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Description of Superdeformed Bands in A ~ 190 Mass Region in Framework of Suggested Three Parameters Nuclear Collective Model

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Because of the importance of moments of inertia in describing and exhibiting the nuclear properties of the collective motion inside the nucleus, a theoretical nuclear model depends on an effective moment of inertia as a function of nuclear spin is introduced for the first time to describe and analyze fourteen superdeformed rotational bands SDRB's of Mercury and Thallium nuclei in the A∼190 mass region. For each band the band-head spins and the model parameters are extracted by fitting the calculated transition energies with experimental ones in order to minimize the root mean square deviations between them. The calculated transition energies are in good agreement with the experimental and other theoretical model results reported in the literature. The values of the adopted model parameters are used to calculate rotational frequencies $\hbar\omega$, kinematic $J^{(1)}$ and dynamic $J^{(2)}$ moments of inertia, and the variation of $J^{(1)}$ and $J^{(2)}$ against $\hbar\omega$ are examined. The ΔI=1 energy staggering presented in the signature partner pairs of the studied SDRB's in odd-mass superdeformed Hg and Tl nuclei are investigated and parameterized by proposing a staggering parameters depends on the two-way cross talk ΔI=1 dipole transition linking the two signature partners. The presence of ΔI=2 energy staggering in six SDRB's of the even-even nucleus ^{194}Hg and odd-odd nucleus ^{194}Tl have been investigated by calculating a staggering parameter which represent the finite difference approximation to the fourth order derivative of the transition energies at a given spin.

1. Introduction

Superdeformation is one of the most interesting examples of collective phenomena in atomic nuclei. Recently with the high efficiency of large detector arrays such Gamma-sphere and Euroball it is possible to identify superdeformed rotational bands (SDRB's) in several mass A∼130,150,190,80,60,70 and 90 regions [1]. The A∼190 mass region is of special interest, since SD states were observed down to quite low spin and also show smooth rise in the dynamical moments of inertia as rotational frequency increase, which is associated [2,3] with the successive gradual alignments of a pair of nucleons occupying specific high N-intruder orbital's in the presence of pairing correlations.

For the SD bands, γ-ray transition energies are the only quantity detected, the exact excitation energies, spins and parities of the levels of SD bands are not known, because it is difficult to observe the link between the SD band and the normally deformed states with known spins. Several theoretical approaches to predict the spins of SD bands were suggested [4–11]. Some SDRB's show unexpected staggering effects in the γ-ray transition energies. At high rotational frequencies ΔI=2 staggering was observed [12, 13]. A SD band is perturbed and two sequences emerge with an energy splitting ranging from some hundred eV to few KeV. The two sequences have spin values $I+4n$ and $I+4n+2$ with n=0,1,2,3,..., respectively. The ΔI=2 staggering effect was interpreted in a variety of ways [14–17]. Recently the ΔI=2 staggering has been shown in the ground bands of normally deformed [ND], nuclei like thorium nuclei [18].

The ΔI=1 staggering is familiar for a long time in ND odd-A nuclei [19,20]. Most of SD bands observed in odd-A nuclei in the mass region A∼190 are signature partner pairs, each pair show a large amplitude ΔI=1 staggering [21–25] and also the band-head moments of inertia of each pair are almost identical. The ΔI=1 staggering has attracted much attention and interest, and has thus become one of most frequently detected subjects. In this paper, we suggested a collective rotational model depends mainly on the moments of inertia as a function of nuclear spin (an effective moments of inertia model). For SDRB's of signature partners of Hg and Tl nuclei, the model is used to assign the level spins, analyze the behavior of kinematic and dynamic moments of inertia and to investigate the ΔI=1 staggering phenomenon using two energy staggering parameters.

2. Outline of the Proposed Model

For axially symmetric deformed nuclei, the low spin states are generally interpreted on the basis of the adiabatic approximation, which assumes that the rotational frequency is small compared with that characterizing the intrinsic structure such that the rotation doesn't strongly perturb the intrinsic motion. For nuclei with a static deformation the rotational energy spectrum in the strong coupling limit corresponds to that for a rigid rotor and follows the Bohr formula

$$E(I) = \frac{\hbar^2}{2J} I(I+1), \tag{1}$$

where J is the moment of inertia. The non-adiabatic corrections, evidenced through the deviation from the $I(I$+1) dependence are usually ascribed for K=0 rotational bands to phenomenon like the rotation vibration interaction, centrifugal stretching, Coriolis anti-pairing, higher order crossing effect etc, and may be included phenomenologically by writing the rotational energy transition as a power series expansion in $I(I$+1) . However, for K≠0 bands, the non-adiabatic effects due to Coriolis interaction term in the Hamiltonian should also be taken into account. In general it introduces another series whose successive individual terms alternate in sign with the level and including decoupling terms. An extension of the rotational formula (1) was chosen in order to take into account the nuclear softness, which is a measure for the relative initial variation of the moment of inertia J with respect to the nuclear spin I.

We will parameterize the level energies of SDRB's by using a new theoretical model depends on an effective moment of inertia J_{eff} as a function of nuclear spin I, the model reads

$$E(I) = \frac{\hbar^2}{2J_{eff}(I)} I(I+1), \tag{2}$$

where

$$J_{eff}(I) = J_o\,[1-Y(I)] \tag{3}$$

with

$$Y(I) = \beta[I(I+1)] + \gamma[I(I+1)]^2 + \delta[I(I+1)]^3. \tag{4}$$

The new model contains four parameters: J_o, β, γ, δ.

For SD bands, γ-ray transition energies $E_\gamma(I)$ are the only quantity detected. The $E_\gamma(I)$ between levels differing by two units of angular momentum is:

$$E_\gamma(I) = E(I) - E(I-2) \tag{5}$$

Using the experimental intraband $E2$ transition energies E_γ(I) one can extract the rotational frequency $\hbar\omega$, dynamic $J^{(2)}$ and kinematic $J^{(1)}$ moments of inertia as follows:

$$\hbar\omega = \frac{1}{4}\left[E_\gamma(I+2 \to I) + E_\gamma(I \to I-2)\right] \quad (MeV), \tag{6}$$

$$J^{(2)} = \frac{4}{E_\gamma(I+2 \to I) - E_\gamma(I \to I-2)} \qquad (\hbar^2 MeV^{-1}), \tag{7}$$

$$J^{(1)} = \frac{2I-1}{E_\gamma(I \to I-2)} \qquad (\hbar^2 MeV^{-1}), \tag{8}$$

where $\Delta E_\gamma(I)$ is the difference between two consecutive γ-ray energies in the cascade

$$\Delta E_\gamma(I) = E_\gamma(I+2) - E_\gamma(I) \tag{9}$$

It is noticed that, while $J^{(1)}$ depends on the spin I preposition $J^{(2)}$ does not.

3. Theory of ΔI=1 Staggering in Signature Partners in SDRB's

Signature is a quantum number specifically appearing in a deformed intrinsic system. It is related to the invariance of a system with quadruple deformation under a rotation of 180° around a principle axis. For an odd-A nuclei the signature quantum number can take two different values $\alpha = (-1)^{I-1/2}$. In SDRB's two rotational bands with sequence of levels differing in spin by one unit $1\hbar$ is divided into two branches each consisting of levels differing in spin by two units $2\hbar$ and classified by the signature quantum number $\alpha = \pm 1/2$ respectively. The energetically forward branches formed by these spin I states that satisfy $I - j$ = even, where j is the total angular momentum of corresponding single-particle state. An interesting phenomenon is the ΔI=1 signature splitting in SD bands in a lot of signature partners depending on the transition energies versus spin, a staggering or zigzag pattern can be seen. These irregularities are attributed to the decoupling effect. To exhibit the ΔI=1 staggering in signature partner pairs off odd SD bands we use two staggering parameters:

(i) The three point formula, one must extract the differences between the average transitions $E_\gamma(I+2 \longrightarrow I)$ and $E_\gamma(I)$ energies in one band and the transition $E_\gamma\,(I+1 \longrightarrow I-1)$ energies in the signature partner

$$\Delta^2 E_\gamma(I) = \frac{1}{2}\left[E_\gamma(I+2) + E_\gamma(I) - 2E_\gamma(I+1)\right] \tag{10}$$

$$= (1/2)[E_\gamma(I+2) + 2E_\gamma(I+1)] - E_{\gamma 2}(I) \tag{11}$$

with $E_{\gamma 2}(I) = E(I) - E(I-2)$. The staggering parameter $\Delta^{(2)}E_\gamma(I)$ includes three consecutive transitions energies.

(ii) The EGOS(I) staggering parameter: It is useful to introduce the quality EGOS(I) which represents the Gamma transitional energy over spin

$$\mathrm{EGOS}(I) = E_{\gamma 1}(1) / 2I \qquad (12)$$

with $E_{\gamma 1}(I) = E(I) - E(I-1)$. For pure rotator $E(I) = \mathrm{A}(I)(I+1)$, EGOS($I$)=2A that is a constant.

4. Test of Δ*I*=2 staggering in Even-Even and Odd-Odd Nuclei

Superdeformed band splits into branches, the difference in spins in each branch is $4\hbar$ and the spin difference between the two branches is $2\hbar$, that is the spin values of the two branches are I, I+4, I+8,..and I+2,I+6,I+10,….respectively. The presence of two regular ΔI =4 families in SD band suggested an explanation based on a fourfold state symmetry. To exhibit the an anomalous ΔI =2 staggering effects which appear in transition energies, we extended the proposed model equation (2) by adding to the excitation energy $E(I)$ a linear and quadratic spin depended behavior $\delta E(I)$. The excitation energy becomes

$$E(I) = \frac{\hbar^2}{2J_{eff}(I)} I(I+1) + \delta E(I), \qquad (13)$$

where the additional term reads

$$\delta E(I) = \begin{cases} aI^2 + bI \quad \text{for} \quad I = 8,12,16,\ldots \\ \qquad\qquad\qquad\qquad 9,13,17,\ldots \\ aI^2 + cI + d \quad \text{for} \quad I = 10,14,16,\ldots \\ \qquad\qquad\qquad\qquad 11,15,19,\ldots \end{cases} \qquad (14)$$

Therefore, we can write the incremental $\delta E_\gamma(I)$ between levels differing by two units of angular momentum as:

$$\delta E_\gamma(I) = \begin{cases} (4a + b - c)I + (-4a + 2c - d) \\ (4a - b + c)I + (-4a + 2b + d) \end{cases} \qquad (15)$$

We notice that the additional term to the excitation energy $\delta E(I)$ contains a linear and quadratic spin dependent, while in transition energy, the additional term $\delta E_\gamma(I)$ contains constant and linear spin dependent. To explore more clearly the ΔI=2 energy staggering for each band, the deviation of the transition energies from a smooth reference $\Delta^{(4)}E_\gamma(I)$ is determined by calculating the finite difference approximation of the fourth order derivative of the transition energies $E_\gamma(I)$ as a given spin I. This smooth reference is given by

$$\Delta^4 E_\gamma(I) = \frac{1}{16}[E_\gamma(I+4) - 4E_\gamma(I+2) + 6E_\gamma(I) - 4E_\gamma(I-2) + E_\gamma(I-4)]. \qquad (16)$$

This formula includes five consecutive E_γ values. For pure rotator, one can easily notice that in this case $\Delta^{(4)}E_\gamma(I)$ vanishes. We limit the staggering parameters $S^{(4)}(I)$ as the difference between the experimental $\Delta^{(4)}E_\gamma^{exp}(I)$ and the calculated reference $\Delta^{(4)}E_\gamma^{cal}(I)$ such that

$$S^{(4)}(I) = \Delta^{(4)}E_\gamma^{exp}(I) - \Delta^{(4)}E_\gamma^{ref}(I). \qquad (17)$$

5. Numerical Results and Discussion

In our calculations all γ-ray transition energies $E_{\gamma 2}(I)$ within each band are assumed to be $\Delta(I) = 2(I_o, I_o + 2, I_o + 4, ...)$. Our selected data set includes four signature partner pairs, namely ^{191}Hg(*SD*2, *SD*3), ^{193}Hg(*SD*1, *SD*2), ^{193}Tl(*SD*1, *SD*2), and ^{193}Tl(*SD*1,*SD*2), three bands SD1,SD2,SD3 in the even-even nucleus ^{194}Hg and three bands SD1,SD3,SD5 in the odd-odd nucleus ^{191}Tl. To parameterize the level spins and the model parameters for each SD band, we assumed various values of band-head spin I_o and then the parameters I_o, β, γ, δ in formula (2) can be adjusted by using a computer simulated search program in order to obtain a minimum root mean square deviation of the calculated transition energies $E_{\gamma}^{cal}(I)$

$$\chi = \sqrt{\frac{1}{N}\sum_{i=1}\left[\frac{\left|E_{\gamma}^{cal}(I_i) - E_{\gamma}^{exp}(I_i)\right|}{\delta E_{\gamma}^{exp}(I_i)}\right]^2}$$

from the experimental energies $E_{\gamma}^{\exp}(I)$ where N is the number of data points considered and $\delta E_{\gamma}^{\exp}(I)$ are the experimental errors in $E_{\gamma}(I)$. The fitted procedure is repeated with spin I_o fixed at the nearest half integer. Table 1 gives the band-head spin I_o and the adopted best model parameters. The calculated results of the γ-ray transition energies $E_{\gamma}(I)$ and the comparison with experimental data are illustrated in Figure 1 for the four signature partners pairs. Very good agreement between calculation and experiment is obtained. The calculated transition energies $E_{\gamma}(I)$ with our present proposed effective moment of inertia model are consistent with other theoretical models reported in references [6,17,23]. Using the assigned spin values and the calculated transition energies $E_{\gamma}(I)$ and with the help of equations (6,7,8), the rotational frequencies $\hbar\omega$, the dynamic $J^{(2)}$ and the kinematic $J^{(1)}$ moments of inertia of our selected four signature partner pairs SDRB's are also obtained. The moments of inertia $J^{(1)}$ and $J^{(2)}$ are plotted as a function of rotational frequency $\hbar\omega$ in Figure 2 compared to the experimental ones. It is seen that $J^{(1)}$ is smaller than $J^{(2)}$ and $J^{(2)}$ of most SD bands increases with increasing $\hbar\omega$ due to the alignment of angular momentum of paired nucleons in high - j low **Ω**-intruder orbital's with the collective rotation, and the gradual disappearance of pairing correlation with increasing frequency [26,27].

Table 1. The estimated bandhead spin I_0 and the adopted best model parameters J_o, β, γ, δ obtained from the fitting procedure for the studied signature partners in the A-odd superdeformed Hg and Tl nuclei. The experimental lowest transition energies $E_{\gamma}(I_0+2\rightarrow I_0)$ for each SD band is also given in [1].

SD bands / parameters	^{191}Hg SD2	^{191}Hg SD3	^{193}Hg SD1	^{193}Hg SD2	^{193}Tl SD1	^{193}Tl SD2	^{195}Tl SD1	^{195}Tl SD2
$E_{\gamma}(I_0+2\rightarrow I_0)$(KeV)	252.4	272.0	233.2	254.0	206.6	227.3	146.2	167.5
I_0 (ħ)	10.5	11.5	9.5	10.5	8.5	9.5	5.5	6.5
J_0 ($\hbar^2$MeV^{-1})	94.0749	93.8498	89.1852	92.9347	95.5604	95.5059	98.8059	94.8027
β (10^{-5})	-4.2582	-5.0746	-19.237	-5.6552	-4.6006	-4.4057	-4.4057	-4.6227
γ (10^{-8})	0.6559	0.9384	22.549	1.2401	0.8664	1.1846	1.1846	0.5608
δ (10^{-12})	-1.2630	-2.1692	-330.41	-3.3991	-2.0396	-3.9814	-3.9814	-0.8506

To exhibit the ΔI=1 energy staggering in our studied signature partners, the staggering parameters $\Delta^{(2)}E_\gamma(I)$ equation (11) and EGOS (I) equation (12) have been calculated and plotted as a function of nuclear spin I in Figures 3 and 4. Zigzag pattern are observed. It is seen that the signature partners show large amplitude staggering for the three point formula $\Delta^{(2)}E_\gamma(I)$, for all ranges of spin, while for EGOS(I) only for high spin states.

In the framework of our proposed model with the additional spin dependent behavior equation (13) for the gamma-ray transition energies, the seven parameters occurring in equation (13) are obtained by using the above simulation procedure. Table 2 shows bandhead spin Io and the best fitted values of the model parameters β, γ, δ, a, b, c, d. Figure 5 illustrate the calculated transition energies compared to the experimental ones for the three bands SD1,SD2,SD3 in ^{194}Hg and the three bands SD1,SD3,SD5 in ^{194}Tl. Also, the calculated results of kinematic $J^{(1)}$ and dynamitic $J^{(2)}$ moments of inertia as a function of rotational frequency $\hbar\omega$ and compared with experimental $J^{(2)}$ values as shown in Figure 6. We investigated and exhibit the ΔI=2 energy staggering in the even-even and odd-odd nuclei ^{194}Hg and ^{194}Tl by calculating the staggering parameter $S^{(4)}(I)$ for each band and plotting it versus nuclear spin I in Figure 7. A significant staggering has been observed.

Table 2. The adopted best model parameters and bandhead spin I_0 for the six SD bands in even-even ^{194}Hg and odd-odd ^{194}Tl nuclei which exhibit ΔI=2 energy staggering. The experimental lowest transition energies $E_\gamma(I_0+2\rightarrow I_0)$ for each SD band is also given in [1].

SD bands / parameters	^{194}Hg SD1	SD2	SD3	^{194}Tl SD1	SD3	SD5
$E_\gamma(I_0+2\rightarrow I_0)$(KeV)	211.7	200.79	200.0	268.0	240.5	187.9
I_0 (ħ)	10	8	9	12	10	8
J_0 ($\hbar^2 MeV^{-1}$)	89.4338	94.0650	94.9974	99.7319	95.2703	101.5143
β (10^{-5})	-7.0688	-4.3151	-3.1054	-2.6195	-3.7740	-2.4573
γ (10^{-8})	0.7376	0.3595	0.3857	0.1372	0.2848	1.2076
δ (10^{-12})	-11.1990	-0.3811	-0.7188	-0.0898	-0.2687	-0.0741
a (10^{-4})	1.481(10^{-4})	-2.9 (10^{-3})	-4.25(10^{-3})	1.0187 (10^{-3})	1.75 (10^{-3})	-0.5181(10^{-3})
b (10^{-4})	8.886 (10^{-4})	-1.74(10^{-3})	-2.55(10^{-3})	6.1125(10^{-3})	10.5 (10^{-3})	-3.1090 (10^{-3})
c (10^{-4})	2.9627(10^{-4})	-0.58(10^{-3})	-8.5 (10^{-4})	2.0375(10^{-3})	3.5 (10^{-3})	-1.0363 (10^{-3})
d (10^{-4})	23.702(10^{-4})	-4.64(10^{-3})	-6.8 (10^{-3})	16.3 (10^{-3})	28.0 (10^{-3})	-8.2908 (10^{-3})

5. Conclusion

Four pairs of signature partners in Hg and Tl nuclei are examined in framework of new theoretical model including four parameters and depend on effective moment of inertia as a function of nuclear spin. The spins of the levels and the model parameters for each band are extracted by using fitting search program. The calculated γ-ray transition energies with the present proposed model agree very well with the experimental data. The variations of the kinematic $J^{(1)}$ and dynamic $J^{(2)}$ moments of inertia as a function of the rotational frequency $\hbar\omega$ have been examined and analyzed. We noticed that $J^{(2)}$ is larger than $J^{(1)}$ and both increases with increasing $\hbar\omega$. We investigated the ΔI =1 staggering in signature partners by two staggering parameters. The first represent the difference between the average transitions $I+2\rightarrow I$ and $I\rightarrow I-2$ energies in one band and the transition $I+1\rightarrow I-1$ energies in the other band of the signature partner pair. The second staggering parameter represents the γ energy over spin. The four signature partner pairs exhibit staggering with large amplitudes. The presence of ΔI =2 energy staggering in six SDRB's of ^{194}Hg and ^{194}Tl have been examined in the rotation of additional spin dependent term to the transition energy and introducing a staggering parameter including five consecutive transition energies.

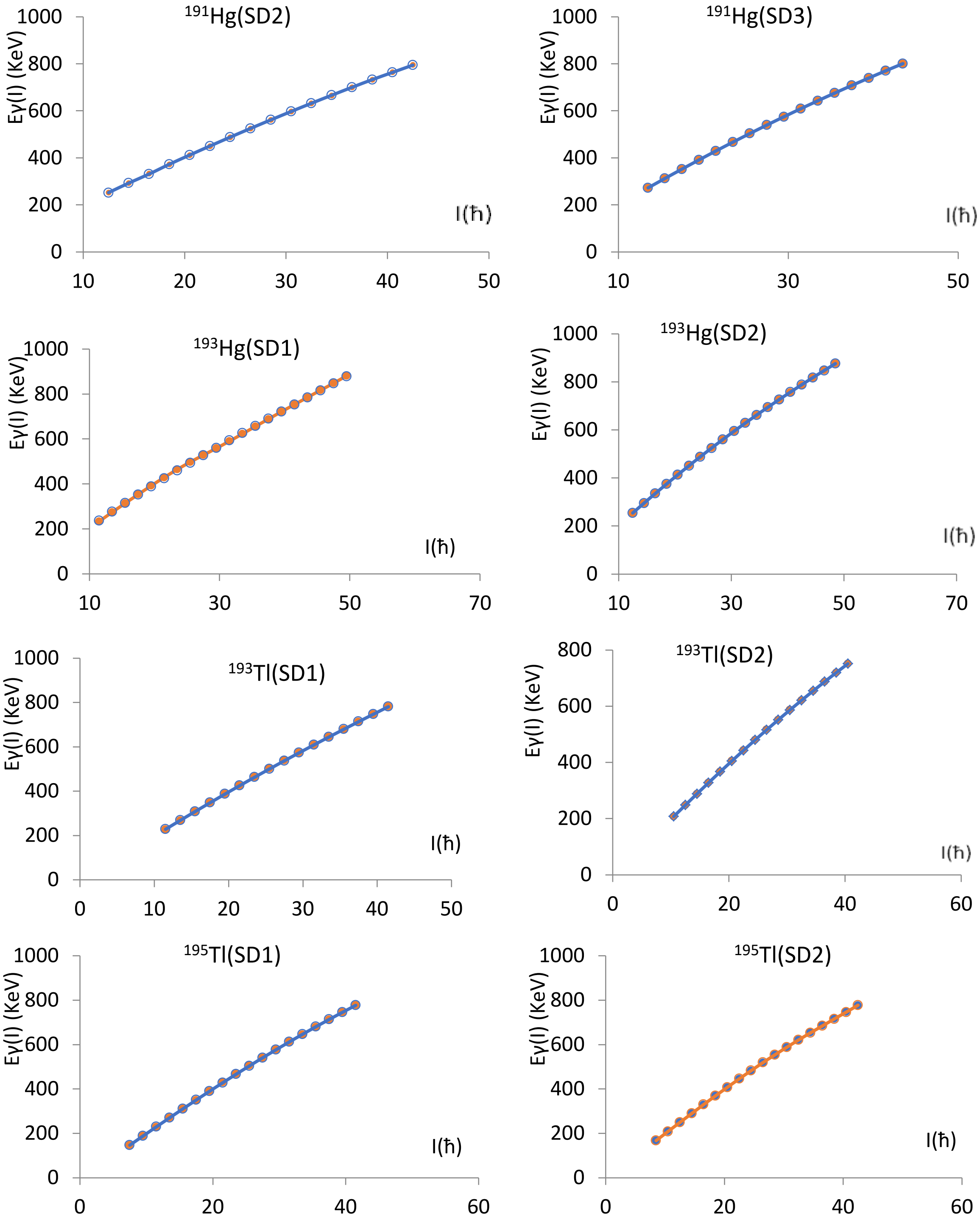

Figure 1. Calculated γ-ray transition energies $E_\gamma(I)$ versus spin I for the 4 pairs of signature partners SD bands ^{191}Hg (SD2,SD3) and ^{193}Hg (SD1,SD2), ^{193}Tl (SD1,SD2) and ^{195}Tl(SD1,SD2) and compared to the experimental ones [1]**.** Solid curves indicate calculated $E_\gamma(I)$ and closed circles indicate experimental values.

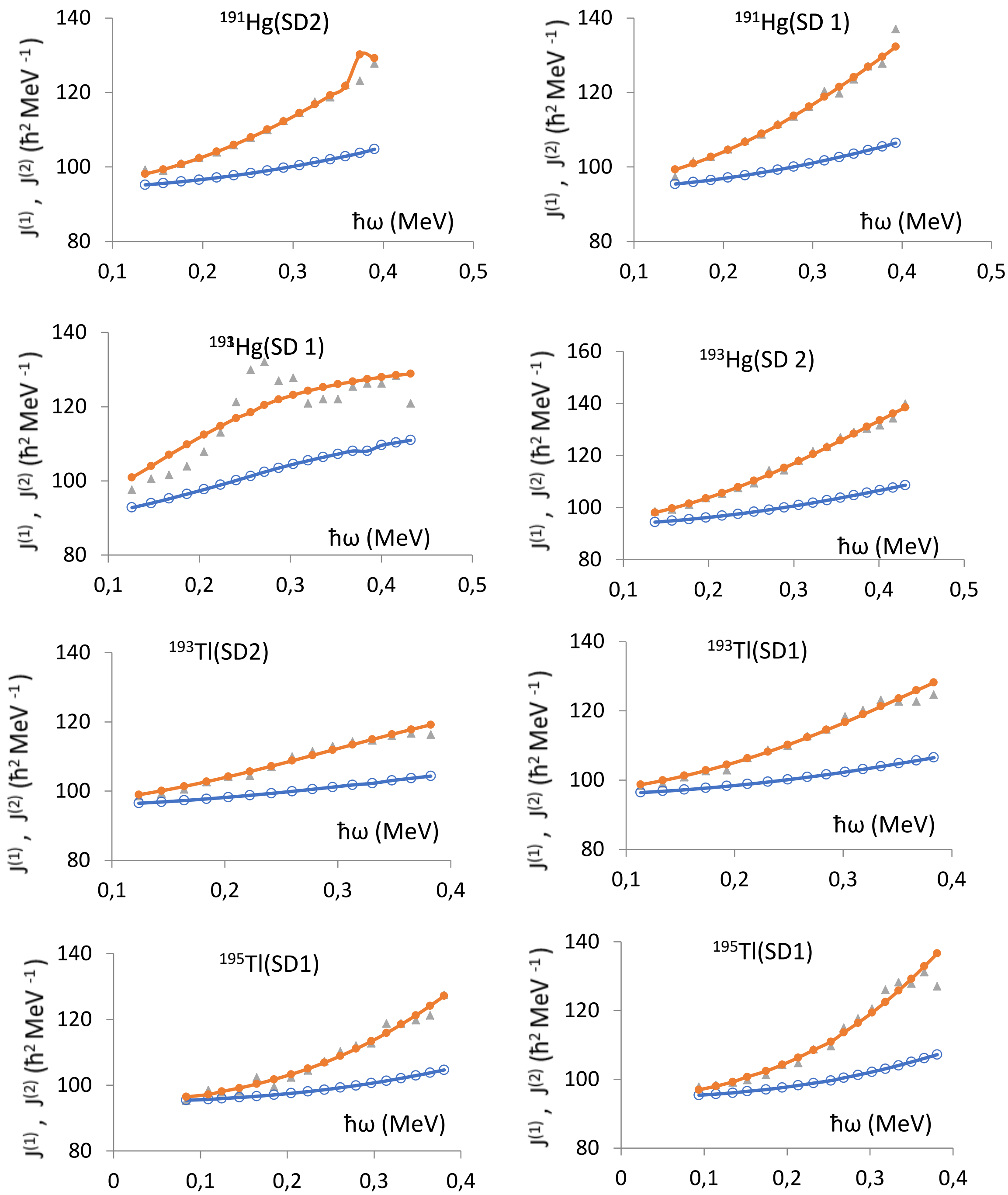

Figure 2. Calculated kinematic $J^{(1)}$(open circles) and dynamic $J^{(2)}$ (closed circles) moments of inertia as a function of rotational frequency ħω for ^{191}Hg, ^{193}Hg, ^{193}Tl and ^{195}Tl nuclei. The experimental data are marked by closed circles with error bars and are taken from [1].

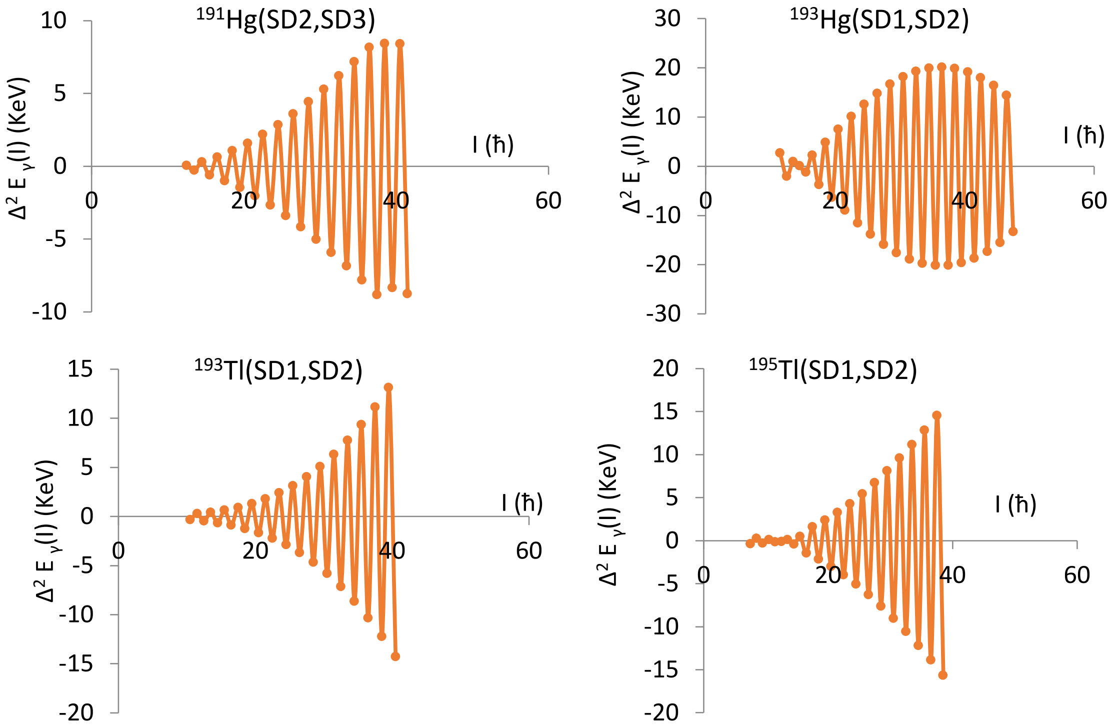

Figure 3. The calculated $\Delta I = 1$ staggering parameter $\Delta^2 E_\gamma(I)$ as a function of nuclear spin I for the signature partner pairs ^{191}Hg(SD2,SD3) ,^{193}Hg (SD1,SD2),^{193}Tl (SD1,SD2) and ^{195}Tl(SD1,SD2).

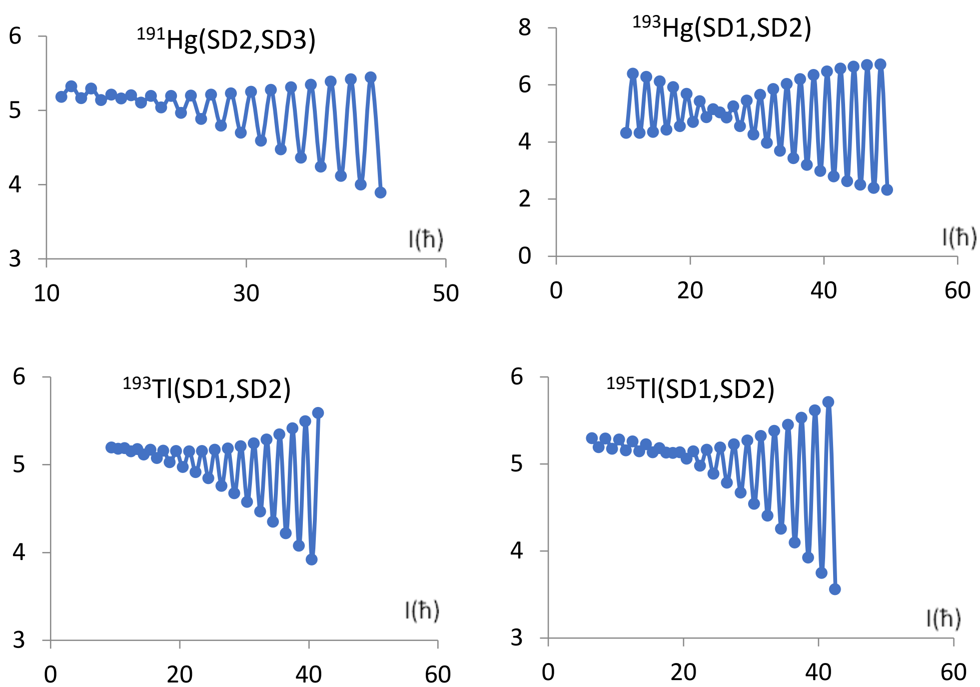

Figure 4. The calculated ΔI=1 staggering index EGOS(I) as a function of nuclear spin I for the signature partner pairs ^{191}Hg(SD2,SD3),^{193}Hg(SD1,SD2),^{193}Tl(SD1,SD2) and ^{195}Tl(SD1,SD2).

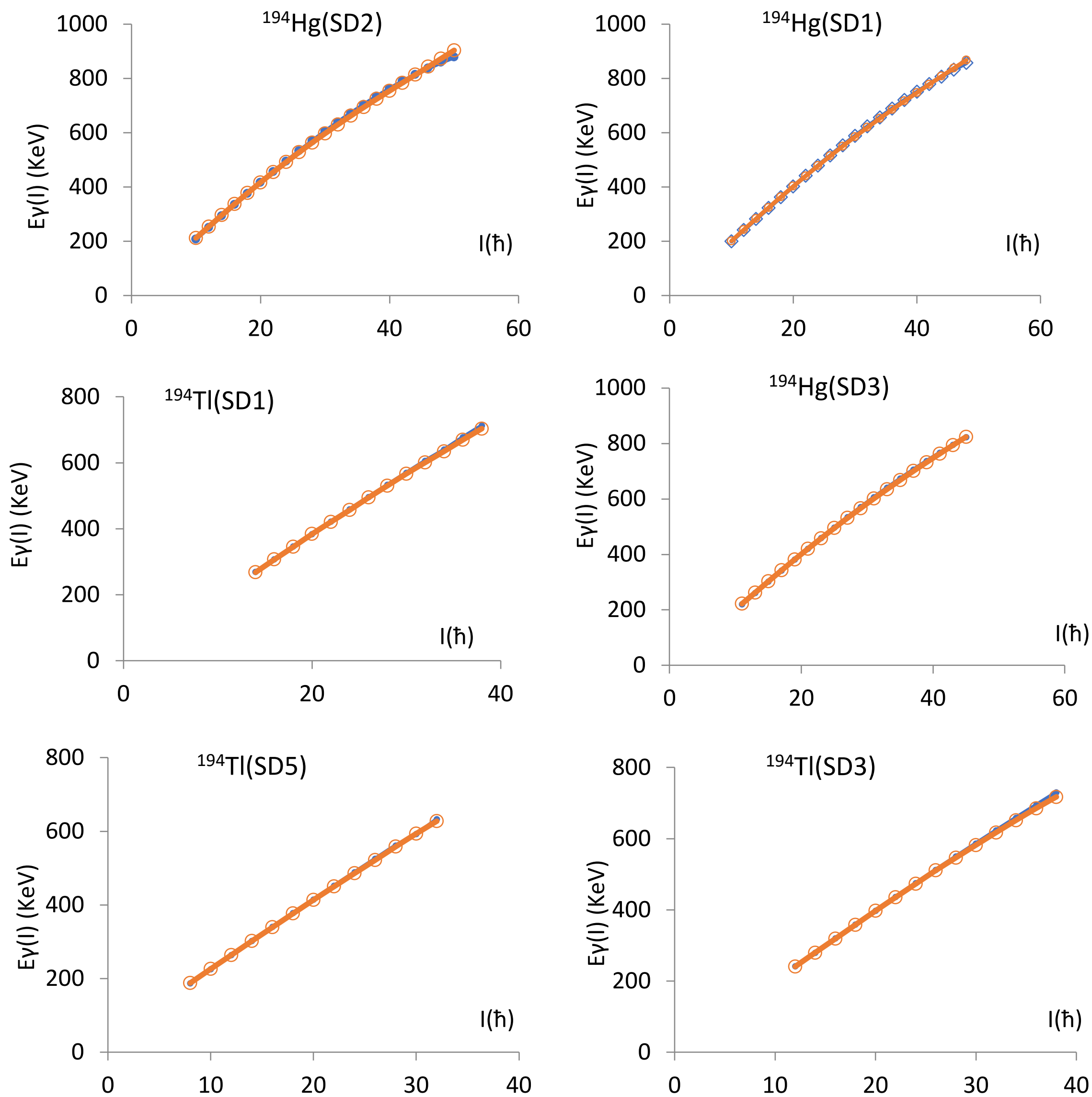

Figure 5. Calculated γ-ray transition energies $E_\gamma(I)$ versus spin I for the three SD bands ^{194}Hg(SD1,SD2,SD3) and ^{194}Tl(SD1,SD3,SD5) and compared to the experimental ones [1]**.** Solid curves indicate calculated $E_\gamma(I)$ and closed circles indicate experimental values**.**

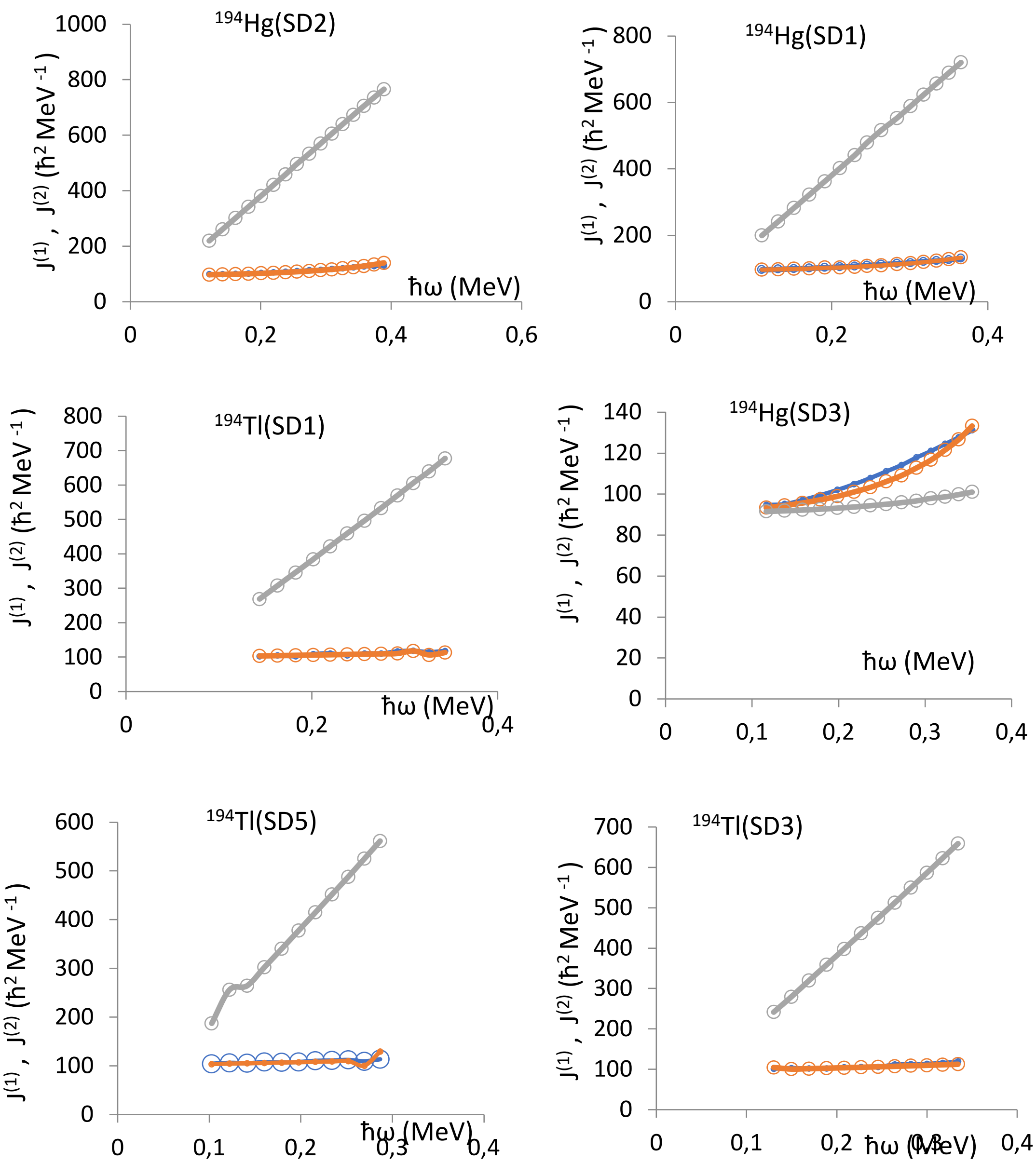

Figure 6. Calculated kinematic $J^{(1)}$(open circles) and dynamic $J^{(2)}$ (closed circles) moments of inertia as a function of rotational frequency ħω for ^{194}Hg(SD1,SD2,SD3) and ^{194}Tl(SD1,SD3,SD5) nuclei. The experimental data are marked by closed circles with error bars and are taken from [1].

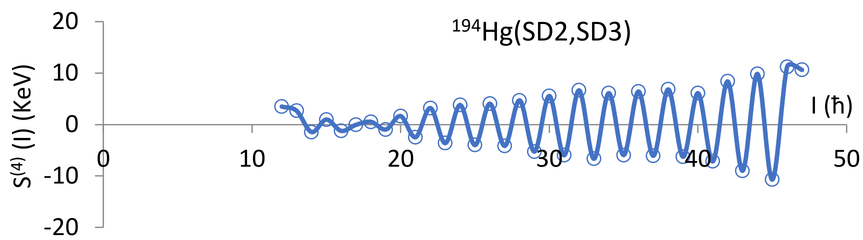

Figure 7. The calculated ΔI = 1 staggering parameter $S^{(4)}(I)$ as a function of nuclear spin *I* for the signature partner pairs ^{194}Hg(SD2,SD3).

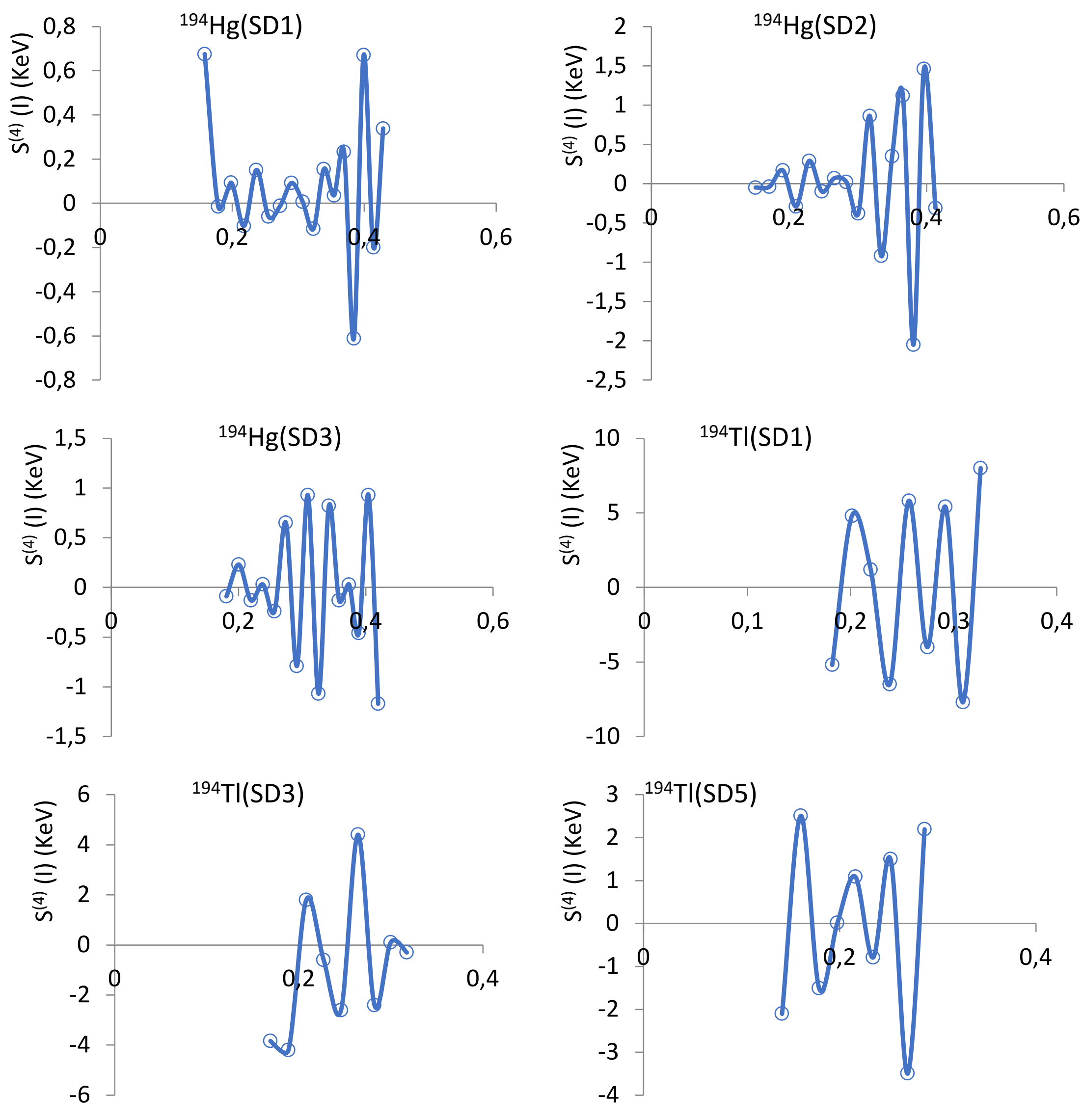

Figure 8. The calculated ΔI =2 staggering parameter $S^{(4)}(I)$ as a function of nuclear spin *I* for the three SD bands 1,2,3 in even-even ^{194}Hg nucleus and the three SD bands 1,3,5 in odd-odd ^{194}Tl nucleus.

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