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Theoretical Works of G.C. Wick in Neutron Physics in the 30-ies

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The name of the Italian theorist Gian-Carlo Wick is well known in the *particle* physics. Nevertheless, in the middle of 30-ies, working in the Enrico Fermi group in Rome, he published several important works in another field – in the *neutron* physics. These works are little known by the present young physicists. Our report will deal only with results on the thermal neutrons albedo problem and his basic theory of the inelastic thermal neutrons scattering by condensed matter. This theory became the first such theory in this field. The history of these works, their physical essence and the relevance to the neutron physics will be discussed.

1. Introduction

The name of the Italian theorist Gian-Carlo Wick is well known in the *particle physics*. Meanwhile in the years from 1932 to 1937, when he was working in the Physics Institute of the University ”La Sapienza” in Rome, he published, in Italian and German languages, several important works in the *neutron physics*. Eighty year later, in his reminiscent paper “Physics and physicists in the Thirties” [1] he wrote (citation from the page 567): “When, in the fall of 1932 I was offered an assistantship by Fermi, I became Fermi’s pupil and I worked in almost daily contact with him for about five years, and I have learned more from him then from anyone else”. And further, on the page 575, G.C. Wick added that his own works there were inspired “in a particular way by the experimental work in Rome that I could watch every day with my own eyes. As an example, the evaluation of the results of work with slow neutrons required the solution of diffusion problems; I found an extremely simple technique to solve some these problems.”

At the very beginning, just after arrival to Rome in 1933, Wick solved the suggested to him by E. Fermi problem of the magnetic moment of the hydrogen *molecule* [2] in the Otto Stern experiments, which was important for the determination of then unknown value of the *proton* magnetic moment. Simultaneously he participated [3], [4] in the physical interpretation of the new phenomenon discovered in 1933 by Emilio Segre and Edoardo Amaldi in the atomic physics – the displacement of high spectral lines of alkali atoms in the atmosphere of a foreign gas (details on the Enrico Fermi explanation of this effect and the role of G.C. Wick in it, are discussed by us in [5]). With publication in 1934 of the E. Fermi theory of beta-decay, G.C. Wick showed that this theory well describes not only the emission of electrons but also the emission of positrons just discovered by Frederic Joliot and Irene Curie; even more, in [6] he theoretically predicted the phenomenon of K-electron capture in radioactive nuclei, which was experimentally demonstrated only in 1937 in Berkeley, USA, by Luis W. Alvarez. As to the Rome, the physicists of the Fermi group, including G.C. Wick, switched to the neutron physics studies which gave the fateful results to the group itself and to the pure and applied nuclear science everywhere. Namely, by spring of 1934, the artificial radioactivity induced by *fast* neutrons, which were produced by a nuclear reaction, was discovered, and then, by autumn of 1934, the effect of multiple amplification of the radioactivity, when it induced by the *thermal* neutrons, was discovered. The details of these events and the paramount role of Enrico Fermi in this discovery have been recently described by F. Guerrra, M. Leone and N. Robotti in [24].

This report will deal only with Wick's results on thermal neutron scattering, in particular on the neutron albedo problem, and with his basic theory of the inelastic thermal neutrons scattering in condensed matter, that became the first such theory in this field. The history of these works, their physical essence and the relevance to the neutron physics will be discussed.

2. Thermal neutrons albedo problem

'Albedo' is Latin word meaning 'whiteness'. It was introduced into optics by Johhann Heinrich Lambert in his 1760 work "Photometria" [7]. In simple words, it means the diffused light reflection coefficient of the surface of a medium. Numerically, for example, for the plane-parallel beam, this is the ratio of the flux reflected in all directions by the unit area of the surface to the incident *flux density* (the flux per unit area of the beam). We will see in this section that for thermal neutrons the situation with albedo is more involved.

On the problem of albedo G.C. Wick and E. Fermi were working both, though separately, using different approaches. As for a media, both use an infinite half-space of a hydrogenous material. Wick applied to neutrons the Boltzmann transport equation of radiation, known to be valid for the 3D geometry, while E. Fermi invented his own intuitive 'one dimensional model' of neutron diffusion. G.C Wick published the result first [8]. E. Fermi published several months later [9] and made a comparison of their results. G.C. Wick resumed to work on the albedo problem later and published in 1943 [10] a more elaborated results with extended discussion.

G.C. Wick's approach was to use the Boltzmann integro-differential equation for probability density function $f(\boldsymbol{r},\boldsymbol{v};t)$ (here and later symbols in bold are vectors) to describe the thermal neutron distribution in a hydrogen media during diffusion. However, he greatly simplified mathematics by using a specific physical model of diffusion, assuming, following E Fermi, that the thermal neutron scattering during neutron collisions with the nuclei is the spherically symmetric. Moreover, he also assumed that all neutrons have the same constant speed corresponding to the thermal energy of about 30 meV. Physically this is valid on average because neutrons are in thermal equilibrium with hydrogen molecules, they diffusion can be taken as a stationary process, therefore the transport equation can be taken as not dependent on time. The x-axis was chosen as the symmetry axis and the x coordinate was the only spatial variable instead of $\boldsymbol{r}$. The only angular variable was the angle θ, actually the cosine $u = cos\theta$, between the direction of the neutron velocity $\boldsymbol{v}$ (with the value normalized to v = 1) and the positive direction of the x-axis. With these assumptions, his Boltzmann transport equation had the form:

$$u\frac{df(x,u)}{dx}+f(x.u)-\frac{\mathrm{N}}{2(\mathrm{N}+1)}\int_{-1}^{+1}f(x,u)du=0. \qquad (1)$$

When neutrons fall on a paraffin medium, occupying half-space from the boundary surface x = 0 to $x \rightarrow \infty$, some of them come back through the boundary surface x=0 after collisions with nuclei in the paraffin, which means that they are reflected. Others are captured instead. The probability density at the boundary is represented by $f(0,u)$ with the positive u-values for ingoing and the negative – for outgoing cases. In general, the quantity $f(x,u)d\tau du$ has the physical meaning of the number of neutrons in the volume dτ having 'velocities' values from u to $u+du$. The *N* is a *physical characteristics* of the medium, namely the ratio of the thermal neutron cross sections for scattering and capture by the Hydrogen nuclei, $N = \sigma_s/\sigma_c$. The physical meaning of *N* is the number of the neutron mean free paths between collisions.

Besides the albedo problem, G.C. Wick also calculated the angular distribution of thermal neutrons inside and outside of paraffin, as well as the radioactivity induced by neutrons in the paraffin media. For these problems the equation (1) contained the additional term – the source of fast neutrons inside paraffin, which we omit. G.C. Wick solved his integro-differential equation by expressing the *integral in Eq. (1)* as the weighted *sum* of several terms: $\sum_i^{2n} p_i\, f(x_i, x_i)$, where the weights p_i and the optimal discrete coordinates x_i should be calculated following the Gauss's formula for the numerical quadratures and mathematical procedures developed by GC Wick. It this way the problem is converted to the solution of the system of several linear differential equations. The desired accuracy of such approximation depends on the number of terms in the sum. Three terms were enough for the albedo problem.

G.C. Wick initially pointed out [7] and later argued in details that two kinds of albedo should be considered for describing the diffusion of thermal neutrons. First one, albedo β, called the *current albedo*, was introduced in the accordance with a classical definition as the ratio of the number of reflected neutrons to the number of the incident neutrons. In terms of the probability density function, the albedo β, is then the ratio of neutron currents:

$$\beta = -\int_{-1}^{0} uf(0,u)du \Big/ \int_{0}^{1} uf(0,u)du. \tag{2}$$

The result of the β calculation depends on the initial function *f(*0*,u)* at the boundary surface for $0<u<1$, that is, on the angular distribution of the incident neutrons. For the *isotropic* flux density, G.C. Wick obtained the 'strong' estimate $\beta=1-4/(\sqrt{(3N)})$ to be compared with the E. Fermi result $\beta=1-2/(\sqrt{(N)})$. However, the neutron angular distribution measurements were not made yet in the 30-ies, no comparison of these expression with the experiment have been made.

The second definition, albedo β_m, was introduced by E. Amaldi and E. Fermi [12]. It was deduced from neutron densities measured by with the 1/v activation detectors and a thin absorber inside the paraffin cube, which was cut in two parts. Wick calls it the *measurable albedo* (the index 'm' in the β_m), which he defines in his theory as:

$$\beta_m = \int_{-1}^{0} f(0,u)du \Big/ \int_{0}^{1} f(0,u)du. \tag{3}$$

His theoretical approximation reported in [10] is $\beta_m=1-2/(\sqrt{(N+1)}+1)$. The Fermi-Amaldi experimental values were $\beta_m=0.82$ and $N=125$. Using this formula and the modern values 50 barn for the scattering and 0.33 barn for the capture cross sections, one obtains $\beta_m=0.85$.

The main Wick's achievement in his 'albedo series' papers is the development of an alternative mathematical method of the solution of the transport integro-differential equation. Subrahmanyan Chandrasekhar, the Nobel prize winner, gave the credit to Wick for the new method and started to use it for solving various problems of the radiative transfer in the theory of stellar atmospheres [11]. The next merit of papers [8],[10] as well as [9] was that they served for development and clarification of the physical understanding of the results obtained by the Fermi group in the experiments on slow neutron diffusion in the paraffin and water. Finally, they were well important for defending the Fermi group position in the discussions on the controversy [12],[13],[14],[15] which have arisen on the albedo problem.

There was the criticism given by authors [14] of the albedo interpretation, and correspondingly, of experimental results in the Rome. E. Fermi, E. Amaldi and G.C. Wick refuted it in the publication [13].

3. Wick's theory of neutron inelastic scattering by crystalline media

To the year 1937, all members of the Fermi group, but E. Amaldi and G.C. Wick, left Rome and Wick himself has got the offer from Bruno Rossi and Gilberto Bernardini to collaborate on the analysis of their cosmic-rays experiments, which he accepted. Nevertheless in 1937 he developed, on his own, the basic principles of the theory of inelastic slow neutrons scattering in the condensed media, and published them in papers [16], [17], [18]. In short, the Wick's initional point of view consisted in suggestion that in crystalline media slow neutrons scatter differently that is usually accepted, e.g., for the scattering in gases, because thermal neutrons should exchange energy and momentum with *phonons*, which are the quanta of crystal lattice vibrations. For such a case, the equations for the conservation of energy and the wave vectors are different than for the scattering by free nuclei. Wick had starting the discussion with the case of the crystal lattice at zero temperature – for the 'cold' crystal. For such a case the slow neutron inelastic scattering in crystals may only *excite* (nothing to absorb!) the acoustic phonons of the frequency ν and the wavelength λ, and the two conservation equations can be written, as the first one,

$$\boldsymbol{p_1} - \boldsymbol{p} = h\boldsymbol{s},$$

here **p** and $\mathbf{p_1}$ are vectors of neutron momenta before (p) /after (p_1) scattering, **s**- phonon wave vector with the value $s = 1/\lambda$), and the second one

$$\frac{\boldsymbol{p}_1^2}{2m} + h\nu = \boldsymbol{p}^2/2m.$$

It follows from these equations that, for inelastic scattering with the phonons creation (neutrons lose energy), the initial neutron momentum should satisfy the condition

$$p > m\nu\lambda + h/2\lambda.$$

Because this condition is usually happens for many crystals in thermal energy region, G.C. Wick wrote about it in more details using the known concept of Brillouin zones and the Debye model for crystals. As to an another inelastic process – the scattering by neutron absorbing the acoustic phonons of crystals – G.C. Wick makes a short comment that this is expected to decrease with the decreasing of the crystal temperature and, at practically attainable temperatures, the crystal becomes transparent for neutrons with the wavelength $\lambda_n > 2d$ (here d is the crystal lattice constant).

Finally Wick calculates the total cross section of slow neutrons inelastic scattering in crystals, basically in the frame of the theory developed by E. Fermi for the neutron scattering by protons bound in molecules, but applying this time the specific of the wave functions in crystals. The final result is given as the cross section containing an integral over the 'eigen-frequencies' of the crystal vibrations (equation (14) in [18]). For the "cold crystal", $\lambda_n < 2d$ and p>~h/2d Wick estimates the partial inelastic cross section with the neutron energy lost (photons emitted) between ε and $\varepsilon + d\varepsilon$ as:

$$d\sigma = \frac{\sigma_0 6d}{MhV^3}\,\varepsilon d\varepsilon,$$

where the σ_0 is the neutron interaction cross section with free nuclei, M is their mass and $V = \nu d$ is a particular velocity of phonons. Also, G.C. Wick deduced an 'approximate' expression for inelastic neutron scattering cross section with the neutron energy gain (absorption of phonons) for the case of the crystal at low temperature and $\lambda_n < 2d$, as:

$$\sigma = \sigma_0 \frac{3h^3}{16\pi M p^2 V d^3} T^3 \theta^{-3}.$$

It should be note, that Wick distinguishes two possible parts of neutron scattering cross section: *coherent* (the interfering part) and *incoherent* (noninterfering part).
The expressions above are for fully incoherent cross sections.

We would like to conclude this Section by quotation from E. Amaldi's fundamental review [19] of the history of nuclear and neutron physics in 30ies: the Wick's papers "paved the way for a very important and presently widely used technique for measuring the dispersion relations of the phonons in crystals and providing direct information on the dynamics of solid state substances".

4. Further developments in neutron spectroscopy of condensed matter

The first theorist after Wick, who steps on the way of the theory of neutron solid state physics, was Isaak Pomeranchuk in Russia [20]. He examined, in more details than Wick, the influence of low temperature upon scattering of slow neutrons by crystals. Soon, following their ideas, many theoreticians started to develop new approaches to the description of slow neutrons inelastic scattering in the condensed matter. The full enough reference list of their works up to the 50-ies can be found in [21]. However, the really new epoch in this field started with publication by George Placzek and Leon Van Hove of their papers [21],[22] on the crystal dynamics, the dispersion relations and correlation functions, in which it was shown how they can be studied theoretically and experimentally using slow neutrons. Also, about that time the intense neutron beams became available at the nuclear research reactors and began to be used as a very powerful neutron sources for neutron spectroscopy of the condensed matter. Though, for the pioneering contributions to the developing of neutron scattering techniques for studies of condensed matter, Bertram N. Brockhouse [23] and Clifford G. Shull obtained Nobel Prize in 1994 only.

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