray intensity of the produced radioisotope, ε_d is the efficiency of the detector. In equation 1, the decay time, irradiation time and counting time for the monitor and sample foils are shown by t_c , t_b and t_m respectively.

Uncertainties in the measured cross-sections were determined by using covariance analysis. The covariance is a mathematical tool that can help to describe the detailed uncertainties with the cross-correlation between different measured quantities. The covariance matrix of cross-section I_σ can be written as

$$I_\sigma = H_x C_x H_x^T \quad (2)$$

Here, I_σ covariance matrix with order $m \times m$, C_x covariance matrix of different attributes with order $n \times n$. The H_x is sensitivity matrix which is given in the following equation

$$H_x^{ij} = \frac{\partial \sigma_i}{\partial x_j}, \quad (i = 1, 2, 3, \dots, m; j = 1, 2, 3, \dots, n) \quad (3)$$

Results and Conclusions

In this section, the measured cross sections, correlation matrices and their uncertainties of nuclear reaction $^{nat}\text{Sn}(\alpha, x)^{122}\text{Sb}$ in the energy range 24 – 40 MeV are presented. To investigate the excitation function of the $^{nat}\text{Sn}(\alpha, x)^{122}\text{Sb}$ nuclear reaction, we measured the 564.24 keV γ -ray with an intensity of 70.68 %, which is emitted during the decay of ^{122}Sb . The radionuclide ^{122}Sb has a half-life of 2.72 days. In Figure 2, the measured excitation function for the $^{nat}\text{Sn}(\alpha, x)^{122}\text{Sb}$ nuclear reaction is presented. This figure also includes available experimental data from EXFOR and theoretical results from TALYS for comparison. From this figure, it is observed that our measured reaction cross sections for the $^{nat}\text{Sn}(\alpha, x)^{122}\text{Sb}$ nuclear reaction are higher than those reported in previous study by Rebeles et al. [13]. The measured cross section for the $^{nat}\text{Sn}(\alpha, x)^{122}\text{Sb}$ nuclear reaction aligns with the trend observed in the theoretical results of Idmodel-6. In Table 2, the measured cross sections for the $^{nat}\text{Sn}(\alpha, x)^{122}\text{Sb}$ nuclear reaction, along with their associated uncertainties and correlation matrix are given.

Table 2. The measured cross sections for the $^{nat}\text{Sn}(\alpha, x)^{122}\text{Sb}$ nuclear reaction, along with their associated uncertainties and correlation matrix

E_α (MeV)	Cross section (mb)	Correlation matrix						
25.07 ± 1.88	4.78 ± 0.29	1						
26.09 ± 0.90	2362 ± 0.33	0.023	1					
28.84 ± 1.73	17.84 ± 0.96	0.054	0.025	1				
32.29 ± 1.58	32.87 ± 1.76	0.055	0.056	0.061	1			
35.50 ± 1.33	39.99 ± 2.12	0.055	0.026	0.062	0.062	1		
38.52 ± 0.92	35.90 ± 1.90	0.055	0.026	0.062	0.062	0.062	1	

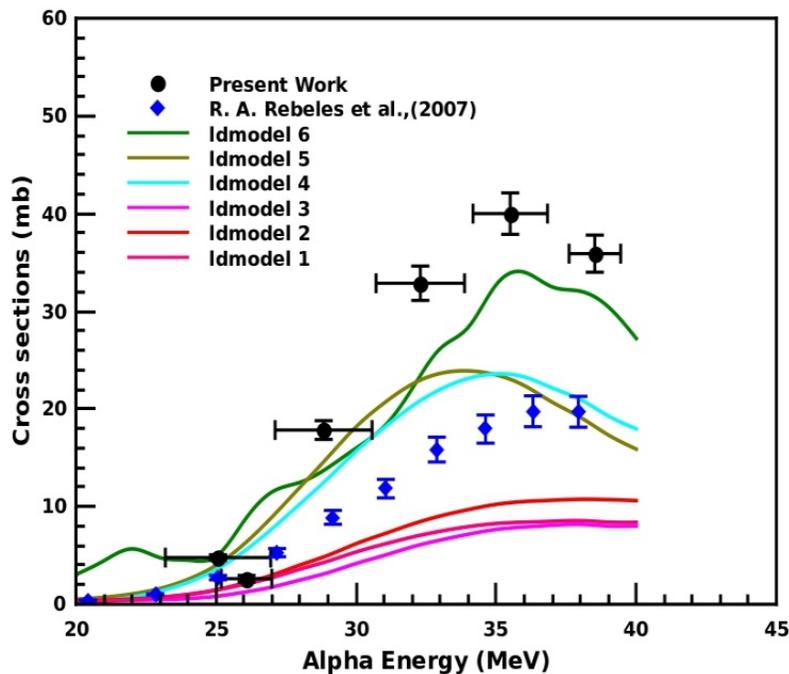


Figure 2. The measured cross sections for the $^{nat}\text{Sn}(\alpha,x)^{122}\text{Sb}$ nuclear reaction along with previous experimental results from EXFOR and theoretical calculations from the TALYS nuclear code.

Acknowledgement: Mahesh Choudhary is grateful for financial support in the form of Senior Research Fellowship from the Council of Scientific and Industrial Research (CSIR), Government of India, (File No 09/013(882)/2019-EMR-I) The IUAC-UGC, Government of India (Sanction No. IUAC/XIII.7/UFR-71353) and Institutions of Eminence (IoE) BHU [Grant No. 6031] are also gratefully acknowledged by one of the authors A. Kumar. We would also like to express our gratitude to VECC's Cyclotron (K-130) staff for providing us with high quality beams throughout the experiment.

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